Monetary Policy in Open Economies with Production Networks[1]

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Abstract

This paper studies monetary policy design in small open economies with cross-border and input-output linkages. We derive the divine coincidence (DC) Phillips curve linking the output gap to a DC index that weights each sector's inflation by sectoral contents in domestic consumption and exports, and of domestic labor. Output gap targeting can be implemented by stabilizing the DC index, which assigns larger weights to sectors that supply more inputs directly or indirectly to domestic output and face larger expenditure-switching effects. Disregarding openness or treating the economy as one sector *overemphasizes* inflation in sectors that export directly or indirectly, and *underemphasizes* inflation in sectors facing large expenditure-switching effects. We quantify our theoretical results using the World Input-Output Database, showing that the Phillips curves are steeper in open than closed economies, and that output gap targeting is near-optimal as in closed economies and outperforms alternative policies ignoring cross-border or input-output linkages.

Keywords: production networks, small open economy, Phillips curve, monetary policy, inflation targeting.
JEL: C67, E52, F41.

1. Introduction

Modern production revolves around intricate input-output (IO) relations within domestic firms and between domestic and foreign firms, and the position and import-export intensity of each domestic firm along the production networks are critical for an economy's response to shocks and the efficacy of stabilizing economic policies. Disruptions to the global supply chain during trade tensions between China and the US since the introduction of the "China Section 301-Tariff Actions" in 2018, the COVID-19 pandemic, and at the outset of the Trump administration's trade actions in 2025 exemplify the primal role of international

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input-output linkages for the changes in output and prices and the stance of monetary policy.[3]

Yet, there is no systematic research focused on the design of monetary policy in open economies with both cross-border and input-output relations —despite two strands of literature providing distinct insights on the issue. On the one hand, in a one-sector small open economy (SOE) model with nominal price rigidities and without input-output relations as per Galí and Monacelli (2005), the optimal monetary policy stabilizes domestic inflation, taking into account the terms of trade. On the other hand, in a multi-sector closed economy with IO linkages as per Rubbo (2023), the monetary policy closing the output gap should target a weighted average of sectoral inflation with the weights proportional to Domar weights (i.e., sectoral sales-to-GDP ratio) to account for the propagation of sectoral distortions along input-output linkages.

In light of these separate findings, it remains unknown what the policy prescription should be for a monetary authority that operates in an open, multi-sector economy with both input-output and cross-border relations between firms. Our paper sheds light on this outstanding issue by revealing the separate roles of input-output and cross-border linkages in the design of monetary policy, and studying the pitfalls of monetary policy that disregards either type of linkages.

We study these issues by developing a small open economy model with production networks between domestic and foreign sectors that are subject to nominal rigidities. Our model combines the one-sector open economy framework in Galí and Monacelli (2005) with the multi-sector, production network framework similar to Rubbo (2023), La'O and Tahbaz-Salehi (2022), and Ghassibe (2021b). In our multi-sector economy with nominal rigidities, inflation in the different sectors relates to sectoral markup wedges that prevent attainment of allocations in the flexible-price equilibrium. The cross-border and input-output linkages further propagate these sectoral markup wedges throughout the economy, resulting in the output gap —defined as the difference between the aggregate output in the sticky-price and the flexible-price equilibria.

We show that the output gap is proportional to a weighted average of sectoral markup wedges. The weight assigned to a sector's markup wedge —which we call the sectoral OG weight— crucially depends on the interplay of cross-border and input-output linkages. The size of the sectoral OG weight is determined by three channels that rely on the distinct roles of the sector for the aggregate output in the network economy: (i) the *Consumer Price Index* (*CPI*), (ii) the *expenditure-switching*, and (iii) the *profit* channels. While the *CPI channel* is also present in closed economies, the *expenditure-switching* and *profit* channels are unique to open economies.[4]

In the *CPI channel*, negative sectoral markup wedges are associated with a lower CPI than in the flexible-price equilibrium. The negative CPI gap links to a positive output gap through a higher real wage that increases domestic labor supply and a higher real exchange rate (i.e., a depreciation of the domestic currency) that increases the income from foreign countries in units of domestic consumption. In the *expenditure-*

[3] See Auray et al. (2024) and Bai et al. (2024, 2025) for discussions on the impacts of trade barriers and Covid-19 on output and monetary policy.

[4] The *expenditure-switching* channel is standard in the international macroeconomic literature. See Engel (2002) for a review.

switching channel, a negative sectoral markup wedge reduces the prices of domestic relative to foreign products and induces a switching of domestic and foreign expenditures from foreign to domestic goods, thereby increasing domestic labor income and relating to a positive output gap. The *profit channel* comprises two sub-channels of export profit and imported factor, respectively. In the export profit sub-channel, negative sectoral markup wedges lower the domestic prices that are the (opportunity) costs of exported goods, thereby increasing export profits and relating to a positive output gap. In the imported factor sub-channel, negative sectoral markup wedges raise domestic sectors' imported-factor costs relative to sales, thus reducing producers' profits and linking to a negative output gap.

The sizes of the three foregoing channels are determined by the different roles of the sector in the open-economy input-output network as a supplier of inputs to both domestic and foreign demand, as well as a customer for domestic labor inputs —which we measure using different sectoral relevance metrics. Because the CPI is the price of aggregate output, the size of the *CPI channel* is determined by the sector's direct and indirect (via the *downstream* sectors) contribution to domestic aggregate output as a supplier of inputs —which we measure using the sectoral *total content in domestic consumption*. The size of the *expenditure-switching channel* —measured by the sectoral *generalized expenditure-switching elasticity*— is proportional to three components: (i) the direct and indirect (via downstream sectors) impacts of sectoral markup wedges on domestic sectors' prices; (ii) the impacts of domestic sectors' prices on the domestic and foreign expenditures on domestic goods —which we measure using the *expenditure-switching elasticity*; and (iii) the sector's direct and indirect (via upstream sectors) use of domestic labor factor —which we measure using the sectoral *total content of domestic labor*.[5] The size of the *export profit sub-channel* is proportional to two components: (i) the direct and indirect (via downstream sectors) impacts of sectoral markup wedges on domestic sectors' prices, and (ii) the share of the sector's export value in domestic output. Finally, the size of the *imported factor sub-channel* is also proportional to two components: (i) the share of the sector's sales in domestic output (i.e., the sectoral *Domar weight*), and (ii) the sector's direct and indirect (via upstream sectors) use of imported intermediate inputs —measured by the sectoral *total content of foreign factor*.

Our sectoral relevance metrics and OG weights encompass those in the closed economy framework with production networks à la Rubbo (2023) and La'O and Tahbaz-Salehi (2022), showing that the OG weight is equal to the total content in domestic consumption and the Domar weight in closed economies that abstract from cross-border linkages, where the expenditure-switching and profit channels are absent.

Using the sectoral OG weights that link sectoral markup wedges to the output gap, we derive the *divine coincidence Phillips curve* (DC Phillips curve), which links the output gap to an aggregate inflation index — the *divine coincidence index* (DC index)— thereby allowing for the simultaneous stabilization of domestic

[5]The (generalized) expenditure-switching elasticity corresponds to the standard expenditure switching effect in the international macroeconomic literature.

inflation and the output gap, and achieving the divine coincidence. The DC index is a weighted average of domestic sectoral inflation, with each sector's relative weight given by its *normalized sectoral OG weight*. This *normalized OG weight* equals the product of a sector's OG weight and its price rigidity —which links positive sectoral inflation to the negative sectoral markup wedge— divided by a normalizer given by the sum of price-rigidity-adjusted sectoral OG weights. While the *normalized OG weights* determine the relative sectoral weights in the DC index, the *sum of price-rigidity-adjusted sectoral OG weights* is inversely related to the slope of the DC Phillips curve and determines its size through the different channels in the OG weight. At the sector level, the sectoral Phillips curves include both output-gap and cost-push driven inflation, similar to those in closed economies with production networks.

We show that the slopes of the DC and the sectoral Phillips curves in open relative to closed economies depend on the balance between two main countervailing forces: (i) in the CPI channel, the domestic sectors' content in domestic consumption is smaller in open than in closed economies, reducing the elasticity of the output gap to domestic inflation and thereby producing a *steeper* slope in open economies; and (ii) the positive expenditure-switching channel, present only in open economies, increases the elasticity of the output gap to domestic inflation and hence produces a *flatter* slope in open economies.

The DC Phillips curve implies that the monetary policy of *output gap targeting* can be implemented by targeting the DC index to zero. Accordingly, we examine how cross-border and input-output linkages determine output gap targeting through their influence on the *normalized OG weights* in the DC index, focusing on the pitfalls of two alternative output gap targeting policies: (i) one that abstracts from *input–output linkages*, as in the one-sector small open economy literature, and (ii) one that abstracts from *cross-border linkages* and targets the closed-economy DC index based on normalized Domar weights —i.e., the Producer Price Index (PPI) targeting policy in closed economies with production networks à la Rubbo (2023).[6]

In the one-sector SOE model —which abstracts from input-output linkages— output gap targeting is the optimal monetary policy, as it simultaneously stabilizes domestic inflation, consistent with the "divine coincidence" result in Galí and Monacelli (2005). However, in the one-sector SOE literature, output gap targeting is implemented by targeting the PPI inflation, which weights sectoral inflation by sectoral sales — proportional to the Domar weights rather than our OG weights. We contribute to this literature by deriving the appropriate sectoral weights in the domestic aggregate inflation index needed to close the output gap, which differ from the PPI weights and account for the interplay of cross-border and input–output linkages.

Output gap targeting that abstracts from cross-border linkages adopts normalized Domar rather than OG weights, and it can overemphasize the relevance of a domestic sector's inflation for the (domestic) output gap. This is because in open economies, a sector's Domar weight is proportional to its *total* sales —which encompasses its direct and indirect (via downstream sectors) contributions to not only *domestic* but also

[6]For simplicity throughout the paper, we refer to the monetary policy that targets the DC index using price-rigidity-adjusted PPI (CPI) weights as the *PPI (CPI) targeting*.

foreign demand. Output gap targeting that abstracts from cross-border linkages can also underestimate the relevance of a domestic sector's inflation by ignoring its effect on domestic-to-foreign prices and, in turn, on the demand for domestic products and labor (i.e., by neglecting the expenditure-switching channel).

Overall, we find that output gap targeting policies both used in the one-sector SOE literature and disregarding cross-border linkages are equivalent to the PPI targeting —which uses normalized Domar instead of OG weights. The extent to which the PPI targeting over- or under-emphasizes the relevance of a sector's inflation depends on the quantitative strength of the aforementioned countervailing sectoral forces for a sector relative to the other sectors. This theoretical possibility motivates our quantitative analysis, which calibrates the model to major economies based on data from the World Input-Output Database (WIOD) to assess the different channels across different economies.

We derive the welfare loss function and the resulting optimal monetary policy in our small open economies with production networks. We show that the welfare loss (up to the second-order approximation) comprises the losses from the output gap misallocation and the within- and across-sector misallocation —similar to those in closed economies à la La'O and Tahbaz-Salehi (2022) and Rubbo (2023)— as well as the cross-border misallocation that is unique to the open economy. The optimal monetary policy —which minimizes the welfare loss subject to the sectoral Phillips curves— cannot simultaneously eliminate the output gap, the within- and across-sector, and the cross-border misallocations, and thus needs to trade off among them. In other words, the "divine coincidence" that holds in one-sector SOEs à la Galí and Monacelli (2005) breaks down in our multi-sector SOEs.

Input-output and cross-border linkages enter the welfare loss function and, therefore, play an important role in optimal monetary policy. In one-sector SOEs *without input–output linkages*, only within-sector and cross-border misallocations remain, and the output gap is proportional to domestic inflation, making welfare loss proportional to the squares of domestic inflation and hence achieving the divine coincidence as in Galí and Monacelli (2005). In multi-sector closed economies *without cross-border linkages*, the cross-border misallocation in the welfare loss function is absent.

To quantify the relevance of the different channels and countervailing forces in our model for the DC Phillips curve and the relative sectoral weights in the DC index, as well as to determine the welfare differences across alternative monetary policies, we calibrate the model to the data from the WIOD comprising 43 countries with 56 major sectors for the year 2014. We show that both the DC and sectoral Phillips curves are steeper in open economies relative to closed economies. The steeper slopes are mainly driven by the smaller domestic sectors' contents in domestic consumption in open than closed economies, which reduces the CPI channel across all sectors and steepens the Phillips curves.

We show that the CPI and expenditure-switching channels explain the bulk of the variation in the normalized OG weights, with the importance of these two channels decreasing and increasing with the openness of the economy, respectively. The percentage difference between the normalized Domar and OG weight —which captures the pitfalls in the output gap targeting that disregards cross-border linkages— is primarily driven by the sector's *export intensity* component —which captures the sector's direct and indirect (via downstream sectors) contribution to exports— and the expenditure-switching component.

We use regression analysis to study the rule-of-thumb combinations of sectoral relevance metrics to approximate the normalized sectoral OG weights. We show that the normalized sectoral OG weights can be closely approximated by a linear combination of *total content in domestic consumption* and *generalized expenditure-switching elasticity*. We also reveal that the normalized Domar-OG differences —capturing the pitfall in the output gap targeting that disregards cross-border linkages, or equivalently, the PPI targeting policy in the one-sector SOE literature— can be closely approximated by a linear combination of *export intensity* and *ratio of generalized expenditure-switching elasticity to the Domar weight*. Therefore, our regression analysis implies that output gap targeting should assign larger weights to inflation in sectors that supply more inputs directly or indirectly (i.e., via the downstream sectors) to domestic consumption, and that face larger expenditure-switching effects. Disregarding cross-border linkages or treating the economy as a one-sector SOE overstates inflation in sectors that export intensively —directly and indirectly— and understates inflation in sectors facing large expenditure-switching effects.

Finally, we compare the welfare under alternative monetary policies, showing that output gap targeting is close to the optimal monetary policy —as in closed economies with production networks à la Rubbo (2023)— and it outperforms three alternatives: (i) the PPI targeting that targets the aggregate inflation index using normalized Domar weights (and therefore abstracts from the cross-border linkages); (ii) the output gap targeting that accounts for cross-border but abstracts from input-output linkages; and (iii) the CPI targeting that targets the aggregate inflation index using normalized CPI weights (and thus abstracts from both cross-border and input-output linkages). For instance, in Mexico, output gap targeting outperforms the PPI targeting and output gap targeting that ignores IO linkages by 67% and 99%, respectively, toward the optimal monetary policy in terms of welfare. In the more open economy of Luxembourg, welfare improvements by output gap targeting are larger at 95% and 99%, respectively. In the relatively closed economy of the US, however, the welfare difference between the output gap and PPI targeting is limited, indicating that international trade plays a minor role in monetary policy design for countries with low openness. Accordingly, our quantitative analysis underscores the importance of accounting for both input–output and cross-border linkages in the design of monetary policy in small open economies with production networks.

Related literature. Our paper is related to four separate strands of literature. First, we relate to literature on the design of monetary policy in closed economies with production networks. Rubbo (2023), La'O and Tahbaz-Salehi (2022), and Xu and Yu (2025) show that in closed economies, output gap targeting is nearly optimal, and that it weights inflation in the different sectors according to the sectoral Domar weights that account for the structure of the domestic production network. La'O and Tahbaz-Salehi (2025) study the optimal fiscal and monetary policies in a closed economy. Compared to the foregoing studies, we show that output gap targeting in open economies is nearly optimal as in closed economies, but it needs to account for the interplay between cross-border and input-output linkages.

Second, our paper relates to literature that investigates the aggregation of sectoral distortions and shocks. Chari et al. (2007) use labor and efficiency wedges to characterize the aggregation of disaggregated shocks and distortions. Acemoglu et al. (2012) show that with input–output linkages, idiosyncratic microeconomic shocks can propagate into aggregate fluctuations. Bigio and La'O (2020) extend that analysis to study a closed economy with production networks; they reveal that the efficiency wedge does not include first-order distortions and that only the labor wedge is critical to first-order economic efficiency. We generalize their results to an open economy with international production networks. Baqaee and Farhi (2024) study distortions in a global economy with interconnected countries and sectors. Elliott and Jackson (2024) examine the propagation of supply chain disruption in international production networks. Compared to their work, we investigate the distortions in SOEs and focus on the design of monetary policy.

Third, our paper relates to literature on the transmission of monetary policy in production networks. Ghassibe (2021a,b) and Afrouzi and Bhattarai (2023) develop an analytical characterization of the transmission mechanism of monetary policy in closed economies with production networks. Nakamura and Steinsson (2010) and Pasten et al. (2020) provide a numerical characterization of the effect of monetary policy on aggregate output and inflation. Silva (2024) explores how the production network alters the propagation of sectoral shocks into the consumer price index in small open economies. Kalemli-Ozcan et al. (2025) develop a New Keynesian open economy model incorporating global production networks and trade distortions to study the interaction between monetary policy and trade. Compared to these works, we focus on the design rather than the transmission of monetary policy in network economies.

Fourth, our paper is linked to the numerous studies on the design of monetary policies in small open economies without production networks. While earlier work focuses on one-sector small open economies (e.g., De Paoli, 2009; Galí and Monacelli, 2005; Soffritti and Zanetti, 2008), more recent work —Matsumura (2022) and Wei and Xie (2020)— explore small open economy models with multiple sectors. Compared to these foregoing studies, we derive closed-form solutions for the output gap targeting and optimal monetary policies and provide a comprehensive analysis of the design of monetary policies in small open economies with fully-fledged cross-border and input-output linkages.

Outline. The remainder of the paper is organized as follows. Section 2 describes our model of a small open economy with production networks. Section 3 studies the sectoral OG weights that link sectoral markup wedges to the output gap. Section 4 derives the Phillips curves and analyzes the output gap targeting policy. Section 5 derives the welfare loss function and optimal monetary policy. Section 6 quantifies the theoretical results using data and compares the welfare under alternative monetary policies. Section 7 concludes the paper.

2. Small open economy with production networks

2.1. Environment

The static, small open economy is populated by a representative household consuming domestic and imported sectoral products and supplying labor in exchange for wage income, a government that levies sector-specific taxes and manages the aggregate demand by controlling the money supply, and producers that operate in $N \in \mathbb{N}_+$ different sectors, indexed by $i \in \{1, 2, \cdots, N\}$.

Each sector i comprises two types of producers: (i) a unit mass of monopolistically competitive firms indexed by $f \in [0, 1]$ that transform labor and intermediate inputs into differentiated goods, and (ii) a unit mass of perfectly competitive firms that pack the differentiated goods of each sector into a domestic sectoral product —which is both used domestically and exported to foreign countries. Each domestic sectoral product has a counterpart foreign sectoral product available for import. Consumption and intermediate inputs comprise domestic and foreign sectoral products.

2.2. Producers

Monopolistically competitive firms. Within each sector i, monopolistically competitive firms use a common constant-returns-to-scale production technology to transform labor and intermediate inputs into differentiated goods. The production technology of each firm f in sector i is

$$Y_{if} = A_i \cdot \left(\frac{L_{if}}{\alpha_i}\right)^{\alpha_i} \prod_{j=1}^{N} \left(\frac{X_{if,j}}{\omega_{i,j}}\right)^{\omega_{i,j}}, \tag{1}$$

where A_i is the sector-specific productivity shock, Y_{if} is the output of firm f in sector i, L_{if} is its labor input, and $X_{if,j}$ is the intermediate input acquired from sector j. Parameter α_i is the share of labor, and $\omega_{i,j}$ is the share of intermediate inputs from sector j. The collection of $\{\omega_{i,j}\}_{i,j}$ characterizes the input-output table. Constant returns-to-scale implies that $\alpha_i + \sum_{j=1}^{N} \omega_{i,j} = 1$.

The openness of the economy is reflected in the composition of $X_{if,j}$, which is aggregated from a domestic sectoral product $X_{Hif,Hj}$ and an imported foreign sectoral product $X_{Hif,Fj}$ according to the following constant-elasticity-of-substitution technology:

$$X_{if,j} = \left(\upsilon_{x,i,j}^{\frac{1}{\theta_j}} X_{Hif,Hj}^{\frac{\theta_j - 1}{\theta_j}} + (1 - \upsilon_{x,i,j})^{\frac{1}{\theta_j}} X_{Hif,Fj}^{\frac{\theta_j - 1}{\theta_j}}\right)^{\frac{\theta_j}{\theta_j - 1}}, \tag{2}$$

where θ_j is the elasticity of substitution between domestic and foreign sectoral products in intermediate input $X_{if,j}$. $\upsilon_{x,i,j}$ is the home bias parameter, which in equilibrium is equal to the steady-state expenditure share of $X_{Hif,Hj}$ in the composite intermediate input $X_{if,j}$.

The total cost of inputs used by the firm is

$$TC_{if} = WL_{if} + \sum_{j=1}^{N}(P_j X_{Hif,Hj} + S \cdot P^*_{IM,Fj} X_{Hif,Fj}), \tag{3}$$

where W is the nominal wage rate, P_j is the domestic sectoral price, $P^*_{IM,Fj}$ is the exogenous sectoral import price denominated in the foreign currency, and S is the nominal exchange rate. Given output Y_{if} and the production technology in equation (1), the firm optimally chooses labor and intermediate inputs to minimize the total cost TC_{if}, which yields the marginal cost of production that equals the average cost due to the constant-return-to-scale technology. Moreover, because all firms f in each sector i share the same production technology and face the same input prices, the marginal cost of production is identical across all firms in sector i, and we denote it by Φ_i.

We model nominal rigidity as a static Calvo-pricing friction where only firms indexed by $f \leq \delta_i \in [0, 1]$ can choose their desired price $P_i^{\#}$ and the remaining firms maintain the price at the steady-state level. We refer to $(1 - \delta_i)/\delta_i$ as the price rigidity of sector i. In each sector i, firms operate in a monopolistically competitive market and receive a sectoral subsidy rate τ_i on sales. Those firms that can adjust their prices set the desired price to maximize profit.

In each sector i, the perfectly competitive and identical sectoral goods packers transform the differentiated goods that the monopolistically competitive firms produce into a sectoral product using a constant-elasticity-of-substitution technology, with the within-sector elasticity of substitution between different firms' products equal to $\varepsilon_i > 1$. The price of the domestic sector i's products —denoted by P_i— is the selling price of its sectoral goods packer. We define the sectoral markup and the desired sectoral markup as $\mu_i \equiv P_i/\Phi_i$ and $\mu_i^{\#} \equiv P_i^{\#}/\Phi_i$, respectively. We further define the *sectoral markup wedge* for domestic sector i as the log deviation of the sectoral markup from the desired markup, *viz.*, $\ln(\mu_i) - \ln(\mu_i^{\#})$. Shown in Appendix A are the expressions for the nominal profit, demand function, and desired prices of the firms, as well as the sectoral product and price index.

2.3. Households

The preference of the representative household is described by the utility function defined over domestic aggregate consumption C and labor supply L:

$$u(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\varphi}}{1+\varphi}, \tag{4}$$

where σ is the coefficient of relative risk aversion, and φ is the inverse of the Frisch elasticity of labor supply. In our static model without investment, domestic aggregate consumption is equivalent to the (domestic) aggregate output; thus, we refer to C as the aggregate output throughout the paper.

The (domestic) aggregate output C combines sectoral consumption $\{C_i\}_i$ that comprises domestic and imported components, C_{Hi} and C_{Fi}, respectively, for each sector i, represented by:[7]

$$C = \prod_{i=1}^{N} \left(\frac{C_i}{\beta_i}\right)^{\beta_i}, \quad \text{where} \quad C_i = \left(v_i^{\frac{1}{\theta_i}} C_{Hi}^{\frac{\theta_i - 1}{\theta_i}} + (1 - v_i)^{\frac{1}{\theta_i}} C_{Fi}^{\frac{\theta_i - 1}{\theta_i}}\right)^{\frac{\theta_i}{\theta_i - 1}}. \tag{5}$$

Vector $\{\beta_i\}_i$ is the set of consumption shares satisfying $\sum_{i=1}^{N} \beta_i = 1$, and v_i is the home bias parameter for the consumption of sectoral products. P_C is denoted as the price index of the aggregate output C —*viz.*, the CPI. The budget constraint of the household is:

$$P_C C = \sum_{i=1}^{N} \left(P_i C_{Hi} + S \cdot P_{IM,i}^{*} C_{Fi}\right) \leq WL + \sum_{i=1}^{N} \int_0^1 \Pi_{if} df + T, \tag{6}$$

where Π_{if} is the profit of firm f in sector i, and T is the lump-sum transfer of tax revenues to the household. To purchase the consumption goods, households demand the following amount of money as the medium of exchange: $M_d = P_C C$. Cost minimization by the household yields the CPI:

$$P_C = \prod_{i=1}^{N} \left(v_i P_i^{1-\theta_i} + (1 - v_i)(S \cdot P_{IM,Fi}^{*})^{1-\theta_i}\right)^{\frac{\beta_i}{1-\theta_i}}. \tag{7}$$

2.4. International trade

In addition to the sales subsidy $\{\tau_i\}_i$, the government also imposes sector-specific export tax $\{\tau_{EX,i}\}_i$ on the products exported to foreign countries. The no-arbitrage condition implies that there is no difference between the prices that producers receive from exporting (i.e., $(1-\tau_{EX,i})P_{EX,i}$) or from selling domestically (i.e., P_i): $(1 - \tau_{EX,i})P_{EX,i} = P_i, \ \forall i \in \{1, 2, \cdots, N\}$.

The export demand for sector i's product is modeled as the reduced-form demand function:[8]

$$Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D_{EX,Fi}^{*}, \tag{8}$$

where $D_{EX,Fi}^{*}$ is the exogenous component of foreign demand, $P_{EX,i}/S$ is the price of the exported domestic sector i goods in units of foreign currency, and the export demand is inversely related to domestic goods' export price, with $\theta_{F,i}$ as the price elasticity of export demand.

Trade is balanced in the static economy, which requires the value of exports to be exactly identical to

[7] As we show in equation (34) of Proposition 2 in Section 4.1, the aggregate consumption gap ($\widehat{C}^{gap}$) relates to sectoral inflation in the sectoral Phillips curves. For consistency with the terminology used in the optimal monetary policy literature, and with a slight abuse of notation, we refer to $\widehat{C}^{gap}$ as the output gap throughout the paper.

[8] In general, the export demand in equation (8) can be written as $Y_{EX,i} = [P_{EX,i}/(S \cdot P_{EX,Fi}^{*})]^{-\theta_{F,i}} D_{EX,Fi}^{*}$, where $P_{EX,Fi}^{*}$ is the exogenous price for foreign-produced sector i's product in foreign markets, and $D_{EX,Fi}^{*}$ is the exogenous foreign demand given the prices. Therefore, $D_{EX,Fi}^{*}$ in equation (8) captures the effects of both $P_{EX,Fi}^{*}$ and $D_{EX,Fi}^{*}$ on export demand.

the value of imports in the whole economy, resulting in the following:[9]

$$\sum_{i=1}^{N} P_{EX,i} Y_{EX,i} = S \sum_{i=1}^{N} P_{IM,Fi}^{*} \left(\sum_{j=1}^{N} \int_{0}^{1} X_{Hjf,Fi} df + C_{Fi} \right). \tag{9}$$

This trade balance condition pins down the endogenous nominal exchange rate S in equilibrium.

2.5. Aggregate states

There are three types of exogenous sector-level states in the economy: productivity $\{A_i\}_i$, foreign demand $\{D_{EX,Fi}^{*}\}_i$, and import price $\{P_{IM,Fi}^{*}\}_i$. The aggregate state $\boldsymbol{\xi}$ collects the realized states:

$$\boldsymbol{\xi} \equiv \left\{ A_i, D_{EX,Fi}^{*}, P_{IM,Fi}^{*} \right\}_{i \in \{1,2,\ldots,N\}} \in \boldsymbol{\Xi} = \mathbb{R}_{\geq 0}^{3N}. \tag{10}$$

2.6. Government: fiscal and monetary policies

The government sets fiscal and monetary policies. Fiscal policy includes a pair of non-contingent sectoral sales and export taxes $\{\tau_i, \tau_{EX,i}\}_i$ that do not respond to changes in exogenous states. The lump-sum transfer T to the households satisfies a fiscal budget balance:

$$T = \sum_{i=1}^{N} \left(\tau_i \int_{0}^{1} P_{if} Y_{if} df + \tau_{EX,i} P_{EX,i} Y_{EX,i} \right). \tag{11}$$

The monetary policy is a one-dimensional state-contingent money supply $M(\boldsymbol{\xi})$ contingent on the aggregate state $\boldsymbol{\xi}$. We investigate the design of this monetary policy, with a particular focus on the monetary policy of output gap targeting that closes the output gap.

2.7. Equilibrium definition

The market clearing conditions for product, labor, and money markets are:

$$Y_i(\boldsymbol{\xi}) = C_{Hi}(\boldsymbol{\xi}) + \sum_{j=1}^{N} \int_{0}^{1} X_{Hjf,Hi}(\boldsymbol{\xi}) df + Y_{EX,i}(\boldsymbol{\xi}), \tag{12}$$

$$L(\boldsymbol{\xi}) = \sum_{i=1}^{N} \int_{0}^{1} L_{if}(\boldsymbol{\xi}) df, \qquad M(\boldsymbol{\xi}) = M_d(\boldsymbol{\xi}). \tag{13}$$

Definition 1. *A sticky-price equilibrium is a set of allocations, prices, and policies (i.e., $\{\tau_i, \tau_{EX,i}\}_i$ and $M(\boldsymbol{\xi})$) such that for any realized state $\boldsymbol{\xi} \in \boldsymbol{\Xi}$,*

(i) producers optimally choose inputs to minimize the cost of production;

[9] Engel (2016) advocates using a balanced trade assumption instead of the risk sharing condition in the complete market.

(ii) monopolistically competitive firms $f \in [0, \delta_i]$ set prices to maximize profits subject to their demand functions, and the remaining firms $f \in (\delta_i, 1]$ do not adjust prices;

(iii) the representative household chooses consumption and labor to maximize utility subject to its budget constraint, and the total expenditure determines the money demand;

(iv) the government budget constraint is satisfied;

(v) all markets clear.

We define the *flexible-price equilibrium* as the special case of the *sticky-price equilibrium* in Definition 1 that involves no Calvo-pricing friction, as stated in the following definition:

Definition 2. *A flexible-price equilibrium is a set of allocations, prices, and policies satisfying all of the conditions stated in Definition 1, except that for any sector $i \in \{1, 2, \cdots, N\}$, $\delta_i = 1$,* viz.*, all firms can adjust prices flexibly.*

While the *sticky-price equilibrium* is our focus, the allocation of the *flexible-price equilibrium* serves as a benchmark to define the distortions and welfare losses that nominal rigidities introduce.

2.8. Flexible-price equilibrium as reference equilibrium

As per Woodford (2003) and Galí (2015), we use non-contingent subsidies and taxes to eliminate domestic-market distortion while allowing domestic producers to exert their market power fully in the international market in the flexible-price equilibrium, as defined by the following assumption:[10]

Assumption 1. *The non-contingent tax rates for sales and exports are equal to*

$$\tau_i = -1/(\varepsilon_i - 1) \ \textit{ and } \ \tau_{EX,i} = 1/\theta_{Fi}, \ \textit{ respectively, for } \ \forall i \in \{1, ..., N\}. \tag{14}$$

Under Assumption 1, the *flexible-price equilibrium* yields the optimal allocation for the domestic social planner, as stated in the following lemma:

Lemma 1. *Under Assumption 1, the flexible-price equilibrium implements the optimal allocation for the domestic social planner.*

Proof: See Appendix J.2.

Lemma 1 allows use of *flexible-price equilibrium* as the reference equilibrium for our further analyses of the domestic country's aggregate distortion and welfare loss.

[10] In one-sector closed economies, Woodford (2003) and Galí (2015) show that a sales subsidy eliminates the monopoly distortion and makes the flexible-price equilibrium optimal for the social planner. La'O and Tahbaz-Salehi (2022) and Rubbo (2023) use sector-specific subsidies for the same purpose in a multi-sector closed economy. In small open economies, given that sales subsidies eliminate the monopoly distortion, the monopoly power of domestic producers on the international market needs to be retained for the domestic social planner to restore the optimality of the allocation in the flexible-price equilibrium. Therefore, we use sector-specific subsidies and export taxes to remove the monopoly distortion in the domestic market and exert the monopoly power in the international market, respectively, as in Matsumura (2022).

2.9. *Notations*

This section summarizes the notations in the model to facilitate the tracking of variables, vectors, and matrices.

Deviations from the steady state and flexible-price equilibrium. We define the steady state of the static economy as the equilibrium in which all exogenous states A_i, $P^*_{IM,Fi}$, and $P^*_{EX,Fi}$ are at the steady state. We denote with x^{ss} and x^{flex} the values for the variable x in the steady state and in the flexible-price equilibrium, respectively. We express the log deviation of the variable x from the steady state x^{ss} and the flexible-price equilibrium x^{flex} as:

$$\widehat{x} \equiv \ln(x) - \ln(x^{ss}) \quad \text{and} \quad \widehat{x}^{gap} \equiv \ln(x) - \ln(x^{flex}), \tag{15}$$

respectively.[11] We denote the output gap by $\widehat{C}^{gap}$. The sectoral markup wedge is $\ln(\mu_i) - \ln(\mu_i^{\#}) = \ln(\mu_i) - \ln(\mu_i^{ss}) \equiv \widehat{\mu}_i$ as the steady-state markup is equal to the desired markup.

Table 1: Notations of parameters and steady-state objects

Name	Expression
Consumption shares and home bias	$\boldsymbol{\beta} \equiv (\beta_1, \beta_2, \cdots, \beta_N)^\top$ & $\mathbf{v} \equiv (v_1, v_2, \cdots, v_N)^\top$
Labor shares	$\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \cdots, \alpha_N)^\top$
Intermediate input shares and home bias	$\boldsymbol{\Omega} \equiv \{\omega_{i,j}\}_{i,j\in\{1,2,\cdots,N\}}$ & $\mathbf{V}_x \equiv \{v_{x,i,j}\}_{i,j\in\{1,2,\cdots,N\}}$
Elasticity of home-foreign substitution	$\boldsymbol{\theta} \equiv (\theta_1, \theta_2, \cdots, \theta_N)^\top$ & $\boldsymbol{\theta}_F \equiv (\theta_{F,1}, \theta_{F,2}, \cdots, \theta_{F,N})^\top$
Frequency of price adjustment	$\boldsymbol{\Delta} = diag(\delta_1, \delta_2, \cdots, \delta_N)$
Steady-state sectoral Domar weight	$\boldsymbol{\lambda} \equiv (\lambda_1, \lambda_2, \cdots, \lambda_N)^\top \equiv \left(\frac{P_1^{ss}Y_1^{ss}}{P_C^{ss}C^{ss}}, \frac{P_2^{ss}Y_2^{ss}}{P_C^{ss}C^{ss}}, \cdots, \frac{P_N^{ss}Y_N^{ss}}{P_C^{ss}C^{ss}}\right)^\top$
Steady-state sectoral export-to-GDP ratio	$\boldsymbol{\lambda}_{EX} \equiv (\lambda_{EX,1}, \cdots, \lambda_{EX,N})^\top \equiv \left(\frac{P_1^{ss}Y_{EX,1}^{ss}}{P_C^{ss}C^{ss}}, \cdots, \frac{P_N^{ss}Y_{EX,N}^{ss}}{P_C^{ss}C^{ss}}\right)^\top$
Steady-state economy-wise labor share	$\Lambda_L \equiv W^{ss}L^{ss}/P_C^{ss}C^{ss}$
Total content in domestic consumption & exports	$\widetilde{\boldsymbol{\lambda}}_D \equiv \left(\widetilde{\lambda}_{D,1}, \widetilde{\lambda}_{D,2}, \cdots, \widetilde{\lambda}_{D,N}\right)^\top$ & $\widetilde{\boldsymbol{\lambda}}_F \equiv \left(\widetilde{\lambda}_{F,1}, \widetilde{\lambda}_{F,2}, \cdots, \widetilde{\lambda}_{F,N}\right)^\top$
Total content of domestic labor	$\widetilde{\boldsymbol{\alpha}} \equiv (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \cdots, \widetilde{\alpha}_N)^\top$
Expenditure-switching (ES) & generalized ES elasticities	$\boldsymbol{\rho}_{ES} \equiv (\rho_{ES,1}, \rho_{ES,2}, \cdots, \rho_{ES,N})^\top$ & $\widetilde{\boldsymbol{\rho}}_{ES} \equiv (\widetilde{\rho}_{ES,1}, \widetilde{\rho}_{ES,2}, \cdots, \widetilde{\rho}_{ES,N})^\top$

Parameters and steady-state objects. Summarized in Table 1 are the key parameters and steady-state variables. Throughout the paper, for any variable x, we use bold fonts to denote the corresponding vector or matrix —i.e., $\mathbf{x} \equiv \{x_i\}_i$ or $\mathbf{x} \equiv \{x_{i,j}\}_{i,j}$. For expositional simplicity, the superscript "ss" to denote the steady state is omitted when there is no obvious confusion. In particular, we introduce the open-economy version of the Leontief-inverse matrix: $\mathbf{L}_{vx} \equiv (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1} = \{l_{vx,i,j}\}_{i,j}$, where $l_{vx,i,j} = \mathbf{I}_{i,j} + (\boldsymbol{\Omega} \odot \mathbf{V}_x)_{i,j} + (\boldsymbol{\Omega} \odot \mathbf{V}_x)^2_{i,j} + \cdots$ captures the total elasticity of sector i's cost to a change in domestic sector j's price, directly and indirectly through the use of domestic products as intermediate inputs.

[11] In our static model, the sectoral inflation is identical to the log deviation of the sectoral price from its steady-state level.

2.10. Sectoral relevance metrics in an open economy with networks

To facilitate the study of the link between sectoral inflation (or markup wedges) and the output gap, we define the sectoral metrics below that depend on the cross-border and input-output linkages of the economy and represent the relevance of a sector in the economy across different dimensions.

Definition 3 (Sectoral relevance metrics). *For each domestic sector i, the* ***total content in domestic consumption*** *$\widetilde{\lambda}_{D,i}$ and the* ***total content in exports*** *$\widetilde{\lambda}_{F,i}$ are defined as:*[12]

$$\widetilde{\lambda}_{D,i} \equiv \sum_r \beta_r \upsilon_r l_{vx,r,i} \quad \textit{and} \quad \widetilde{\lambda}_{F,i} \equiv \sum_r \lambda_{EX,r} l_{vx,r,i}, \quad \textit{respectively.} \tag{16}$$

The ***total content of domestic labor*** *$\widetilde{\alpha}_i$ and the* ***total content of foreign factor*** *are:*

$$\widetilde{\alpha}_i \equiv \sum_r l_{vx,i,r} \alpha_r \quad \textit{and} \quad 1 - \widetilde{\alpha}_i, \quad \textit{respectively.} \tag{17}$$

The ***expenditure-switching elasticity*** *$\rho_{ES,r}$ is:*

$$\rho_{ES,r} \equiv \underbrace{(\theta_{F,r} - 1)\,\lambda_{EX,r}}_{\textit{foreign expenditure}} + \underbrace{(\theta_r - 1)\left[\beta_r \upsilon_r (1 - \upsilon_r) + \sum\nolimits_s \lambda_s \omega_{s,r} \upsilon_{x,s,r} (1 - \upsilon_{x,s,r})\right]}_{\textit{domestic expenditure}}, \tag{18}$$

based on which the ***generalized expenditure-switching elasticity*** *$\widetilde{\rho}_{ES,i}$ is defined as:*

$$\widetilde{\rho}_{ES,i} \equiv \sum_r \rho_{ES,r} \widetilde{\alpha}_r l_{vx,r,i}. \tag{19}$$

The *total content in domestic consumption* $\widetilde{\lambda}_{D,i}$ (vs. *total content in exports* $\widetilde{\lambda}_{F,i}$) of a domestic sector i in equation (16) encapsulates the importance of a domestic sector in the network economy as both a direct and an indirect supplier (via downstream sectors) —captured by the Leontief inverse $l_{vx,r,i}$— for the domestic aggregate consumption or output (vs. exports).[13] As a result, a sector's total content in domestic consumption decreases in the import shares of the sector and its downstream sectors, as shown in Proposition G.1 of Appendix G. The *total content of domestic labor* $\widetilde{\alpha}_i$ (vs. *total content of foreign factor* $1 - \widetilde{\alpha}_i$) of a domestic sector i in equation (17) summarizes the sector's role in the network economy as both a direct and an indirect customer (via upstream sectors) —captured by the Leontief inverse $l_{vx,i,r}$— of domestic labor factor (vs. imported foreign factor).

The *expenditure-switching elasticity* $\rho_{ES,i}$ of a domestic sector i in equation (18) —corresponding to the

[12] In matrix forms, the total contents in domestic consumption and exports, the total content of domestic labor, and the generalized elasticity of expenditure switching are equal to $\widetilde{\boldsymbol{\lambda}}_D^\top \equiv (\boldsymbol{\beta}\odot\mathbf{v})^\top \mathbf{L}_{vx}$, $\widetilde{\boldsymbol{\lambda}}_F^\top \equiv \boldsymbol{\lambda}_{EX}^\top \mathbf{L}_{vx}$, $\widetilde{\boldsymbol{\alpha}} \equiv \mathbf{L}_{vx}\boldsymbol{\alpha}$, and $\widetilde{\boldsymbol{\rho}}_{ES}^\top \equiv (\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}})^\top \mathbf{L}_{vx}$, respectively.

[13] For a pair of domestic sectors $r \neq i$, r is defined as a downstream (vs. upstream) sector of i if $l_{vx,r,i} > 0$ (vs. $l_{vx,i,r} > 0$).

standard expenditure switching effect in international macroeconomic literature— captures the elasticity of the domestic and foreign expenditures on domestic products to the relative prices of foreign versus domestic sector i's products. To further capture the impact of the domestic-to-foreign price on domestic labor income through the expenditure switching effect, we define the *generalized expenditure-switching elasticity* $\widetilde{\rho}_{ES,i}$. Specifically, it is equal to the elasticity of domestic labor income —evinced by the total content of domestic labor $\widetilde{\alpha}_r$ in equation (19)— to the relative price of foreign versus domestic sector i, *through* the direct and indirect (via downstream sectors) impact of the foreign-to-domestic price on the expenditures on domestic products —captured by the expenditure-switching elasticity $\rho_{ES,i}$ multiplied by the Leontief inverse $l_{vx,r,i}$. The vector formats for these defined sectoral metrics are summarized in Table 1.

3. Sectoral markup wedges and the output gap

In this section, we derive the output gap as a weighted average of the sectoral markup wedges, and the sectoral weights are functions of the sectoral relevance metrics, as introduced in Section 2.10, thereby depending on the cross-border and input-output linkages.[14]

Under nominal rigidities, as a fraction $(1-\delta_i)$ of sector i's firms cannot adjust prices in response to changes in marginal costs, sectoral inflation is linked to sectoral markup wedges —encapsulating sectoral distortions— through sectoral price rigidities as follows:[15]

$$\widehat{\mu}_i(\boldsymbol{\xi}) = -(1-\delta_i)/\delta_i \cdot \widehat{P}_i(\boldsymbol{\xi}). \tag{20}$$

These negative sectoral markup wedges relate to a positive output gap through three distinct channels, as outlined in the following theorem:

Theorem 1 (Output gap and sectoral markup wedges)**.** *In a sticky-price equilibrium, the output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ is proportional to a weighted average of sectoral markup wedges $\{\widehat{\mu}_i(\boldsymbol{\xi})\}_i$:*[16]

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\sum_{i=1}^{N} \mathcal{M}_{OG,i} \cdot \widehat{\mu}_i(\boldsymbol{\xi}), \tag{21}$$

where the vector of sectoral OG weights ($\boldsymbol{\mathcal{M}}_{OG} \equiv (\mathcal{M}_{OG,1}, \mathcal{M}_{OG,2}, \cdots, \mathcal{M}_{OG,N})$) is equal to:

$$\boldsymbol{\mathcal{M}}_{OG} \equiv \underbrace{\widetilde{\boldsymbol{\lambda}}_D}_{\textit{CPI channel}} + \underbrace{\kappa_{CPI}^{-1} \cdot \widetilde{\boldsymbol{\rho}}_{ES}}_{\textit{expenditure-switching channel}} + \underbrace{\kappa_{CPI}^{-1} \cdot \left(\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right)}_{\textit{profit channel}}, \tag{22}$$

[14] In Appendix B.1, we show that up to the first-order approximation, the aggregate distortion is proportional to the output gap.

[15] Exogenous shocks to sectoral productivity, import prices, and export demand drive sectoral inflation in the sticky-price equilibrium. Shown in Appendix J.6 is the derivation of equation (20).

[16] The negative sign on the RHS of equation (21) indicates that the negative sectoral markup wedges linked to the positive sectoral inflation are associated with a positive output gap.

$$\kappa_{CPI} \equiv \frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}} + 1, \tag{23}$$

$$\kappa_C \equiv \Big[\frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}} \big(\sigma + \varphi/\Lambda_L\big) + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} \big(\sigma + \varphi/\Lambda_L\big) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\Big] / \kappa_{CPI}. \tag{24}$$

Proof: See Appendix K.6.

Equation (21) shows that negative sectoral markup wedges relate to a positive output gap. The sectoral OG weight ($\mathcal{M}_{OG,i}$) in equation (22) measures the relevance of the sector's markup wedge for the output gap, with its size determined by three distinct channels: (i) the *CPI*, (ii) the *expenditure-switching*, and (iii) the *profit* channels. To illustrate the three channels, we first present and interpret the two equilibrium conditions that we combine to derive equation (21), contrasting the cases of closed and open economies.

The first condition is the log-linearized labor supply equation around the flexible-price equilibrium (equation 25) combined with the equation of real wage gap (equation 26) as follows:

$$\begin{aligned} \big(\sigma + \varphi/\Lambda_L\big) \widehat{C}^{gap}(\boldsymbol{\xi}) &= \widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi}) & (25) \\ &= (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) \big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) - \widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). & (26) \end{aligned}$$

The second condition is the log-linearized open-economy budget constraint around the flexible-price equilibrium, given by:[17]

$$\begin{aligned} (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) \widehat{C}^{gap}(\boldsymbol{\xi}) = &\underbrace{-(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big)}_{\text{expenditure-switching channel}} + \underbrace{[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})}_{\text{imported factor channel}} \underbrace{- \boldsymbol{\lambda}_{EX}^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big)}_{\text{export profit channel}} \\ &+ \underbrace{(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) \big(\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\big)}_{\text{real exchange rate mechanism}} + o(\|\widehat{\boldsymbol{\xi}}\|), \qquad (27) \end{aligned}$$

where the gaps in domestic sectoral prices and CPI relative to foreign prices are both functions of the gap in domestic wage relative to foreign prices ($\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})$) and sectoral markup wedges as follows:

$$\begin{aligned} \widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) &= \widetilde{\boldsymbol{\alpha}} \big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) + \mathbf{L}_{vx} \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), & (28) \\ \widehat{P}_C^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi}) &= \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} \big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) + \widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). & (29) \end{aligned}$$

The first condition in equation (25) is similar to that in closed economies, by relating the output gap to the real wage gap through labor supply and linking the real wage gap to sectoral markup wedges (equation 26).

[17]The nominal exchange rate (S) is the endogenous component in foreign goods prices (in units of the domestic currency) that relates to sectoral markup wedges and, therefore, monetary policy. As such, the exchange rate gap ($\widehat{S}^{gap}$) reflects the prices of foreign products or the values of foreign demand in the log-linearization of equilibrium conditions around the flexible-price equilibrium. The unitary vector $\mathbf{1}$ in equations (27) and (28) indicates that a depreciation of domestic currency (i.e., an increase in $\widehat{S}^{gap}$) uniformly raises the prices of foreign products or the values of foreign demand in units of domestic currency.

In particular, while the real wage gap is a function of only sectoral markup wedges in closed economies — where the total share of domestic labor in domestic output $\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} = 1$— it also relates to the gap in domestic wage relative to foreign prices ($\widehat{W}^{gap} - \widehat{S}^{gap}$) in open economies. The second condition in equation (27) is specific to open economies. It relates the output gap to changes in domestic income through the switch of expenditures from foreign to domestic products, changes in expenditures on imported foreign factors and export profits, and the real exchange rate gap. In closed economies, condition (27) does not affect the output gap, and condition (26) of the real wage gap alone determines the relationship of the output gap to sectoral markup wedges. Using the equilibrium conditions in equations (26) and (27), we now interpret each of the three channels in the OG weight in Theorem 1.

(i) CPI channel. The *CPI channel* describes the relationship between sectoral markup wedges and the output gap through distortion in the price of the aggregate output —i.e., the CPI. Specifically, negative sectoral markup wedges are associated with a lower CPI in the sticky-price relative to the flexible-price equilibrium, and the negative CPI gap relates to a positive output gap through two complementary mechanisms. One operates through the real wage —a standard mechanism in closed economies with nominal rigidities— where a lower CPI increases the real wage (W/P_C) and stimulates a higher supply of domestic labor, thereby being linked to a positive output gap, as evinced by the negative coefficient $-\widetilde{\boldsymbol{\lambda}}_D^\top$ in equation (26). The second mechanism operates through the real exchange rate and is specific to open economies, where a lower CPI increases the domestic income from foreign countries in units of domestic consumption —as evinced by the term of the real exchange rate gap $(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})(\widehat{S}^{gap} - \widehat{P}_C^{gap})$ in equation (27).

For a domestic sector in an open economy with production networks, the size of the CPI channel is proportional to the domestic sector's total content in domestic consumption $\widetilde{\lambda}_{D,i}$ —as shown in the CPI channel in the OG weight of equation (22). In particular, both the CPI channel and the OG weight in equation (22) reduce to the corresponding Domar weight in closed economies à la Rubbo (2023).[18]

(ii) Expenditure-switching channel. The *expenditure-switching channel* —specific to the open economy— is standard in the international macroeconomic literature. It describes how domestic sectoral markup wedges are linked to the (domestic) output gap, by changing the relative price of domestic to foreign products and generating a switch of domestic and foreign expenditures from foreign toward domestic products.

Equation (28) shows that negative sectoral markup wedges —directly and indirectly via the Leontief inverse $\mathbf{L}_{vx}$— reduce the prices of domestic products relative to foreign products, which are captured by

[18]The OG weight reduces to the Domar weight in closed economies under our normalization by the scalar κ_{CPI} in equation (23). κ_{CPI} captures the size of the CPI channel relative to the CPI gap, and it is equal to the sum of the following two elasticities relating to the real wage and exchange rate mechanisms, respectively: (i) the elasticity of the real wage gap to the CPI gap that equals unity, multiplied by the elasticity of the output gap to the real wage gap in equation (27) (with equations 26, 28, and 29 substituted in) —which equals $(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}/(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}$; and (ii) the elasticity of the real exchange rate gap $(\widehat{S}^{gap} - \widehat{P}_C^{gap})$ to the CPI gap that equals unity, multiplied by the elasticity of the output gap to the real exchange rate gap in equation (27) —which equals the share of income from foreign countries in total domestic income $(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})$.

the difference between the sectoral inflation gap $\widehat{\mathbf{P}}^{gap}$ and the exchange rate gap $\mathbf{1}\widehat{S}^{gap}$. This reduction in domestic-to-foreign goods prices generates a switch of both domestic and foreign expenditures from foreign to domestic products —as evinced by the negative coefficient $-\boldsymbol{\rho}_{ES}$ in equation (27). As a result, domestic labor income from international trade increases —as captured by the total content of domestic labor $\widetilde{\boldsymbol{\alpha}}$— thereby connecting negative sectoral markup wedges with a positive output gap, as shown by the expenditure-switching term in equation (27).

Consequently, for a domestic sector of an open economy with production networks, the size of the expenditure-switching channel is determined jointly by the magnitudes of (i) the Leontief inverse $\mathbf{L}_{vx}$ in equation (28) —which links negative sectoral markup wedges to the gaps in domestic-to-foreign goods prices and (ii) the elasticity of domestic labor income to the domestic-to-foreign goods prices $(-(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^{\top}$ in equation 27). These two sub-components are combined to yield the generalized expenditure-switching elasticity in the expenditure-switching channel in the OG weight of equation (22).

(iii) Profit channel. The *profit* channel —also specific to the open economy— describes how domestic sectoral markup wedges relate to the output gap through both the profits from exports and the costs of imported foreign inputs. In the export profit sub-channel, negative sectoral markup wedges directly and indirectly reduce domestic prices or, equivalently, the (opportunity) costs of exported goods, increasing the profits from exports and relating to a positive output gap. In the imported factor sub-channel, negative sectoral markup wedges are associated with higher sectoral costs of imported foreign factors in production (relative to sectoral sales) and lower domestic producers' profits, thus linking to a negative output gap.[19]

For a domestic sector in an open economy with production networks, the size of the export profit sub-channel is determined by both the Leontief inverse $\mathbf{L}_{vx}$ in equation (28) —which links negative sectoral markup wedges to the gaps in domestic prices— and the share of the sector's exports in domestic output —i.e., $\boldsymbol{\lambda}_{EX}^{\top}$ in equation (27). The size of the imported factor channel is determined by both the sector's size and its direct and indirect (via upstream sectors) use of imported inputs —captured by the product of the sectoral Domar weight (i.e., λ_i) and the sectoral content of foreign factor (i.e., $1 - \widetilde{\alpha}_i$) in equation (27). These two sub-channels are combined to yield the profit channel in the OG weight of equation (22).

Role of the exchange rate in the output gap. Negative markup wedges lower the domestic-to-foreign prices relative to the flexible-price equilibrium, thus improving trade conditions through the expenditure-switching effect. Under our *key assumption of balanced trade*, the domestic currency (relative to domestic wage) appreciates (i.e., $\widehat{S}^{gap} - \widehat{W}^{gap}$ decreases) —as we show in equation (K.29) of Appendix K.7— to attenuate the positive output gap and preserve the trade balance in two ways. First, the currency appreciation

[19] In general, the two sub-channels of export profits and imported factors do not perfectly offset one another. For example, in the special case of a multi-sector small open economy that imports foreign goods only for consumption but not as intermediate inputs, the export profit sub-channel is nonzero while the imported factor sub-channel reduces to zero, thereby allowing sectoral markup wedges to link positively to the output gap through the profit channel. Discussed in Appendix C is why the optimization by the domestic planner does not imply a zero profit channel in general.

can attenuate the positive output gap by increasing the domestic-to-foreign price gaps in equation (27), which reduces export profit and domestic labor income through expenditure switching. Second, currency appreciation can attenuate the positive output gap by reducing the income from foreign countries in units of domestic consumption through the lower real exchange rate.[20]

In Appendix B.2, we show that in a dynamic environment with the standard assumption of a *complete (asset) market* à la Galí and Monacelli (2005) —such that the Backus-Smith and uncovered interest rate conditions hold— the consumption gap (i.e., $\widehat{C}^{gap}$) is proportional to the real exchange rate gap according to the international risk-sharing condition à la Galí and Monacelli (2005) and Corsetti et al. (2010), rather than determined by the open-economy budget constraint in equation (27) that is based on the trade balance condition. Using the risk-sharing condition, we derive the sectoral OG weight in complete markets and show that the share of the CPI channel in the OG weight of complete markets is smaller than that in our baseline case of balanced trade.[21]

4. The Phillips curves and the output gap targeting policy

In this section, we use the sectoral OG weight introduced in the previous section to derive an aggregate-level Phillips curve that links an aggregate inflation index to the output gap, which allows for the divine coincidence.[22] We follow Rubbo (2023) to refer to this aggregate Phillips curve as the divine coincidence Phillips curve and to the associated aggregate inflation index as the divine coincidence index.

In subsection 4.1, we derive the DC and sectoral Phillips curves. We show that the slope of the DC Phillips curve is inversely related to the *sum of the sectoral OG weights*, while the *relative OG weights* determine the *relative sectoral weights* in the DC index. Subsection 4.2 compares the slopes of the DC and sectoral Phillips curves to the counterfactual slopes in closed economies and without IO linkages. Subsection 4.3 studies how the *relative sectoral weights* in the DC index —which is targeted to zero to implement the policy of output gap targeting— depend on cross-border and input-output linkages.

[20] In contrast, in our multi-sector small open economies, the terms of trade (an important concept in the SOE literature) has a limited role in the output gap. In the special case of one-sector small open economies à la Galí and Monacelli (2005), the terms of trade gap is proportional to the output gap, both of which are equal to zero under the optimal policy of domestic inflation stabilization. In our multi-sector small open economies, as we show in Appendix K.8, the terms of trade gap is equal to a weighted average of sectoral domestic-to-foreign price gaps (i.e., $\left[(\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}))^\top \boldsymbol{\lambda}_{EX}\right]^{-1} (\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX})^\top (\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap})$, where $\odot$ is the Hadamard (element-wise) product). As shown in equation (27), the sectoral domestic-to-foreign price gaps are important components of the expenditure-switching channel in the output gap. However, their relationships to the output gap are captured by their sectoral weights in equation (27) rather than their weights in the terms of trade gap. As such, the relevance of the terms of trade for the output gap in multi-sector open economies with production networks is limited.

[21] According to the analysis in Section 4.1, the smaller share of the CPI channel in the OG weight in complete markets than under balanced trade implies a smaller (vs. larger) relative sectoral weights in the DC index in complete markets for sectors with a larger (vs. smaller) CPI channel. In Appendix B.2, we also show that the slope of the DC Phillips curve is flatter in complete markets than under balanced trade. Provided in Appendix B.2 are the detailed DC Phillips curve and DC index, with the economic intuition behind the flatter slope in complete markets.

[22] In multi-sector open economies, sector-level Phillips curves include a residual of exogenous shocks (see subsection 4.1), preventing simultaneous stabilization of inflation and output gap —i.e., the "divine coincidence" fails to hold— both at the sector level and under arbitrary aggregation, similar to the case of closed economies à la Rubbo (2023).

4.1. The divine coincidence and sectoral Phillips curves

The divine coincidence Phillips curves. Based on the sectoral OG weights in equation (22) that relate sectoral markup wedges to the output gap, we construct the following divine coincidence index as a weighted average of sectoral inflation.

Definition 4 (**Divine coincidence index**). *Assume that no sector has perfectly rigid prices (i.e., $\delta_i \neq 0\ \forall i$). The* divine coincidence index (DC index for short) *weights sectors according to their sectoral OG weights —adjusted by the sectoral price rigidity— as in the following equation:*

$$\pi_{DC} \equiv \sum_{i=1}^{N} \widetilde{\mathcal{M}}_{OG,i} \widehat{P}_i, \tag{30}$$

where the normalized OG weight *($\widetilde{\mathcal{M}}_{OG,i}$) is equal to:*

$$\widetilde{\mathcal{M}}_{OG,i} \equiv \frac{\mathcal{M}_{OG,i} \cdot (1-\delta_i)/\delta_i}{\sum_{i'=1}^{N} \mathcal{M}_{OG,i'} \cdot (1-\delta_{i'})/\delta_{i'}}, \tag{31}$$

and we denote the normalization factor by κ_{OG} as follows:

$$\kappa_{OG} \equiv \sum_{i'=1}^{N} \mathcal{M}_{OG,i'} \cdot (1-\delta_{i'})/\delta_{i'}. \tag{32}$$

Notably, the normalized sectoral OG weight ($\widetilde{\mathcal{M}}_{OG,i}$) in equation (31) is the price-rigidity-adjusted sectoral OG weight normalized by its sum across all sectors κ_{OG}. Thus, the normalized sectoral OG weight sums to one (i.e., $\sum_{i=1}^{N} \widetilde{\mathcal{M}}_{OG,i} = 1$) and reflects the sectoral weight relative to the sum of weights, in contrast to the original sectoral OG weight that determines both the sum of sectoral weights and the relative sectoral weights.[23] Based on the DC index defined in equation (30), we derive the divine coincidence Phillips curve that is consistent with the simultaneous stabilization of domestic aggregate inflation and the output gap, as stated in the next proposition.

Proposition 1 (**Divine coincidence Phillips curve**). *For any realized state $\boldsymbol{\xi} \in \Xi$, the divine coincidence Phillips curve (DC Phillips curve for short) is given by:*

$$\pi_{DC}(\boldsymbol{\xi}) = \frac{\kappa_C}{\kappa_{OG}} \widehat{C}^{gap}(\boldsymbol{\xi}). \tag{33}$$

Proof: Straightforward substitution of equation (20) in equation (21) from Theorem 1.

[23] We distinguish between normalized and original sectoral OG weights because the original OG weight reduces to the Domar weight in closed economies, making our analysis of the DC Phillips curve directly comparable to the case of the closed economy.

The divine coincidence Phillips curve in Proposition 1 links domestic inflation in the form of the DC index in equation (30) to the output gap, allowing for the simultaneous stabilization of domestic aggregate inflation and the output gap that achieves the divine coincidence, as in closed economies à la Rubbo (2023). In particular, equation (31) reveals that the DC index assigns higher weights to sectors with high nominal rigidities —as captured by the sectoral price rigidity $(1-\delta_i)/\delta_i$— which is consistent with the results in closed economies (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). However, in an open economy, equation (31) indicates that the weight assigned to sector i is proportional to the OG weight ($\mathcal{M}_{OG,i}$) defined in equation (22), which internalizes the structure of both input-output and cross-border linkages, as stated in Theorem 1. Our DC index in equation (30) nests the DC index of Rubbo (2023) in the case of the closed economy with production networks.

Equation (33) shows that, in our small open economy with production networks, the slope of the DC Phillips curve is equal to κ_C/κ_{OG}, which is inversely related to the *sum of the price-rigidity-adjusted OG weights* (i.e., κ_{OG}). In contrast, the *relative sectoral OG weights* determine the shares of sectoral inflation in the DC index, as evinced by the normalized OG weights in equation (31).

In Appendix E, we extend our baseline model of producer-currency pricing (PCP) to the setting of foreign-currency pricing —which encompasses the alternatives of local-currency pricing (LCP) and dominant-currency pricing (DCP). We show that the divine coincidence index under foreign-currency pricing includes sectoral inflation of both domestic-market prices and export prices in the foreign market (Corollary E.1). Particularly, while the CPI and profit channels depend on domestic inflation, the expenditure-switching channel depends on both domestic and export price inflation.

The sectoral Phillips curves. In addition to the DC Phillips curve that simultaneously stabilizes domestic aggregate inflation and the output gap, we also derive the sectoral Phillips curves linking sectoral inflation to both the output gap and the exogenous sectoral shocks, as stated in the next proposition.

Proposition 2 (Sectoral Phillips curves). *In the sticky-price equilibrium, the following sectoral-level Phillips curves hold:*

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \underbrace{\boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi})}_{\textit{output-gap-driven inflation}} + \underbrace{\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}}}_{\textit{cost-push inflation}} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{34}$$

where $\widehat{\mathbf{P}}(\boldsymbol{\xi})$ is an N-by-1 vector of sectoral inflation, and parameters $\boldsymbol{\mathcal{B}}$ (an N-by-1 vector) and $\boldsymbol{\mathcal{V}}$ (an N-by-3N matrix) are the slopes of Phillips curves and the coefficients of exogenous shocks, respectively.

Proof: See Appendix L.1.

In Proposition 2, the slopes of the sectoral Phillips curves are equal to:

$$\boldsymbol{\mathcal{B}} \equiv \boldsymbol{\Delta}_{\Phi}\Big\{\underbrace{\boldsymbol{\alpha}\left(\sigma+\varphi/\Lambda_L+\Gamma_{CPI,C}\right)}_{\text{nominal wage component}} + \underbrace{\left(\boldsymbol{\Omega}\odot\mathbf{V}_{\mathbf{1}-\boldsymbol{x}}\right)\mathbf{1}\Gamma_{S,C}}_{\text{nominal exchange rate component}}\Big\}, \tag{35}$$

$$\mathbf{\Delta}_\Phi \equiv \left[\mathbf{\Delta}^{-1} - \mathbf{\Omega} \odot \mathbf{V}_x - \boldsymbol{\alpha}\mathbf{\Gamma}_{CPI,P}^\top - (\mathbf{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\mathbf{\Gamma}_{S,P}^\top\right]^{-1} \tag{36}$$

where the scalar $\Gamma_{CPI,C} \equiv \left(\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(1 - \boldsymbol{\beta}^\top \mathbf{v})$ and vector $\mathbf{\Gamma}_{CPI,P}$ are the elasticities of the CPI to the output gap and domestic sectoral inflation, respectively. The scalar $\Gamma_{S,C} \equiv \left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(1 + \Gamma_{CPI,C})$ and the vector $\mathbf{\Gamma}_{S,P}$ are the elasticities of the nominal exchange rate to the output gap and domestic sectoral inflation, respectively.[24]

Consistent with the sectoral OG weight in equation (22), the slopes of sectoral Phillips curves —which also link domestic inflation to the output gap— include the same three channels. The terms representing the domestic contents in domestic consumption (i.e., $\boldsymbol{\beta}^\top \mathbf{v}$ and $\boldsymbol{\beta} \odot \mathbf{v}$ in $\Gamma_{CPI,C}$ and $\mathbf{\Gamma}_{CPI,P}$, and $\mathbf{V}_x$ in $\mathbf{\Delta}_\Phi$) capture the *CPI channel* in the slopes of sectoral Phillips curves. The terms $\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}}$ and $\boldsymbol{\lambda}_{EX}$ in the vector $\boldsymbol{\mathcal{M}}_p \equiv (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})$ and the vector $\boldsymbol{\mathcal{M}}_\mu \equiv (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\mathbf{\Delta}^{-1} - \mathbf{I})\left[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right]$ capture the relations of domestic sectoral inflation to the output gap in the open-economy budget constraint, through the *expenditure-switching*, *export profit*, and *imported factor channels*, respectively.

Equation (35) shows that sectoral inflation is linked to the output gap *via* the positive nominal wage and nominal exchange rate components, thereby making the slopes of sectoral Phillips curves positive for all sectors. The nominal wage component demonstrates that a positive output gap is linked to a higher wage via the labor supply —as captured by the term $(\sigma + \varphi/\Lambda_L)$— thus relating to higher sectoral inflation. The nominal exchange rate component demonstrates that a positive output gap relates to an increased nominal expenditure and a worsened current account, hence occurring with a depreciation of the domestic currency and an increase in the real exchange rate, captured by the term $\left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}$ in $\Gamma_{S,C}$. The increase in the nominal exchange rate propagates into the cost of imported inputs and accordingly into sectoral inflation, captured by the term $(\mathbf{\Omega} \odot \mathbf{V}_{\mathbf{1}-\boldsymbol{x}})\,\mathbf{1}$. The nominal wage and exchange rate components are in nominal terms and, therefore, affected by the CPI —as captured by the term $\Gamma_{CPI,C}$ in both components.[25]

4.2. The slopes of Phillips curves

In this section, we study the slopes of the DC and sectoral Phillips curves, comparing them to their counterparts in the counterfactual cases of closed economies and no input-output linkages.

Equation (33) in Proposition 1 shows that the slope of the DC Phillips curve is inversely related to the *sum of the price-rigidity-adjusted OG weights*, thereby depending on the three channels that comprise the sectoral OG weight in equation (22). Similarly, the slopes of the sectoral Phillips curves in equation (36) also depend on these three channels, as we discussed in the previous subsection. In closed economies, the slopes of the DC and sectoral Phillips curves reduce to $(\sigma + \varphi)\,/\,(\sum_i \lambda_i(1 - \delta_i)/\delta_i)$ and $(\mathbf{\Delta}^{-1} - \mathbf{\Omega} -$

[24] The vectors $\mathbf{\Gamma}_{CPI,P}$ and $\mathbf{\Gamma}_{S,P}$ are equal to: $\mathbf{\Gamma}_{CPI,P} \equiv \left(\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}\left[\left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)(\boldsymbol{\beta} \odot \mathbf{v}) + (1 - \boldsymbol{\beta}^\top \mathbf{v})(\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu)\right]$ and $\mathbf{\Gamma}_{S,P} \equiv \left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu + \mathbf{\Gamma}_{CPI,P})$, respectively. Appendix L.1 reports the definition of the matrix $\boldsymbol{\mathcal{V}}$.

[25] Sectoral inflation also directly and positively links to the nominal CPI and exchange rate —as evinced by the positive vectors $\mathbf{\Gamma}_{CPI,P}$ and $\mathbf{\Gamma}_{S,P}$ in the denominator of $\mathbf{\Delta}_\Phi$ in equation (36)— which generates a positive feedback effect that increases the elasticity of domestic inflation to the output gap (i.e., flattens the slope of the sectoral Phillips curve).

$\boldsymbol{\alpha}\boldsymbol{\beta}^{\top})^{-1}\boldsymbol{\alpha}(\sigma+\varphi)$, respectively, consistent with the results of Rubbo (2023).

Compared to the closed-economy case, the slopes of the DC and sectoral Phillips curves in open economies can be flatter or steeper, depending on the quantitative strength of two main countervailing forces. First, in the CPI channel, the content of domestic sectoral goods in domestic consumption is smaller in open than in closed economies —as shown by $\widetilde{\boldsymbol{\lambda}}_D^{\top} < \boldsymbol{\lambda}^{\top}$ in the *sum of the CPI channels of the OG weights* in equation (D.4) for the DC Phillips curve.[26] Therefore, the elasticity of the (domestic) output gap to domestic sectoral inflation through the CPI channel is smaller in open economies, thereby *steepening* the slopes of both the DC and sectoral Phillips curves relative to the closed-economy case. Second, the positive, open-economy-specific expenditure-switching channel increases the elasticity of the output gap to domestic sectoral markups and inflation —as shown by the *sum of the expenditure-switching channels* in equation (D.1) for the DC Phillips curve— thereby *flattening* the slopes in open relative to closed economies.[27]

In Section 6.1, we calibrate our model using the WIOD data and show that the first force —namely, the smaller content of domestic goods in domestic consumption in open relative to closed economies— dominates in most cases, generally making the slopes of both the DC and the sectoral Phillips curves steeper in open economies. In Figure D.2 of Appendix D.1, we also show that the DC Phillips curve slope in the baseline economy with IO linkages is flatter than that in one-sector SOEs and in multi-sector SOEs without IO linkages, consistent with the results in closed economies à la Rubbo (2023). Intuitively, with the introduction of IO linkages, domestic sectoral goods are not only *directly* but also *indirectly* used by domestic output, which increases the elasticity of the domestic output gap to domestic sectoral inflation, thereby *flattening* the slope of the DC Phillips curve. In particular, the figure shows that the introduction of IO linkages flattens the slope more in closed (relative to open) economies for those economies that are relatively open.

Takeaways for the DC Phillips curve slope. Overall, our analysis shows that the *sum of the sectoral OG weight* —through the sum of each of the three channels across sectors— determines the slope of the DC Phillips curve —which, therefore, depends on cross-border and input-output linkages.

4.3. Relative sectoral weights in the divine coincidence index for output gap targeting

In this section, we show that the monetary policy of output gap targeting that closes the output gap is implemented by targeting the divine coincidence index —which depends on the *relative sectoral OG weights*— to zero. Therefore, we study the role of input-output and cross-border linkages for output gap targeting through their impacts on the *relative sectoral weights in the DC index*. We do so by focusing on the pitfalls of the normalized sectoral OG weights in alternative output gap targeting that: (i) disregards the

[26] For sectoral Phillips curves, this is shown by $\boldsymbol{\beta}^{\top}\mathbf{v} < 1$, $\boldsymbol{\beta}\odot\mathbf{v} < \mathbf{1}$ in $\Gamma_{CPI,C}$ and $\boldsymbol{\Gamma}_{CPI,P}$, and $\mathbf{V}_x < \mathbf{I}$ in $\boldsymbol{\Delta}_{\Phi}$ in equation (35).

[27] For sectoral Phillips curves, this is shown by $\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}} > \mathbf{0}$ in $\boldsymbol{\mathcal{M}}_p$ which comprises $\Gamma_{CPI,C}$, $\boldsymbol{\Gamma}_{CPI,P}$, $\Gamma_{S,C}$, and $\boldsymbol{\Gamma}_{S,P}$ in equation (35).

role of input-output linkages as in the one-sector small open economy literature, and (ii) disregards the role of cross-border linkages and targets the output gap in closed economies with production networks.

Output gap targeting. Proposition 1 implies that the DC index is a sufficient statistic for the output gap in multi-sector open economies with production networks, as in closed economies à la Rubbo (2023). Therefore, the monetary policy of *output gap targeting* that fully closes the output gap can be implemented by targeting the DC index to zero. As we show later in the paper, although output gap targeting is not optimal in multi-sector open economies because the divine coincidence fails to hold (Section 5), it remains a useful policy that closely approximates the optimal monetary policy in minimizing welfare loss, similar to the case of multi-sector closed economies à la Rubbo (2023) (Section 6.3).

Pitfall in output gap targeting in one-sector small open economy literature. A well-established result in one-sector SOE models without input-output linkages is that optimal monetary policy should stabilize domestic inflation (Galí and Monacelli, 2005), which simultaneously closes the output gap, as well as the terms-of-trade gap —i.e., the divine coincidence holds. This result is consistent with our theoretical finding in the special case of the one-sector version of our model (Section 5).

However, in the one-sector SOE literature, the optimal policy of output gap targeting is usually implemented by targeting the PPI inflation, where domestic sectoral inflation rates are weighted by the sectoral sales that are proportional to the sectoral Domar weights. Below, we analyze the pitfalls of using Domar weights in place of OG weights for output gap targeting. Thus, our results are consistent with the findings in one-sector SOE models that domestic inflation should be stabilized. We contribute to this line of research by deriving the appropriate sectoral weights for output gap targeting in open economies with production networks.

Pitfall in output gap targeting that disregards cross-border linkages. To assess the relevance of openness for output gap targeting, we study the pitfalls of using OG weights that ignore cross-border linkages in the DC index.

As a first step, we determine the OG weight in closed economies. In a closed economy, only domestic demand exists; consequently, the total content of domestic goods in exports is zero (i.e., $\widetilde{\lambda}_{F,i} = 0, \ \forall i$). Moreover, the *expenditure-switching* and *profit channels* are equal to zero. Thus, *total content in domestic consumption* uniquely determines the closed-economy OG weight, which is equal to the Domar weight and consistent with the results of Rubbo (2023), as summarized in the next lemma.[28]

Lemma 2. *In a closed economy, the OG weight of each sector $i \in \{1, 2, \cdots, N\}$ reduces to the Domar weight, i.e., $\mathcal{M}_{OG,i} = \lambda_i$. In the open economy, the Domar weight of each sector i equals the sum of the*

[28]Our standard assumption of a Cobb-Douglas production function is crucial for establishing the equivalence between the sectoral *total content in domestic consumption* and the Domar weight, as discussed in Baqaee (2018).

sectoral total contents in domestic consumption and in exports, viz.:

$$\lambda_i = \widetilde{\lambda}_{D,i} + \widetilde{\lambda}_{F,i}. \tag{37}$$

Proof: See Appendix L.2.

Lemma 2 implies that output gap targeting that ignores the cross-border linkages will adopt the Domar weight in place of the OG weight. Thus, we construct the normalized sectoral Domar weights for closed economies as $\widetilde{\lambda}_i \equiv (\lambda_i(1-\delta_i)/\delta_i)/\kappa_\lambda$, where $\kappa_\lambda \equiv \sum_{i'} \lambda_{i'}(1-\delta'_i)/\delta'_i$ —similar to the normalized sectoral OG weights for open economies in equation (31). The corresponding monetary policy that targets the aggregate inflation index using the normalized Domar weights —or, equivalently, the *(price-rigidity-adjusted) PPI targeting* policy— coincides with output gap targeting in closed economies à la Rubbo (2023). For simplicity, we refer to the price-rigidity-adjusted PPI targeting policy as "*PPI targeting*" throughout the paper.[29]

Equation (37) in Lemma 2 further shows that, unlike in closed economies, the Domar weight in open economies includes not only *total content in domestic consumption* ($\widetilde{\lambda}_{D,i}$), but also *total content in exports* ($\widetilde{\lambda}_{F,i}$) as domestic output in open economies is supplied to both domestic and foreign customers. Combining Lemma 2 and Theorem 1 gives the percentage deviation of the normalized closed-economy OG weight (i.e., the Domar weight) from the normalized open-economy OG weight, as stated in the following proposition:

Proposition 3. *The percentage deviation of the normalized Domar weight relative to the normalized OG weight is:*

$$\frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i} = \frac{\kappa_\lambda}{\kappa_{OG}}\Bigg[\underbrace{\frac{\widetilde{\lambda}_{F,i}}{\lambda_i}}_{\text{export intensity}} \underbrace{-\kappa_{CPI}^{-1} \cdot \frac{\widetilde{\rho}_{ES,i}}{\lambda_i}}_{\text{expenditure switching}} + \underbrace{\kappa_{CPI}^{-1} \cdot \left((1-\widetilde{\alpha}_i) - \widetilde{\lambda}_{F,i}/\lambda_i\right)}_{\text{profit}}\Bigg] + \left(1 - \frac{\kappa_\lambda}{\kappa_{OG}}\right). \tag{38}$$

Proof: Straightforward result from Lemma 2 and Theorem 1.

Proposition 3 shows that the percentage deviation of the normalized Domar weight from the normalized OG weight is equal to the percentage deviation of the Domar from OG weights (in brackets) —rescaled by the ratio of the sums of (price-rigidity-adjusted) Domar to OG weights ($\kappa_\lambda/\kappa_{OG}$)— plus the sector-invariant constant $1 - \kappa_\lambda/\kappa_{OG}$. Proposition 3 demonstrates that the PPI targeting that fails to consider cross-border linkages and uses Domar weights can *either* overstate *or* understate the inflation of a domestic sector, depending on the magnitudes of *two main countervailing forces*.[30]

[29] Similarly, throughout the paper, we refer to the price-rigidity-adjusted CPI targeting policy as "*CPI targeting*," which targets the aggregate inflation index using the normalized CPI weights, i.e., $\widetilde{\beta}_i \equiv (\beta_i(1-\delta_i)/\delta_i)/\kappa_\beta$, where $\kappa_\beta \equiv \sum_{i'} \beta_{i'}(1-\delta'_i)/\delta'_i$.

[30] As we show in the quantitative section 6.2, the magnitude of the profit channel is close to zero.

First, Domar weights in open economies capture domestic sectors' supply of inputs to foreign countries in addition to domestic output —summarized by the sectoral *export intensity*, which we define as the ratio of a sector's total content in exports to its Domar weight (i.e., the first component in the brackets on the RHS of equation 38). Thus, the output gap targeting that disregards cross-border linkages and uses the Domar weights overemphasizes the contribution of the domestic sector to the domestic output as a supplier, thereby overstating the relative weights of sectors that export more directly and indirectly in the DC index.

Second, the normalized Domar weights —which disregard cross-border imports and exports— can understate the importance of a domestic sector's inflation by failing to consider its impact on the domestic-to-foreign prices and, in turn, the demand for domestic goods and labor —as captured by the negative *expenditure-switching component* (i.e., the second component in the brackets on the RHS of equation 38). Accordingly, the output gap targeting that ignores openness understates the relative weights of sectors that face a large expenditure-switching effect and a large total content of domestic labor in the DC index.

Takeaways for relative sectoral weights in output gap targeting. Overall, we find that the output gap targeting policies used in the one-sector SOE literature and disregarding cross-border linkages both implement the PPI targeting —which uses normalized Domar instead of OG weights. The extent to which the PPI targeting over- or under-states the relevance of a sector's inflation depends on the *strengths* of the two major countervailing forces —namely, an overestimation from overstating the sector's total content in domestic output versus an underestimation from ignoring the expenditure-switching channel— for this sector *relative to* other sectors.[31]

5. Welfare loss and optimal monetary policy

In this section, we study the welfare loss function and optimal monetary policy. As in Woodford (2003) and Galí (2015), we derive the closed-form solution of the policy that minimizes welfare losses up to the second-order approximation.

Welfare loss. Under the assumption of non-contingent subsidy and tax rates in Lemma 1, the *flexible-price equilibrium* represents the optimal allocation for the domestic social planner. We define *welfare loss* as the utility gap of the representative household between the *sticky* and *flexible-price equilibria*, $u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi})$, and approximate it to the second order, as stated in the following proposition:

Proposition 4 (Welfare loss). *Given the realized state $\boldsymbol{\xi} \in \Xi$, the welfare loss can be decomposed as:*

$$u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = \underbrace{-\frac{1}{2}\Big(\sigma - 1 + \frac{\varphi + 1}{\Lambda_L}\Big)\widehat{C}^{gap}(\boldsymbol{\xi})^2}_{\text{output-gap misallocaton}} \underbrace{-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top(\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})\widehat{\mathbf{P}}(\boldsymbol{\xi})}_{\text{within- and across-sector, and cross-border misallocations}}, \tag{39}$$

[31] In Appendix F, we study the impacts of introducing input–output linkages into a multi-sector, horizontal SOE versus a multi-sector, horizontal closed economy on normalized sectoral OG weights. We show that the increase in the sectoral normalized OG weights due to the introduction of IO linkages is positively linked to the sector's share of the CPI channel in the OG weight.

where the within-sector, across-sector, and cross-border misallocations are equal to

$$-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^{within}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = -\frac{1}{2}\sum_i \lambda_i \varepsilon_i \frac{1-\delta_i}{\delta_i}\widehat{P}_i(\boldsymbol{\xi})^2, \tag{40}$$

$$\begin{aligned}-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^{across}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = &-\frac{1}{2}\sum_{i=1}^n \beta_i \big[\widehat{C}_i^{gap}(\boldsymbol{\xi}) - \widehat{C}^{gap}(\boldsymbol{\xi})\big]^2 - \frac{1}{2}\sum_{i=1}^n \lambda_i\alpha_i\big[\widehat{L}_i^{gap}(\boldsymbol{\xi}) - \widehat{Y}_i^{gap}(\boldsymbol{\xi})\big]^2 \\ &-\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n \lambda_i\omega_{i,j}\big[\widehat{X}_{i,j}^{gap}(\boldsymbol{\xi}) - \widehat{Y}_i^{gap}(\boldsymbol{\xi})\big]^2,\end{aligned} \tag{41}$$

$$\begin{aligned}-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^{cb}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = &-\frac{1}{2}\sum_{i=1}^n \frac{\beta_i}{\theta_i} v_i(1-v_i)\big[\widehat{C}_{Hi}^{gap}(\boldsymbol{\xi}) - \widehat{C}_{Fi}^{gap}(\boldsymbol{\xi})\big]^2 \\ &-\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n \frac{\lambda_i\omega_{i,j}}{\theta_j} v_{x,i,j}(1-v_{x,i,j})\big[\widehat{X}_{Hi,Hj}^{gap}(\boldsymbol{\xi}) - \widehat{X}_{Hi,Fj}^{gap}(\boldsymbol{\xi})\big]^2 \\ &-\frac{1}{2}\sum_{i=1}^n \frac{\lambda_{EX,i}}{\theta_{F,i}-1}\Big[\frac{\theta_{F,i}-1}{\theta_{F,i}}\widehat{Y}_{EX,i}^{gap}(\boldsymbol{\xi})^2 - \Lambda_L \widehat{L}^{gap}(\boldsymbol{\xi})^2\Big].\end{aligned} \tag{42}$$

Proof: See Appendix M.1.

Equation (39) shows that, to a second-order approximation, *welfare loss* consists of the sum of losses from the output gap misallocation, within- and across-sector misallocation —similar to those in closed economies à la La'O and Tahbaz-Salehi (2022) and Rubbo (2023)— as well as the cross-border misallocation, which is specific to the open economy. Specifically, the within-sector misallocation is the sum of the misallocation arising from inflation dispersion in all sectors, which is similar to its counterpart in one-sector economies. The across-sector misallocation includes those arising from the disproportional sectoral consumption relative to aggregate consumption (first term on the RHS of equation 41), as well as those arising from the disproportional use of sectoral labor and intermediate inputs relative to sectoral output across different sectors i (second and third terms on the RHS of equation 41, respectively). The cross-border misallocation includes distortions arising from the disproportional use of domestic versus foreign goods for both consumption and intermediate inputs (first and second terms on the RHS of equation 42). The cross-border misallocation also includes distortions arising from disproportionate exports relative to the use of domestic labor, which cause domestic producers' monopoly power in international markets to deviate from the optimal level (the third term on the right-hand side of equation 42).

Role of input-output and cross-border linkages in the welfare loss. In what follows, we analyze how input–output and cross-border linkages affect welfare loss using two special cases of our framework: (i) the workhorse model of the one-sector small open economy without IO linkages, as in Galí and Monacelli (2005), and (ii) the multi-sector closed economy without cross-border linkages à la Rubbo (2023).

In the one-sector small open economy, the welfare loss in equation (39) reduces to the sum of the output gap, within-sector misallocation, and cross-border misallocation, the latter two terms being proportional to the square of domestic inflation, as shown in equation (39). In addition, in the one-sector economy, the output gap is proportional to domestic inflation, as can be seen by substituting equation (20) into (21). As a result, the welfare loss is proportional to the square of domestic inflation, allowing the optimal monetary policy to achieve the first-best allocation by fully stabilizing domestic inflation (i.e., the divine coincidence) as in Galí and Monacelli (2005). In multi-sector closed economies with production networks, the welfare loss in equation (39) reduces to the cases in La'O and Tahbaz-Salehi (2022) and Rubbo (2023). In other words, the OG weights $\boldsymbol{\mathcal{M}}_{OG}$ in the output-gap misallocation reduce to the Domar weight, and cross-border misallocation is absent.

Optimal monetary policy. Next, we provide an analytical characterization of the optimal monetary policy.

Definition 5 (Optimal monetary policy). *For any aggregate state $\boldsymbol{\xi} \in \Xi$, the optimal monetary policy sets the money supply $M(\boldsymbol{\xi})$ —which is equivalent to choosing the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ in equilibrium— to minimize the welfare loss in equation (39)* subject to *the sectoral Phillips curves in equation (34).*

Consistent with Definition 5, we derive the aggregate inflation index that the monetary authority should target to implement the optimal monetary policy, as stated in the following proposition:

Proposition 5 (Implementation of the optimal monetary policy). *The optimal monetary policy is implemented by setting the following aggregate inflation index to zero:*

$$\left\{\left[\sigma - 1 + (\varphi + 1)/\Lambda_L\right] \kappa_C^{-1} \boldsymbol{\mathcal{M}}_{OG}^{\top}(\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \boldsymbol{\mathcal{B}}^{\top}(\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})\right\} \widehat{\mathbf{P}} = 0, \tag{43}$$

for any realized state $\boldsymbol{\xi} \in \Xi$.

Proof: See Appendix M.2.

Equation (43) shows that the optimal policy accounts for *both* the output gap misallocation —as evinced by the OG weights $\boldsymbol{\mathcal{M}}_{OG}^{\top}$, as the first term in the brackets— *and* the within- and across-sector, and cross-border misallocation generated by sectoral distortions —as captured by the second term $\boldsymbol{\mathcal{B}}^{\top}(\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})$ in the curly brackets. In contrast, output gap targeting closes the output gap, but it does not simultaneously eliminate the within- and across-sector, and cross-border misallocations, because the sectoral inflation underlying these misallocations is not proportional to the output gap according to sectoral Phillips curves (34). Therefore, the "divine coincidence" —which holds in the workhorse model of one-sector SOEs, as in Galí and Monacelli (2005)— fails to hold in our multi-sector open economies, similar to the case of the multi-sector closed economies in La'O and Tahbaz-Salehi (2022) and Rubbo (2023). However, as we show in Section 6.3, the optimal monetary policy is well approximated by output gap targeting in terms of welfare loss, as in the case of multi-sector closed economies à la Rubbo (2023).

Takeaways for welfare loss and optimal monetary policy. Overall, our analysis demonstrates that treating the economy as a one-sector SOE and as a closed economy with production networks ignores the across-sector and cross-border distortions, respectively, in welfare losses and in the formulation of optimal monetary policy.

6. Quantitative analysis

In this section, we quantify our theoretical results by calibrating the model to the input-output matrices of major economies in the WIOD. Subsection 6.1 studies the slopes of the DC and sectoral Phillips curves in open economies relative to closed economies, focusing on the relevance of different channels for the differences between open-economy and closed-economy slopes. Subsection 6.2 examines the relevance of the different channels for the normalized OG weights and for the differences between normalized Domar and OG weights. It then uses the rule-of-thumb combinations of sectoral relevance metrics to approximate the normalized OG weights and Domar-OG differences, revealing the relevance of cross-border and IO linkages for the normalized OG weights. Subsection 6.3 investigates the welfare of alternative monetary policies, showing that output gap targeting is close to the optimal policy, and enhances welfare over alternative policies that ignore cross-border or input-output linkages.

Our quantitative analysis uses the National Input-Output Tables (NIOTS) for 43 economies (28 EU and 15 OECD countries, each of them comprising 56 sectors) from the WIOD for the year 2014. We calibrate the input-output matrix and import and export shares of each country using its NIOTS sector-level data.[32] Table 2 shows the calibration of the key parameters in our model. Appendix I.1 presents the calibration of the full set of parameters and provides additional details on the WIOD.

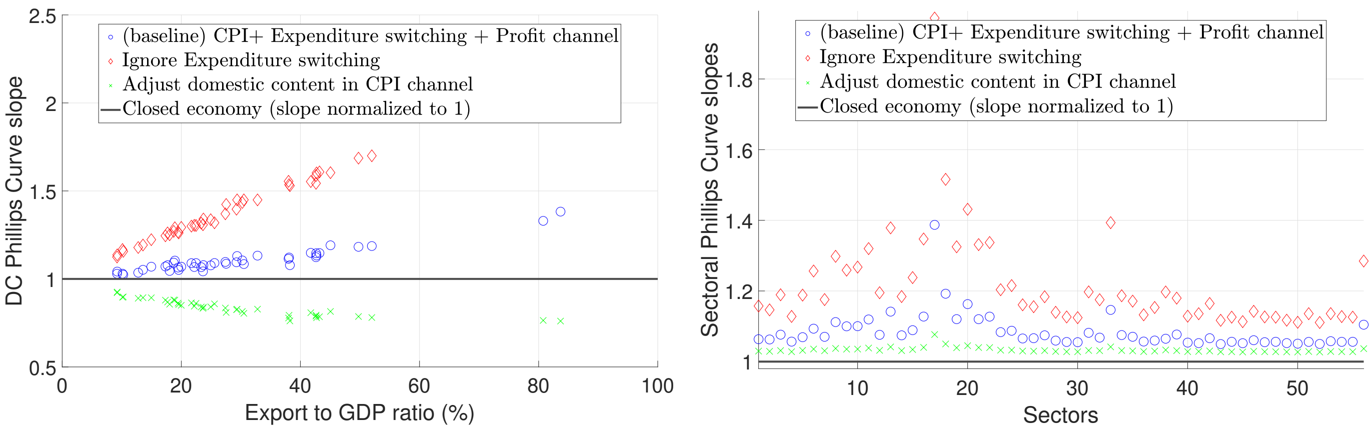
Table 2: Model calibration

Parameters	Data variables/moments used
Common across all countries	
Risk aversion, $\sigma = 2$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Labor supply elasticity, $\varphi = 1$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Elasticity of substitution (EOS) across varieties, $\varepsilon_i = 8$	Atkeson and Burstein (2008)
EOS. btw. domestic and foreign goods, $\theta_i = \theta_{Fi} = 5$	Head and Mayer (2014)
Sector-level frequency of price adjustment, δ_i	Pasten et al. (2024)
Frequency of wage adjustment, δ_0	Beraja et al. (2019) and Barattieri et al. (2014)
Country specific	
Input-output matrix, $\mathbf{\Omega}$	Sectoral gross output, intermediate goods from both domestic and foreign
Home bias for firms' import, $\mathbf{V}_x$	Intermediate goods from both domestic and foreign
Labor share, $\boldsymbol{\alpha}$	Sectoral gross output, labor compensation
Export to foreign countries in steady state, $D^*_{EX,Fi}$	Sectoral exports to foreign countries
Consumer consumption share, $\boldsymbol{\beta}$	Sectoral consumption from both domestic and foreign, and GDP
Consumer consumption home bias, $\boldsymbol{v}$	Sectoral consumption from both domestic and foreign

[32] Data source: https://www.rug.nl/ggdc/valuechain/wiod/?lang=en. The release of the WIOD in 2016 provides information for the period 2000-2014. In our analysis, we use the latest available year, 2014. The NIOTS provides each country's sector-level imports from the Rest of the World (RoW) and exports to the RoW, which are aggregates of the country's imports from and exports to all other countries, respectively, including those countries that are not listed in the WIOD. Using the NIOTS data, we calibrate each country individually as a small open economy relative to the rest of the world, rather than calibrating all countries simultaneously within a global equilibrium.

6.1. *The slopes of Phillips curves in open versus closed economies*

Figure 1: Slopes of Phillips curves (relative to a closed economy)



(a) The DC Phillips curve

(b) Sectoral Phillips curves (Mexico)

Notes: Shown in panel (a) is the ratio of the slope of the DC Phillips curve in open economies to its counterpart in closed economies, for each country in the sample. Shown in panel (b) are the ratios of the slopes of the sectoral Phillips curves in open economies to their counterparts in closed economies, for Mexico. The closed-economy slopes are normalized to one, corresponding to the bold black horizontal line at one. In both panels, the blue circles indicate the baseline open-economy Phillips curve slopes, the red diamonds indicate the counterfactual case that ignores the expenditure-switching channel, and the green crosses indicate the counterfactual slope that replaces the open-economy sectoral content in domestic consumption with its closed-economy counterpart in the CPI channel (i.e., in panel a, change $\widetilde{\boldsymbol{\lambda}}_D$ to $\boldsymbol{\lambda}$; in panel b, change $\boldsymbol{\beta} \odot \boldsymbol{v}$ to $\boldsymbol{\beta} \odot \mathbf{1}$, $\boldsymbol{\beta}^\top \mathbf{v}$ to 1, and $\mathbf{V}_x$ to $\mathbf{I}$ in $\boldsymbol{\Delta}_\Phi$).

Shown in panels (a) and (b) in Figure 1 are the ratios of the slopes of the DC Phillips curve and the sectoral Phillips curves in open economies to their counterparts in closed economies à la Rubbo (2023), for each country in the sample and for Mexico, respectively. In both panels, we consider the following three cases: (i) the baseline open-economy DC and sectoral Phillips curves in equations (33) and (35), respectively (blue circles); (ii) the counterfactual case that ignores the expenditure-switching channel, keeping only the CPI channel (red diamonds); and (iii) the counterfactual case that replaces the open-economy content in domestic consumption with its closed-economy counterpart in the CPI channel —as shown on the RHS of equation (D.4) (green crosses).[33] In both panels, because we plot the ratios of the slopes of the Phillips curves to their counterparts in the closed economy, the slopes of the closed-economy Phillips curves are indicated by the bold black horizontal line at one; a value above (vs. below) one indicates that the Phillips curve is steeper (vs. flatter) in open than in closed economies.

As shown in panels (a) and (b) of Figure 1, the slopes of both the DC and sectoral Phillips curves in open economies are generally *steeper* than those in closed economies —as shown by the blue circles above the black horizontal line of unity— which result from the combination of two countervailing forces. On the one hand, the smaller content of domestic goods in domestic consumption in open than in closed economies

[33]The slopes of the DC and the sectoral Phillips curves for the closed economy and the counterfactual cases (ii) and (iii) are shown in Appendix D.1.

generates a smaller elasticity of the output gap to domestic sectoral inflation and, therefore, *steeper* slopes of the DC and sectoral Phillips curves. This is captured by the blue circles for the baseline open-economy case (i) lying *above* the counterfactual case (iii) of green crosses —where the open-economy content in domestic consumption in the CPI channel is replaced by the closed-economy counterpart. On the other hand, the positive, open-economy-specific channel of expenditure switching increases the elasticity of the output gap to domestic sectoral inflation, thereby *flattening* the slopes of the DC and sectoral Phillips curve in open relative to closed economies. This is shown by the blue circles for the baseline open-economy case (i) lying *below* the counterfactual case (ii) of red diamonds —where the open-economy-specific channel of expenditure switching is absent. Quantitatively, the force of the smaller content of domestic goods in domestic consumption in the open-economy CPI channel dominates the countervailing force that arises from the positive expenditure-switching channel, thus leading to *steeper* slopes of both the DC and sectoral Phillips curves in open economies than in closed economies, consistent with our analyses in Section 4.2.

6.2. Quantifying and approximating normalized OG weights and Domar-OG differences

In this section, we quantify and approximate the normalized OG weights and the differences between normalized Domar and OG weights. The normalized Domar weights correspond to the normalized OG weights in closed economies and coincide with the sectoral weights in the PPI targeting policy used in one-sector SOE literature, as discussed in Section 4.3. We show that the *CPI* and *expenditure-switching channels* explain the bulk of the variation in normalized sectoral OG weights across sectors for all countries, and that the contribution of the *CPI* (vs. *expenditure-switching*) channel decreases (vs. increases) with the openness of the economy (panel a of Figure I.1 in Appendix I.2). In contrast, the contribution of the *profit channel* is marginal. We also show that the pitfalls in the output gap targeting ignoring openness —measured by the differences between the normalized Domar and OG weights— are mostly driven by the *export intensity* and the *expenditure switching* components, and that the contribution of the *export intensity* (vs. *expenditure switching*) components decreases (vs. increases) with the openness of the economy (panel b of Figure I.1).[34]

Then, we use panel regressions to study the rule-of-thumb combinations of the sectoral relevance metrics in Definition 3 to approximate the normalized sectoral OG weights and the difference between the normalized Domar and OG weights. We show that the normalized OG weights and the normalized Domar-OG differences can be approximated by the linear combination of *total content in domestic consumption* and *generalized expenditure-switching elasticity* and the linear combination of *export intensity* and the *ratio of generalized expenditure-switching elasticity to Domar weight*, respectively. We also show that ignoring IO linkages leads to an inaccurate approximation of normalized sectoral OG weights.

We study the combinations of sectoral relevance metrics to approximate the normalized OG weights and

[34] Presented in Appendix I.2 are the details of the method, figures, and results of the variance decomposition analysis.

Domar-OG differences using the following regressions:[35]

$$y_{c,i} = \mathbf{X}_{c,i}^{\top}\boldsymbol{\beta} + \eta_c + \epsilon_{c,i}, \quad \text{with} \quad y_{c,i} \in \{\widetilde{\mathcal{M}}_{OG,c,i}, (\widetilde{\lambda}_{c,i} - \widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}\}, \tag{44}$$

where the dependent variable $y_{c,i}$ is either the level of the normalized OG weight ($\widetilde{\mathcal{M}}_{OG,c,i}$) or the percentage difference between the normalized Domar and OG weights ($(\widetilde{\lambda}_{c,i} - \widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}$) for sector i and country c. The variable $\boldsymbol{X}_{c,i}$ includes our sectoral relevance metrics for the regressions (see Tables 3 and 4), and η_c is the country fixed effect.

Table 3: Sectoral relevance metrics and the normalized OG weights in the data

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Total content in dom. consumption	0.483***				0.460***	0.468***		
	(0.0121)				(0.00663)	(0.0122)		
Import share		-0.000109*						
		(5.79e-05)						
Import Intensity			-0.000431***					
			(4.20e-05)					
Generalized ES elasticity				0.175***	0.139***			
				(0.00856)	(0.00452)			
Total content of dom. labor						0.00309***		
						(0.000627)		
Domar weight							0.409***	
							(0.0226)	
Total content in dom. consumption w/o IO								0.712***
								(0.0333)
Generalized ES elasticity w/o IO								0.546***
								(0.0992)
Observations	601	601	601	601	601	601	601	601
R-squared	0.841	0.005	0.251	0.236	0.988	0.847	0.899	0.658
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Shown in the table are regression results based on equation (44), which regresses the level of the normalized sectoral OG weight $\widetilde{\mathcal{M}}_{OG,c,i}$ over the sectoral relevance metrics characterized by Definition 3 in Section 2.10. Total content in domestic consumption, generalized expenditure-switching (ES) elasticity, total content of domestic labor, and the Domar weight are multiplied by the sectoral price rigidities, $(1-\delta_i)/\delta_i$. The analysis includes the subsample of 11 relatively open economies —in terms of the economy-wise export-to-GDP ratio— out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Approximation of relative sectoral weights in the DC index. Shown in Table 3 are the estimates of the equation (44) with the normalized sectoral OG weight ($\widetilde{\mathcal{M}}_{OG,c,i}$) as the dependent variable; the sectoral relevance metrics on the RHS are multiplied by the sectoral price rigidities ($(1-\delta_i)/\delta_i$) to align with the normalized OG weight on the LHS. As shown in column (1), the price-rigidity-adjusted total content in domestic consumption is positively related to the normalized OG weight of the sector with a coefficient equal to 0.48 and an R-squared of 0.84. As shown in column (4), the price-rigidity-adjusted generalized

[35] We focus on a subsample of 11 relatively open economies —in terms of the economy-wise export-to-GDP ratio— out of all 43 economies. Results are robust, albeit less strong, for less open economies. We do not include sectoral fixed effects in the regression, as our main purpose is to explore the variations in normalized OG weights across different sectors.

expenditure-switching elasticity is positively related to the normalized OG weight of the sector, with a coefficient of 0.175 and a medium-sized R-squared of 0.24.

To validate the negative impact of the import shares on the sectoral total content in domestic consumption and normalized OG weights —as discussed in Section 2.10 and Appendix G— we define the *import intensity* of a sector i as $1 - \widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i}$, where $\widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i}$ captures the domestic demand for i's goods in the baseline economy with imports ($\widetilde{\lambda}_{D,i}$) relative to that without imports ($\widetilde{\lambda}_{All,D,i}$).[36] Accordingly, the import intensity of a sector measures the impact of the entire economy's import shares on the domestic demand for this sector's goods. As shown in column (3), a sector's normalized OG weight significantly decreases with the import intensity, thereby validating the negative impact of the direct and indirect import shares of a sector on its normalized OG weight.

As shown in column (5), the linear combination of the price-rigidity-adjusted total content in domestic consumption and generalized expenditure-switching elasticity provides a precise approximation of the normalized sectoral OG weights, with a large R-squared of 0.99 that is significantly higher than those with each of the two sectoral metrics in columns (1) and (4). As shown in column (6), the total content of domestic labor is significantly and positively related to the normalized OG weight, but adding it to the total content in domestic consumption does not improve the approximation accuracy —as shown by the R-squared of 0.847 that is very close to the 0.841 in column (1). Comparing columns (6) and (5) shows that the expenditure switching effect is more important than the domestic labor content for the generalized expenditure-switching elasticity to approximate the normalized OG weights.

As shown in column (7), the normalized Domar weight —which is the nearly optimal normalized OG weight in closed economies à la Rubbo (2023) and La'O and Tahbaz-Salehi (2022)— has a weaker explanatory power than the linear combination of the price-rigidity-adjusted total content in domestic consumption and generalized expenditure-switching elasticity in column (5), with a smaller R-squared of 0.90. The comparison between columns (5) and (7) illustrates the importance of considering openness in approximating the normalized sectoral OG weights in the DC index.

The results in Table 3 imply that the output gap targeting should assign larger weights to sectors that supply more inputs directly or indirectly (i.e., via the downstream sectors) to domestic consumption and that face a large expenditure-switching effect.

Approximation of pitfalls in the output gap targeting abstracting from openness. Presented in Table 4 are the results for the version of the regression in equation (44) with the percentage difference between the normalized Domar and OG weights ($(\widetilde{\lambda}_{c,i} - \widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}$) as the dependent variable, which measures the pitfalls in the policy of output gap targeting that ignores openness, that is, the PPI targeting policy. As

[36] The term $\widetilde{\lambda}_{All,D,i}$ is the i-th entry of the vector $\boldsymbol{\beta}^{\top}(\mathbf{I} - \boldsymbol{\Omega})^{-1}$ and captures the domestic demand that reaches the domestic sector i directly and indirectly (via downstream sectors) *if* the entire economy —including sector i and its downstream sectors— does not import from abroad (i.e., $v_r = 1$ for all r and $v_{x,r,s} = 1$ for all r and s).

Table 4: Sectoral relevance metrics and the difference between normalized Domar and OG weights

	(1)	(2)	(3)	(4)
Export Intensity	0.387***		0.878***	0.127***
	(0.0244)		(0.00863)	(0.0247)
Generalized ES elasticity over Domar		-0.0318***	-0.255***	
		(0.00813)	(0.00380)	
Total content of domestic labor				-0.730***
				(0.0501)
Observations	601	601	601	601
R-squared	0.366	0.043	0.939	0.570
Country FE	Yes	Yes	Yes	Yes

Notes: Shown in the table are regression results based on equation (44), which regresses the normalized sectoral Domar-OG percentage difference $(\widetilde{\lambda}_{c,i} - \widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}$ over the sectoral relevance metrics characterized by Definition 3 in Section 2.10. The generalized ES elasticity over Domar is the ratio of the sectoral generalized expenditure-switching elasticity to the Domar weight. The analysis includes the subsample of 11 relatively open economies —in terms of the economy-wise export-to-GDP ratio— out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

shown in column (1), the export intensity is positively related to the normalized Domar-OG difference with a coefficient of 0.39 and a medium-sized R-squared of 0.37. As shown in column (2), the ratio of the generalized expenditure-switching elasticity to the Domar weight is negatively related to the normalized Domar-OG difference with a coefficient of -0.03 and a small R-squared of 0.04.

Despite the limited explanatory power of each individual component shown in columns (1) and (2), column (3) shows that the linear combination of the export intensity and the ratio of the generalized expenditure-switching elasticity to the Domar weight explains most of the variations in the normalized Domar-OG difference, as evinced by the large R-squared of 0.94. This R-squared is significantly larger than those in columns (1) and (2), revealing the complementarity between the two components in approximating the normalized Domar-OG difference. As shown in column (4), adding the total content of domestic labor to the export intensity only partially improves the approximation accuracy —as evinced by the R-squared of 0.57, which is larger than 0.37 in column (1) but much smaller than 0.94 in column (3). This implies that the expenditure-switching mechanism in the generalized expenditure-switching elasticity plays an important role in explaining the normalized Domar-OG difference.

The results in Table 4 support Proposition 3 and show that the sector-level pitfalls in the output gap targeting that abstract from cross-border linkages can be well approximated by the rule-of-thumb, linear combination of the export intensity and the ratio of the generalized expenditure-switching elasticity to the Domar weight. These results demonstrate that monetary policy adopting normalized Domar weights, or the policy that targets PPI inflation in the one-sector SOE literature, overstates inflation in sectors with either large export intensity or limited generalized expenditure-switching elasticity. To account for the openness of the economy in the closed-economy PPI targeting policy, the monetary authority should assign smaller weights to sectors that export intensively directly and indirectly, and assign larger weights to sectors that face large expenditure-switching effects.

Relevance of input-output linkages for relative sectoral weights in the DC index. Our theoretical analysis in Section 2.10 shows that IO linkages are important drivers of the sectoral relevance metrics that underpin the sectoral OG weights. In an SOE without production networks, domestic sectors are only direct, rather than indirect, suppliers to domestic and foreign demand, so import intensity simplifies to the import share.

Comparing columns (2) and (3) in Table 3 shows that import intensity explains more variation in normalized OG weights than the import share, as evinced by the larger R-squared for import intensity (0.25) than the almost negligible R-squared for import share (0.01). Moreover, as shown in column (8) in Table 3, the counterfactual sectoral metrics of total content in domestic consumption and generalized expenditure-switching elasticity that abstract from IO linkages explain only 66% of the variation in the normalized sectoral OG weights, which is much smaller than that considering IO linkages in column (5).[37] Thus, indirect imports via both upstream and downstream sectors —as captured by sectoral relevance metrics with IO linkages rather than without— are important for the approximation of normalized OG weights, hence supporting the relevance of input-output linkages for monetary policy.

We conclude that the structure of input-output linkages interplays with the cross-border linkages of the small open economy to determine the metrics of sectoral relevance in Definition 3 and, in turn, the relative sectoral weights in the DC index for output gap targeting. Ignoring production networks results in an imprecise approximation of the correct sectoral weights required to close the output gap.

6.3. *Welfare comparison of alternative monetary policies*

In this section, we quantitatively compare the welfare losses of the economy —using equation (39) in Proposition 4 of Section 5— under alternative monetary policies, and show that output gap targeting performs close to the optimal monetary policy, as in closed economies à la Rubbo (2023), and outperforms policies that ignore either cross-border or input-output linkages.[38]

Specifically, we compare the welfare loss under the following five alternative monetary policies: the optimal policy, output gap targeting, PPI targeting, output gap targeting without IO linkages, and CPI tar-

[37] In multi-sector small open economies without IO linkages, the Leontief inverse reduces to a diagonal matrix, with $l_{vx,r,i} = 1$ for $r = i$ and $l_{vx,r,i} = 0$ for $r \neq i$, and sectoral labor cost shares reduce to $\alpha_i = 1$. Thus, the total content in domestic consumption and the generalized expenditure-switching elasticity reduce to $\widetilde{\lambda}_{D,i} = \beta_i v_i$ and $\widetilde{\rho}_{ES,i} = (\theta_{F,i} - 1)\lambda_{EX,i} + (\theta_i - 1)\,\beta_i v_i\,(1 - v_i)$, respectively.

[38] The welfare loss represents the expected welfare loss in the remaining part of Subsection 6.3. For each case, we compute welfare losses under different monetary policies using the same simulations of log-normal shocks to the import prices of all sectors. For simplicity, we assume that the shocks to different sectors have the same mean. We set the mean of sectoral shocks to generate an average CPI inflation of 2% for each economy to compare —under the same aggregate level of inflation— the welfare losses across different economies with different openness and structures of input-output linkages. The variance-covariance matrix of these shocks is calibrated on Mexico. We simulate the shocks 100,000 times to compute the expected welfare loss under each of the alternative monetary policies. In Appendix I.4, we show that our results are similar under shocks to import prices in the manufacturing sectors and under aggregate productivity shocks.

geting. The policy of PPI (vs. CPI) targeting targets an aggregate inflation index where the Domar weight λ_i (vs. consumption share β_i) —after adjusting for sectoral price rigidities (i.e., multiply by $(1-\delta_i)/\delta_i$) and normalized by the sum of sectoral weights— is used as the weight for each sector i's inflation. The output gap targeting without IO linkages weights sectoral inflation with the normalized OG weights that ignore IO linkages.[39] We study the PPI and CPI targeting because they are both widely used policies. While the PPI targeting ignores the openness of the economy and coincides with output gap targeting used in closed economies à la Rubbo (2023) and in one-sector small open economy literature, the CPI targeting ignores both openness and IO linkages. In addition, we examine the output gap targeting without IO linkages to evaluate the relevance of input-output linkages for the welfare implications of monetary policy.

Table 5: Welfare loss under different monetary policies

	(1) Optimal	(2) Output gap targeting	(3) PPI targeting	(4) Output gap targeting w/o IO	(5) CPI targeting
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-1.859	-1.879	-1.922	-4.948	-4.968
Improvement by OG targeting towards optimal			67.1%	99.3%	99.3%
Output gap misallocation	-0.003	0.000	-0.002	-0.385	-0.388
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.024	0.000	-0.041	-2.684	-2.701
— policy-irrelevant	-1.879	-1.879	-1.879	-1.879	-1.879
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-7.742	-7.777	-8.504	-11.551	-10.675
Improvement by OG targeting towards optimal			95.4%	99.1%	98.8%
Output gap misallocation	-0.006	0.000	-0.089	-0.569	-0.427
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.041	0.000	-0.638	-3.205	-2.471
— policy-irrelevant	-7.777	-7.777	-7.777	-7.777	-7.777
U.S. Export-to-GDP ratio: 9%					
Total welfare loss	-1.400	-1.472	-1.476	-6.546	-6.757
Improvement by OG targeting towards optimal			5.4%	98.6%	98.6%
Output gap misallocation	-0.011	0.000	0.000	-0.596	-0.623
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.083	0.000	-0.004	-4.478	-4.662
— policy-irrelevant	-1.472	-1.472	-1.472	-1.472	-1.472

Notes: Shown in the table is the welfare loss —expressed in units of percent of steady-state consumption— under different monetary policy designs. Shown in columns (1) to (5) are the welfare losses under the optimal policy, output gap targeting, PPI targeting, output gap targeting without IO linkages, and CPI targeting policy, respectively. Outlined in Appendix I.3 are the relative sectoral weights of sectoral inflation adopted by the alternative monetary policies.

Presented in Table 5 is the welfare loss expressed as a percentage of the steady-state consumption under the alternative monetary policies. We consider the welfare loss for Mexico, Luxembourg, and the US,

[39] Specifically, output gap targeting without IO linkages targets the alternative DC inflation index that replaces the $\mathcal{M}_{OG,i}$ in the normalized OG weights —including the normalizer— in equation (31) with $\beta_i v_i + \kappa_{CPI}^{-1}[(\theta_{F,i}-1)\lambda_{EX,i} + (\theta_i - 1)\,\beta_i v_i\,(1-v_i)]$.

as these countries represent those with medium, large, and small degrees of openness, respectively —as measured by the economy-wise export-to-GDP ratio (19%, 83%, and 9%, respectively). Using equation (39), we decompose the welfare loss into the *output gap misallocation* and the *within- and across-sector, and cross-border misallocation*. We further use equation (H.1) to decompose the within- and across-sector, and cross-border misallocation into two sub-components: (i) an output-gap-related term and (ii) a policy-irrelevant term.

As shown in Table 5, output gap targeting yields a welfare loss that is close to the optimal policy and significantly outperforms the PPI targeting (column 3), output gap targeting without IO linkages (column 4), and CPI targeting (column 5) —which ignore cross-border linkages, IO linkages, and *both* cross-border *and* IO linkages, respectively. For Mexico, the difference in the welfare loss between the optimal policy and output gap targeting is very small and equal to 0.020 percent of the steady-state consumption (-1.859 vs. -1.879), thereby establishing that output gap targeting is nearly optimal. Important to our analysis, output gap targeting improves the welfare loss over the PPI targeting by 0.043 percent of the steady-state consumption, and it generates an even larger improvement over output gap targeting that ignores IO linkages and the CPI targeting (-1.879 vs. -1.922 vs. -4.948 vs. -4.968). The welfare improvement of output gap targeting over the PPI targeting (vs. output gap targeting without IO linkages) corresponds to 67.1% (vs. 99.3%) of the welfare difference between the optimal and the PPI targeting policy (vs. output gap targeting without IO linkages), thereby exhibiting welfare enhancement if the design of monetary policy accounts for openness and IO linkages of the economy. The welfare improvement of output gap targeting over the PPI targeting —which is the policy used in one-sector SOE literature— also shows the importance of considering input-output linkages in designing monetary policies in SOEs.

Decomposing the total welfare loss into different components illustrates why output gap targeting is close to the optimal policy and improves over policies that ignore cross-border and input-output linkages. Output gap targeting eliminates the welfare losses arising from the output gap misallocation and from the output-gap-related components in the within- and across-sector, and cross-border misallocation. Quantitatively, Table 5 shows that these two components related to the output gap generate large welfare losses in Mexico for the PPI targeting (-0.002 and -0.041), and even larger losses for output gap targeting without IO linkages (-0.385 and -2.684) and the CPI targeting (-0.388 and -2.701). These results support the adoption of output gap targeting that considers both the cross-border and input-output linkages to enhance welfare in small open economies.

Finally, we examine the welfare loss under alternative monetary policies for two additional economies, namely Luxembourg and the US, which represent the polar cases of open and closed economies, respectively. In the most open economy of Luxembourg (the middle panel of Table 5), output gap targeting improves over the PPI targeting by a large 95.4%, compared to a more limited 67.1% for Mexico. The same qualitative results outlined for Mexico hold for Luxembourg and are stronger quantitatively. The bottom panel of Table 5 presents the welfare loss for the nearly closed economy of the US, showing that the output gap and PPI targeting yield similar welfare loss and are equally close to the optimal policy, echoing the results of La'O and Tahbaz-Salehi (2022) and Rubbo (2023) in closed economies. Therefore, we conclude that the difference between the output gap and PPI targeting is significant for open economies, but its importance diminishes in relatively closed economies like the US.[40]

7. Conclusion

This paper investigates the design of monetary policy in small open economies with cross-border and input-output linkages and nominal rigidities. The output gap can be expressed as a weighted average of sectoral markup wedges that encapsulate the inefficiency in each sector, with each sector's weight represented by the sectoral OG weight. We derive the divine coincidence Phillips curve, which links the aggregate inflation of the divine coincidence index to the output gap and allows for the simultaneous stabilization of inflation and the output gap. The relative sectoral OG weights determine the share of each sector's inflation in the DC index, and the sum of the sectoral OG weights determines the slope of the DC Phillips curve. We show that both the DC and sectoral Phillips curves are steeper in open than in closed economies.

The DC Phillips curve implies that the monetary policy of output gap targeting can be implemented by targeting the DC index to zero. The relative sectoral weights in the DC index for output-gap targeting, in turn, depend on the sector's relevance as a supplier of inputs to both domestic and foreign demand, and as a customer of domestic labor within international production networks. We show that output gap targeting should assign larger weights to inflation in sectors that supply more inputs directly or indirectly (i.e., via the downstream sectors) to domestic consumption and face larger expenditure-switching effects. Disregarding openness or treating the economy as a one-sector SOE overstates inflation in sectors that export intensively directly and indirectly, and understates inflation in sectors that face larger expenditure-switching effects.

We derive the closed-form solution for the optimal monetary policy that minimizes the welfare losses, as well as calibrate our model to the WIOD to quantify our theoretical results. We show that the policy of output gap targeting generates welfare losses quantitatively close to the optimal policy as in closed economies, and outperforms alternative monetary policies of the PPI targeting that abstract from cross-border linkages or the output gap targeting that ignores input-output linkages. Overall, our analysis demonstrates that cross-border and input-output linkages are important for the conduct of monetary policy in small open economies with international production networks.

Our study suggests several interesting extensions for future research. First, one could relax the assumption of financial autarky and study the interplay between the incompleteness of the financial market and the

[40]In Appendix I.4, we examine the welfare losses under alternative monetary policies that are optimal when considering and ignoring IO linkages, respectively, in both multi-sector SOEs and closed economies. As shown in Table I.4, the counterpart optimal monetary policies for the multi-sector economies without IO linkages lead to a larger welfare loss than the baseline optimal monetary policies that consider IO linkages. This additional welfare loss of the optimal monetary policy from ignoring IO linkages is larger in open than in closed economies, especially for countries with a large degree of openness, illustrating that IO and cross-border linkages interplay to determine the welfare losses associated with monetary policies.

production networks for the design of monetary policy. Second, the analysis could consider the cases in which fiscal policy fails to offset the first-order distortions with non-contingent subsidies. Such contexts lead to a sub-optimal flexible-price equilibrium for the domestic social planner, as in Baqaee and Farhi (2024), such that the monetary policy needs to account for the interaction between the supply-side effect of monetary policy and the openness of the economy to improve efficiency. Third, the subsequent efforts might explore large open economies where monetary policy would need to account for feedback effects from the responses of foreign economies to the domestic policy —which may interplay with international product networks to determine the impact of the domestic monetary policy. Finally, the analysis could be extended to models incorporating endogenous adjustments in domestic and cross-border input-output linkages, as in Xu et al. (2025). We plan to investigate some of these issues in future work.

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Online Appendix

Monetary Policy in Open Economies with Production Networks

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A. Firm profit, sectoral goods packer, and Calvo pricing

Given the firm's price P_{if} and the sectoral tax (or subsidy if negative) rate τ_i on sales, the nominal profit of firm f in sector i equals:

$$\Pi_{if} = (1 - \tau_i) P_{if} Y_{if} - \Phi_i \cdot Y_{if}. \tag{A.1}$$

Sectoral goods packers. In each sector i, the perfectly competitive and identical sectoral goods packers transform the differentiated goods produced by the monopolistically competitive firms into a sectoral product using the following constant-elasticity-of-substitution technology:

$$Y_i = \left(\int_0^1 Y_{if}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}}, \tag{A.2}$$

where the within-sector elasticity of substitution between different firms' products is $\varepsilon_i > 1$. The cost minimization of the goods packers yields the following sectoral price index and demand function for the firms:

$$P_i = \left(\int_0^1 P_{if}^{1-\varepsilon_i} df \right)^{\frac{1}{1-\varepsilon_i}} \quad \text{and} \quad Y_{if} = \left(\frac{P_{if}}{P_i} \right)^{-\varepsilon_i} Y_i. \tag{A.3}$$

Nominal rigidity and sectoral markup wedges. Denote by $P_i^{\#}$ the price that maximizes the firm's profit in equation (A.1) subject to the demand function in equation (A.3), which is equal to the following:

$$P_i^{\#} = \frac{1}{1-\tau_i} \frac{\varepsilon_i}{\varepsilon_i - 1} \Phi_i \equiv \mu_i^{\#} \cdot \Phi_i, \tag{A.4}$$

where $\mu_i^{\#}$ denotes the desired sectoral (gross) markup. Nominal rigidity is modeled as static Calvo-pricing friction, in which only firms indexed by $f \leq \delta_i \in [0, 1]$ are allowed to choose their desired price $P_i^{\#}$ and the remaining firms maintain their price at the steady-state level. We refer to $(1 - \delta_i)/\delta_i$ as the price rigidity of sector i. The sectoral markup $\mu_i \equiv P_i/\Phi_i$ differs from the desired markup $\mu_i^{\#}$ if the price rigidity of sector i is strictly positive, *viz.*, $(1 - \delta_i)/\delta_i > 0$. We define the *sectoral markup wedge* for domestic sector i as the log deviation of the sectoral markup from the desired markup, *viz*, $\ln(\mu_i) - \ln(\mu_i^{\#})$.

B. Additional results on the relation between sectoral markup wedges and the output gap

B.1. Aggregate wedges and output gap

We follow the approach in Chari et al. (2007) to define the efficiency and labor wedges in the multi-sector, small open economy.

Definition B.1 (Aggregate wedges)**.** *The two wedges $A_{agg} : \Xi \mapsto \mathbb{R}_+$ and $\Gamma_L : \Xi \mapsto \mathbb{R}_+$ allow equilibrium aggregate consumption and labor inputs to satisfy the following equations:*[1]

$$C(\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) L(\boldsymbol{\xi})^{\Lambda_L^{flex}(\boldsymbol{\xi})}, \quad \forall \boldsymbol{\xi} \in \Xi, \tag{B.1}$$

$$\frac{u_L(C(\boldsymbol{\xi}), L(\boldsymbol{\xi}))}{-u_C(C(\boldsymbol{\xi}), L(\boldsymbol{\xi}))} = \Gamma_L(\boldsymbol{\xi}) \frac{\partial C}{\partial L}(\boldsymbol{\xi}), \quad \forall \boldsymbol{\xi} \in \Xi, \tag{B.2}$$

in the economy. We refer to $A_{agg}(\boldsymbol{\xi})$ as the efficiency wedge, or aggregate TFP, and $\Gamma_L(\boldsymbol{\xi})$ as the labor wedge, respectively, for any realized state $\boldsymbol{\xi} \in \Xi$.

The equilibrium of the economy is summarized by the aggregate production function in equation (B.1) and the intratemporal condition between aggregate consumption and labor supply in equation (B.2). The aggregate production function describes the transformation of labor inputs into aggregate consumption, where the transformation ratio equals the economy-wide share of domestic labor inputs in aggregate consumption expenditure in the flexible-price equilibrium ($\Lambda_L^{flex}(\boldsymbol{\xi})$). In our open economy, domestic consumption comprises foreign goods, which are supplied through exports of domestically produced goods in exchange for imports of foreign products. Because the marginal revenue of export strictly decreases with the export quantity and its use of domestic labor inputs, domestic labor supplies foreign products (through exports) in a *decreasing-return-to-scale* way, leading to a lower-than-unitary transformation ratio in the open economy. In contrast, the transformation ratio is equal to one in a closed economy —as in Bigio and La'O (2020)— because all domestic consumption uses domestic products instead of imported foreign goods that are exchanged using exports. The efficiency wedge $A_{agg}(\boldsymbol{\xi})$ captures the shifts in the aggregate production function or the aggregate TFP.

The intratemporal condition in equation (B.2) relates the marginal product of labor for aggregate output (i.e., $\partial C/\partial L$) to the marginal rate of substitution between consumption and labor (i.e., $-u_L/u_C$), and the labor wedge $\Gamma_L(\boldsymbol{\xi})$ encapsulates the distortions that make the marginal product of labor different from the marginal rate of substitution.

Based on the definition of the efficiency wedge in Definition B.1, we establish the following open-economy version of Hulten's theorem:[2]

Lemma B.1 (The open-economy version of Hulten's theorem)**.** *Up to a first-order approximation, the de-*

[1] As shown in Appendix K.2, the marginal product of labor is $(\partial C/\partial L)(\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \Lambda_L^{flex} L(\boldsymbol{\xi})^{\Lambda_L^{flex}(\boldsymbol{\xi})-1}$.

[2] In closed economies with production networks, Bigio and La'O (2020) define a prototype economy and the corresponding efficiency and labor wedges. They also show that Hulten's theorem holds and that sectoral distortions have no first-order effect on the efficiency wedge. While Baqaee and Farhi (2024) decompose the real GDP of an open economy into the efficiency wedge and the labor wedge in an inter-connected global production network, our Lemma B.1 shows the decomposition for small open economies under the assumption of nominal rigidities.

viation of the efficiency wedge from the steady state is a weighted average of sectoral shocks as follows:

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}(\boldsymbol{\xi}) \tag{B.3}$$
$$= \boldsymbol{\lambda}^\top \widehat{\mathbf{A}} - \Big\{ \underbrace{[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top}_{\textit{imported consumption}} + \underbrace{\boldsymbol{\lambda}^\top (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})}_{\textit{imported interm. inputs}} \Big\} \widehat{\mathbf{P}}^*_{IM,F} + \underbrace{[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top}_{\textit{profits from exports}} \widehat{\mathbf{D}}^*_{EX,F}.$$

Proof: See Appendix K.1.

Equation (B.3) shows that deviation of the efficiency wedge from the steady state is linked to the deviations of exogenous sectoral productivity ($\widehat{\mathbf{A}}$), import prices ($\widehat{\mathbf{P}}^*_{IM,F}$), and foreign demand ($\widehat{\mathbf{D}}^*_{EX,F}$) from the steady state. The elasticity of the efficiency wedge to the sectoral productivity is the Domar weight of the sector ($\boldsymbol{\lambda}$), as in a closed economy (Hulten, 1978; Bigio and La'O, 2020). In an open economy, however, the elasticities of the efficiency wedge to import prices and foreign demand depend on the linkages between the domestic and foreign economies. The elasticity of the efficiency wedge to a sector's import price shock equals the share of the sector's imports of consumption goods and intermediate inputs in aggregate output. Such elasticity is negative, as imported inflation materializes as a negative supply shock. The elasticity of the efficiency wedge to a shock to the sector's foreign demand equals the share of the sector's profits from exports in aggregate output. Such elasticity is positive because an increase in the foreign demand for domestic goods raises export profits, which increases domestic income and consumption for a given amount of domestic labor.

Lemma B.1 implies that —similar to the closed economy case— sectoral distortions have no first-order impact on the efficiency wedge in a small open economy with production networks. Therefore, the labor wedge encapsulates sectoral distortions entirely, as stated in the following proposition:

Proposition B.1 (Sectoral distortion, efficiency and labor wedges, and the output gap)**.** *Up to the first-order approximation, the efficiency wedge in the sticky-price equilibrium is the same as the efficiency wedge in the flexible-price equilibrium:*

$$\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}^{flex}_{agg}(\boldsymbol{\xi}) = \widehat{C}^{gap}(\boldsymbol{\xi}) - \Lambda_L \cdot \widehat{L}^{gap}(\boldsymbol{\xi}) = 0. \tag{B.4}$$

The labor wedge, though, deviates from the flexible-price level, and the deviation is proportional to the output gap:[3]

$$\widehat{\Gamma}_L(\boldsymbol{\xi}) - \widehat{\Gamma}^{flex}_L(\boldsymbol{\xi}) = \widehat{\Gamma}_L(\boldsymbol{\xi}) = [\sigma - 1 + (\varphi + 1)/\Lambda_L]\, \widehat{C}^{gap}(\boldsymbol{\xi}). \tag{B.5}$$

[3]The deviation of the labor wedge from the flexible-price equilibrium equals the deviation of the labor wedge from the steady state. This is because the labor wedge equals one in the flexible-price equilibrium for any realized state $\boldsymbol{\xi}$, including the steady state.

Proof: See Appendix K.2.

Proposition B.1 shows that up to the first-order approximation, the efficiency wedge is unaffected by sectoral distortions, but that the labor wedge is different from zero and it summarizes the distortions at the aggregate level. In particular, the deviation of the labor wedge from the flexible-price equilibrium is proportional to the output gap.

B.2. The DC Phillips curve and DC index in dynamic, complete markets

Sectoral OG weight in complete markets. In a dynamic environment with the standard assumption of a *complete (asset) market* à la Galí and Monacelli (2005) —such that the Backus-Smith and uncovered interest rate conditions hold— the consumption gap (i.e., $\widehat{C}^{gap}$) satisfies the international risk-sharing condition à la Galí and Monacelli (2005) and Corsetti et al. (2010) in gap forms as follows:[4]

$$\sigma\widehat{C}^{gap} = \widehat{S}^{gap} - \widehat{P}_C^{gap}, \tag{B.6}$$

rather than log-linearized open-economy budget constraint in equation (27). The labor supply condition still holds, *viz.*:

$$\sigma\widehat{C}^{gap} + \varphi\widehat{L}^{gap} = \widehat{W}^{gap} - \widehat{P}_C^{gap}, \tag{B.7}$$

where the employment gap $\widehat{L}^{gap}$ satisfies the following equation derived from the market clearing condition (K.25):[5]

$$\begin{aligned}\boldsymbol{\lambda}^\top\boldsymbol{\alpha}\widehat{L}^{gap} = {} & \widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}(\widehat{P}_C^{gap} - \widehat{W}^{gap} + \widehat{C}^{gap}) \\ & - (\boldsymbol{\lambda}\odot\widetilde{\boldsymbol{\alpha}})^\top\widehat{\boldsymbol{\mu}} + \widetilde{\boldsymbol{\lambda}}_F^\top\boldsymbol{\alpha}(\widehat{S}^{gap} - \widehat{W}^{gap}) - (\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}})^\top(\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap}),\end{aligned} \tag{B.8}$$

Combining equations (B.6), (B.7), and (B.8) with pricing equations (28) and (29) yields:

$$\widehat{C}^{gap} = \frac{\varphi(\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\left[(\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\widetilde{\boldsymbol{\lambda}}_D + \widetilde{\boldsymbol{\rho}}_{ES} + \boldsymbol{\lambda}\odot\widetilde{\boldsymbol{\alpha}}\right]^\top - \left\{\widetilde{\boldsymbol{\lambda}}^\top\boldsymbol{\alpha} + \varphi\left[(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\widetilde{\boldsymbol{\lambda}}_D^\top + \widetilde{\boldsymbol{\rho}}_{ES}^\top + \widetilde{\boldsymbol{\lambda}}_F^\top\right]\boldsymbol{\alpha}\right\}\widetilde{\boldsymbol{\lambda}}_D^\top}{\sigma\left(\boldsymbol{\lambda}^\top\boldsymbol{\alpha}\right) + \varphi\left(\boldsymbol{\lambda}_D^\top\boldsymbol{\alpha}\right)^2 + \sigma\varphi\left[(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\widetilde{\boldsymbol{\lambda}}_D^\top + \widetilde{\boldsymbol{\rho}}_{ES}^\top + \widetilde{\boldsymbol{\lambda}}_F^\top\right]\boldsymbol{\alpha}}\widehat{\boldsymbol{\mu}}, \tag{B.9}$$

$$\widehat{L}^{gap} = -\frac{(\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\widetilde{\boldsymbol{\lambda}}_D^\top + \sigma\left[(\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\widetilde{\boldsymbol{\lambda}}_D + \widetilde{\boldsymbol{\rho}}_{ES} + \boldsymbol{\lambda}\odot\widetilde{\boldsymbol{\alpha}}\right]^\top}{\sigma\left(\boldsymbol{\lambda}^\top\boldsymbol{\alpha}\right) + \varphi\left(\boldsymbol{\lambda}_D^\top\boldsymbol{\alpha}\right)^2 + \sigma\varphi\left[(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\widetilde{\boldsymbol{\lambda}}_D^\top + \widetilde{\boldsymbol{\rho}}_{ES}^\top + \widetilde{\boldsymbol{\lambda}}_F^\top\right]\boldsymbol{\alpha}}\widehat{\boldsymbol{\mu}}. \tag{B.10}$$

[4] We use the international risk-sharing condition $C^{-\sigma}/P_C = (C^*)^{-\sigma}/(S\,P_C^*)$ à la Galí and Monacelli (2005) —where, for the SOE in our model, the aggregate consumption (C^*) and the CPI (P_C^*) of the rest of the world are exogenous.

[5] Condition (B.8) is derived by combining condition (K.25) with $\widehat{P}_i^{gap} + \widehat{Y}_i^{gap} = \widehat{W}^{gap} + \widehat{L}_i^{gap} + \widehat{\mu}_i$, and then multiplying both sides of the equation by $\boldsymbol{\alpha}$.

Equations (B.9) and (B.10) show that in complete markets without balanced trade, the consumption gap is different from the employment gap, which is in sharp contrast to our baseline case of balanced trade — where the consumption gap is equivalent to the output gap and proportional to the employment gap (equation B.4 in Proposition B.1).

Because the employment gap better captures the output gap than the consumption gap, and is more closely associated with the monetary policy design, we focus on the employment gap rather than the consumption gap, and derive the relation of the employment gap to sectoral markup wedges in complete markets as follows:

$$\kappa_{L,cp} \cdot \widehat{L}_t^{gap} = -\sum_{i=1}^{N} \mathcal{M}_{OG,cp,i} \cdot \widehat{\mu}_{i,t}, \tag{B.11}$$

$$\text{where} \quad \boldsymbol{\mathcal{M}}_{OG,cp} \equiv \underbrace{\widetilde{\boldsymbol{\lambda}}_D}_{\text{CPI channel}} + \underbrace{\kappa_{CPI,cp}^{-1} \cdot \widetilde{\boldsymbol{\rho}}_{ES}}_{\text{expenditure-switching channel}} + \underbrace{\kappa_{CPI,cp}^{-1} \cdot \left[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right]}_{\text{profit channel}}, \tag{B.12}$$

$$\kappa_{CPI,cp} \equiv (1 + \sigma^{-1})\widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha} + 1, \tag{B.13}$$

$$\kappa_{L,cp} \equiv \left\{\boldsymbol{\lambda}^{\top}\boldsymbol{\alpha} + (\varphi/\sigma)\,(\boldsymbol{\lambda}_D^{\top}\boldsymbol{\alpha})^2 + \varphi\left[(1 - \widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha})\widetilde{\boldsymbol{\lambda}}_D^{\top} + \widetilde{\boldsymbol{\rho}}_{ES}^{\top} + \widetilde{\boldsymbol{\lambda}}_F^{\top}\right]\boldsymbol{\alpha}\right\} \big/ \kappa_{CPI,cp}.$$

With a slight abuse of notation, we call the elasticity of the employment gap to sectoral markup wedges the sectoral OG weight —denoted by $\mathcal{M}_{OG,cp,i}$. Equation (B.11) is the counterpart of equation (21) in Theorem (1).

The DC Phillips curve and DC index in complete markets. Rubbo (2023) (in equation 21) shows that in a dynamic environment, the sectoral markup wedges are linked to sectoral inflation in current and next periods as follows:

$$\pi_{i,t} = -\frac{\widetilde{\delta}_i}{1 - \widetilde{\delta}_i}\widehat{\mu}_{i,t} + \beta \mathbb{E}_t \pi_{i,t+1}, \tag{B.14}$$

where β is the discount factor, and

$$\widetilde{\delta}_i \equiv \frac{\delta_i[1 - \beta(1 - \delta_i)]}{1 - \beta\delta_i(1 - \delta_i)}. \tag{B.15}$$

Combining equations (B.14) with (B.11), yields the following DC Phillips curve in dynamic, complete markets that links the employment gap to domestic inflation:

$$\pi_{DC,cp,t} = \frac{\kappa_{L,cp}}{\kappa_{OG,cp}}\widehat{L}_t^{gap} + \beta \mathbb{E}_t \pi_{DC,cp,t+1}, \tag{B.16}$$

$$\text{where} \quad \kappa_{OG,cp} \equiv \sum_{i=1}^{N} \mathcal{M}_{OG,cp,i} \cdot (1 - \widetilde{\delta}_i)/\widetilde{\delta}_i, \tag{B.17}$$

and the DC index in complete markets is defined as:

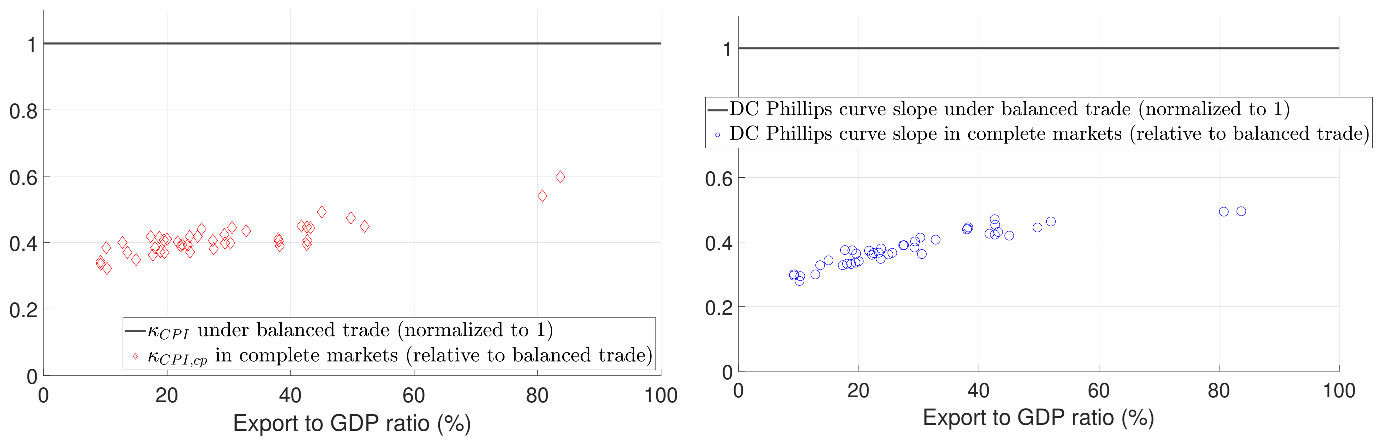
$$\pi_{DC,cp,t} \equiv \sum_{i=1}^{N} \widetilde{\mathcal{M}}_{OG,cp,i}\pi_{i,t}, \tag{B.18}$$

$$\text{where} \quad \widetilde{\mathcal{M}}_{OG,cp,i} \equiv \frac{\mathcal{M}_{OG,cp,i} \cdot (1-\widetilde{\delta}_i)/\widetilde{\delta}_i}{\kappa_{OG,cp}}. \tag{B.19}$$

The counterpart DC Phillips curve in terms of the employment gap in our baseline case of balanced trade is:

$$\pi_{DC} = \frac{\kappa_C \Lambda_L}{\kappa_{OG}} \widehat{L}^{gap}. \tag{B.20}$$

Figure B.1: The ratios of κ_{CPI} and the DC Phillips curve slope in complete markets versus under balanced trade



(a) $\kappa_{CPI,cp}$ and κ_{CPI} in the DC Phillips curve

(b) Slopes of the DC Phillips curve

Notes: Shown in panel (a) is the ratio of $\kappa_{CPI,cp}$ in complete markets (red diamonds) versus κ_{CPI} under balanced trade (normalized to one and indicated by the bold black horizontal line at one), for each country in the sample. Shown in panel (b) is the ratio of the DC Phillips curve slopes in complete markets (blue circles) versus under balanced trade (normalized to one and indicated by the bold black horizontal line at one), for each country in the sample. In panel (b), the price-rigidity parameter $\widetilde{\delta}_i$ is set to δ_i (i.e., imposing $\beta = 0$ in equation B.15) to make the DC Phillips curve slope in complete markets comparable to that under balanced trade.

Comparisons of the DC Phillips curve and DC index in balanced trade versus complete markets. Plotted in panel (a) in Figure B.1 is the ratio of κ_{CPI} in complete markets —given by equation (B.13) and indicated by red diamonds— to that under balanced trade —given by equation (23). In particular, κ_{CPI} under balanced trade is normalized to one and indicated by the bold black horizontal line at one. As shown in panel (a), κ_{CPI} —which captures the relative importance of the CPI channel in the sectoral OG weight— is smaller in complete markets than under balanced trade —as evinced by the red diamonds below the horizontal line at one. This implies that the relative sectoral weights in the DC index are smaller in complete markets than under balanced trade for sectors with a larger (vs. smaller) CPI channel.

Plotted in panel (b) in Figure B.1 is the ratio of the DC Phillips curve slope (in terms of the employment gap) in complete markets —given by $\kappa_{L,cp}/\kappa_{OG,cp}$ in equation (B.16) and indicated by blue circles— to that under balanced trade —given by $\kappa_C \Lambda_L/\kappa_{OG}$ in equation (B.20). In particular, the DC Phillips curve slope under balanced trade is normalized to one and indicated by the bold black horizontal line at one. We set the price-rigidity parameter $\widetilde{\delta}_i$ to δ_i —i.e., imposing $\beta = 0$ in equation (B.15)— to make the DC Phillips curve slope in complete markets comparable to that under balanced trade. As shown in panel (b), the slope of the DC Phillips curve is flatter in complete markets than under balanced trade —i.e., the employment gap is larger in complete markets facing the same negative markup wedges. This is consistent with the intuition in Heathcote and Perri (2002) that production is re-allocated to the country with lower marginal cost under international risk sharing.

C. Discussion on the non-zero profit channel in the sectoral OG weight

In this section, we illustrate that the profit channel is non-zero and explain why it is non-zero. In subsection C.1, we first present a special case of the baseline model in which the profit channel is always positive, thereby demonstrating that the profit channel does not theoretically cancel out. Then, in subsection C.2, in the baseline model, we pinpoint the terms in the output gap that are linked to the solution of the domestic planner's problem, which cancel out according to the envelope theorem. We further show that in the output gap, these terms linked to the planner's problem are distinct from the terms linked to the profit channel. As a result, the optimization by the planner does *not imply* the canceling out of the profit channel in the OG weights.

C.1. A special case of the baseline model where the profit channel is positive

Consider a special case of our baseline multi-sector SOE that imports only consumption goods but no intermediate inputs. In such an economy, $\widetilde{\boldsymbol{\alpha}} = \mathbf{1}$, so that the imported factor sub-channel equals zero, *viz.*:

$$\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}}) = \mathbf{0}.$$

At the same time, with the exports of the economy, the export profit sub-channel is equal to $\widetilde{\boldsymbol{\lambda}}_F$ and satisfies:

$$\widetilde{\boldsymbol{\lambda}}_F > \mathbf{0} \quad \text{and} \quad \widetilde{\boldsymbol{\lambda}}_F \neq \mathbf{0}.$$

As a consequence, the two sub-channels of export profit and imported factor do not cancel out, and the profit channel captured by $\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})$ is always positive in this special case.

C.2. The domestic planner's optimization problem versus the profit channel

In this subsection, we first derive the optimality conditions of the domestic planner in the steady state (*step 1*). Then, we show that in the steady state, the social planner's problem is *equivalent* to maximizing the export profits in terms of the *export prices*. By the envelope theorem, the terms associated with the planner's maximization of export profit should cancel in the output gap (*step 2*). Accordingly, we pinpoint these canceled terms linked to the export profit maximization in the output gap, and demonstrate that they are distinct from the profit channel in the sectoral OG weights (*step 3*).

Step 1: Optimality conditions of the domestic planner in the steady state. Consider the optimization problem of the domestic social planner described in Section J.2, *viz.*:

$$u^{flex}(\boldsymbol{\xi}) = \max_{\{C_{Hi}, C_{Fi}, \{X_{Hi,Hj}, X_{Hi,Fj}\}_j, L_i\}_i} u\Big(\mathcal{C}\big(\{\mathcal{C}_i(C_{Hi}, C_{Fi})\}_i\big), \sum_i L_i\Big),$$
$$\text{s.t.} \qquad \sum_i \left(D^*_{EX,Fi}\right)^{\frac{1}{\theta_{F,i}}} \left(Y^*_{EX,i}\right)^{\frac{\theta_{F,i}-1}{\theta_{F,i}}} - \sum_i P^*_{IM,Fi}\Big(C_{Fi} + \sum_j X_{Hj,Fi}\Big) \geq 0,$$
$$\text{where} \qquad Y^*_{EX,i} \equiv A_i F_i\big(\{L_i, \mathcal{X}_{i,j}(X_{Hi,Hj}, X_{Hi,Fj})\}_j\big) - C_{Hi} - \sum_j X_{Hj,Hi}.$$

Using the export demand function $Y^*_{EX,i} = \left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D^*_{EX,i}$, this problem can equivalently be expressed as a maximization with respect to export price $P_{EX,i}/S$ in place of C_{Hi} as follows:

$$u^{flex}(\boldsymbol{\xi}) = \max_{\{P_{EX,i}/S, C_{Fi}, \{X_{Hi,Hj}, X_{Hi,Fj}\}_j, L_i\}_i} u\Big(\mathcal{C}\big(\{\mathcal{C}_i(C_{Hi}, C_{Fi})\}_i\big), \sum_i L_i\Big), \tag{C.1}$$
$$\text{s.t.} \qquad \sum_i \left(\frac{P_{EX,i}}{S}\right)^{1-\theta_{F,i}} D^*_{EX,i} - \sum_i P^*_{IM,Fi}\Big(C_{Fi} + \sum_j X_{Hj,Fi}\Big) \geq 0, \tag{C.2}$$
$$\text{where} \qquad C_{Hi} \equiv A_i F_i\big(\{L_i, \mathcal{X}_{i,j}(X_{Hi,Hj}, X_{Hi,Fj})\}_j\big) - \left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D^*_{EX,i} - \sum_j X_{Hj,Hi}.$$

The first-order conditions of the maximization problem in equations (C.1) and (C.2) with respect to $P_{EX,i}/S$ and C_{Fi} are:

$$0 = \frac{\partial u}{\partial C_{Hi}} \cdot \theta_{F,i}\Big(\frac{P_{EX,i}}{S}\Big)^{-\theta_{F,i}-1} D^*_{EX,i} + \lambda\,(1-\theta_{F,i})\Big(\frac{P_{EX,i}}{S}\Big)^{-\theta_{F,i}} D^*_{EX,i}, \tag{C.3}$$
$$0 = \frac{\partial u}{\partial C_{Fi}} - \lambda\, P^*_{IM,Fi}, \tag{C.4}$$

with λ being the Lagrange multiplier on the trade balance condition (C.2). Combining equations (C.3) and (C.4) to eliminate the multiplier λ yields:

$$0 = (1-\theta_{F,i})\left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D^*_{EX,i} + P^*_{IM,Fi}\,\frac{\partial u/\partial C_{Hi}}{\partial u/\partial C_{Fi}} \cdot \theta_{F,i}\Big(\frac{P_{EX,i}}{S}\Big)^{-\theta_{F,i}-1} D^*_{EX,i},$$

which —in the steady state where the implied prices satisfy $P^{*,ss}_{IM,Fi}\frac{\partial u/\partial C_{Hi}}{\partial u/\partial C_{Fi}} = \frac{P^{ss}_i}{S^{ss}} = 1$— becomes:[6]

$$0 = (1-\theta_{F,i})\left(\frac{P^{ss}_{EX,i}}{S^{ss}}\right)^{-\theta_{F,i}} D^{*,ss}_{EX,i} + \theta_{F,i}\left(\frac{P^{ss}_{EX,i}}{S^{ss}}\right)^{-\theta_{F,i}-1} D^{*,ss}_{EX,i}. \tag{C.5}$$

Step 2: Domestic planner's problem is equivalent to maximizing the export profits in terms of export prices. As is shown in equation (27) of Section 3, the sectoral OG weight —including the profit channel in it— is primarily derived from the open-economy budget constraint as follows:

$$\begin{aligned} P_C C &= \sum_i \left[P_i Y_i\Big(1-\frac{1-\alpha_i}{\mu_i}\Big) + (P_{EX,i}-P_i)Y^*_{EX,i}\right] \\ &= \sum_i \left\{P_i Y_i\Big(1-\frac{1-\alpha_i}{\mu_i}\Big) + \underbrace{\left[\left(\frac{P_{EX,i}}{S}\right)^{1-\theta_{F,i}} D^*_{EX,i} - \frac{P_i}{S}\left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D^*_{EX,i}\right]}_{\text{export profit}} \cdot S\right\}, \end{aligned} \tag{C.6}$$

where the last equality comes from substituting in the export demand $Y^*_{EX,i} = \left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D^*_{EX,i}$.

The total differentiation of the export profit term in equation (C.6) —evaluated at the steady state— is equal to:

$$\begin{aligned} &\underbrace{\left[(1-\theta_{F,i})\left(\frac{P^{ss}_{EX,i}}{S^{ss}}\right)^{-\theta_{F,i}} D^{*,ss}_{EX,i} + \theta_{F,i}\frac{P^{ss}_i}{S^{ss}}\left(\frac{P^{ss}_{EX,i}}{S^{ss}}\right)^{-\theta_{F,i}-1} D^{*,ss}_{EX,i}\right]\left(\frac{P^{ss}_{EX,i}}{S^{ss}}\right) d\ln\left(\frac{P_{EX,i}}{S}\right)}_{\text{export profit maximization through choices of export prices}} \\ &\quad - \left(\frac{P^{ss}_i}{S^{ss}}\right)\left(\frac{P^{ss}_{EX,i}}{S^{ss}}\right)^{-\theta_{F,i}} D^{*,ss}_{EX,i} d\ln\left(\frac{P_i}{S}\right) + \left[\left(\frac{P^{ss}_{EX,i}}{S^{ss}}\right)^{1-\theta_{F,i}} - \frac{P^{ss}_i}{S^{ss}}\left(\frac{P^{ss}_{EX,i}}{S}\right)^{-\theta_{F,i}}\right] D^{*,ss}_{EX,i} d\ln D^*_{EX,i} \\ &= \underbrace{(-\theta_{F,i}Y^{*,ss}_{EX,i} + \theta_{F,i}Y^{*,ss}_{EX,i}) d\ln\left(\frac{P_{EX,i}}{S}\right)}_{\substack{\text{export profit maximization} \\ \text{through choices of export prices}}} \underbrace{-Y^{*,ss}_{EX,i} d\ln\left(\frac{P_i}{S}\right)}_{\substack{\text{export profit term} \\ \text{linked to domestic sectoral prices}}} + \frac{Y^{*,ss}_{EX,i}}{\theta_{F,i}-1} d\ln D^*_{EX,i}. \end{aligned} \tag{C.7}$$

Because the non-contingent export tax in Assumption 1 allows the competitive equilibrium to implement the domestic planner's allocation in the steady state, the steady-state equilibrium prices in equation (C.7) coincide with those in the steady-state solution to the domestic planner's problem in equation (C.5).[7] As a result, the terms containing $d\ln\left(\frac{P_{EX,i}}{S}\right)$ that are linked to the export profit maximization in equation (C.7) (i.e., the first term on both sides of the equation) are equal to zero, which is consistent with the first-order condition of the planner in equation (C.5). The remaining term $-Y^{*,ss}_{EX,i} d\ln\left(\frac{P_i}{S}\right)$ in equation (C.7) is non-zero, and it relates to the export profit through the domestic sectoral prices P_i/S, which are not set by the

[6] The utility function equation (J.9) implies that the marginal rate of substitution between C_{Hi} and C_{Fi} is $\frac{\partial u/\partial C_{Hi}}{\partial u/\partial C_{Fi}} = \frac{\partial \mathcal{C}_i/\partial C_{Hi}}{\partial \mathcal{C}_i/\partial C_{Fi}}$. Substituting equation (J.21) in this equation of marginal rate of substitution and evaluating it at the steady state yields $P^{*,ss}_{IM,Fi}\frac{\partial u/\partial C_{Hi}}{\partial u/\partial C_{Fi}} = \frac{P^{ss}_i}{S^{ss}} = 1$.

[7] The equality in equation (C.7) is also based on the rate of the non-contingent export tax in Assumption 1 —*viz.*, $\tau_{EX,i} = 1/(\theta_{F,i}-1)$ such that $P^{ss}_{EX,i} = \theta_{F,i}/(\theta_{F,i}-1)$.

domestic planner to maximize the export profit.[8]

Equation (C.7) indicates that, under Assumption 1, the domestic planner's problem is *equivalent* to maximizing the export profit in terms of the choices of export prices in the steady state. Therefore, applying the envelope theorem to the domestic planner's problem, we should expect that the terms linked to the export profit maximization cancel out in the output gap and sectoral OG weights.[9]

Step 3: Identify the canceled terms relating to the export profit maximization in the output gap, and show that they are not the profit channel. Equation (C.7) indicates that the log-linearization of the export profit term in equation (C.6) around the flexible-price equilibrium is equal to:

$$\underbrace{(-\theta_{F,i} Y^{*,ss}_{EX,i} + \theta_{F,i} Y^{*,ss}_{EX,i})(\widehat{P}^{gap}_{EX,i} - \widehat{S}^{gap})}_{\text{export profit maximization through choices of export prices}} - \underbrace{Y^{*,ss}_{EX,i}(\widehat{P}^{gap}_{i} - \widehat{S}^{gap})}_{\text{export profit term linked to domestic sectoral prices}} . \tag{C.8}$$

These terms in equation (C.8) can be found in the log-linearization of the open-economy budget constraint (C.6) around the flexible-price equilibrium as follows:

$$\begin{aligned}\widehat{P}^{gap}_{C} + \widehat{C}^{gap} = & [\boldsymbol{\lambda} \odot (\widehat{\mathbf{P}}^{gap} + \widehat{\mathbf{Y}}^{gap})]^{\top} \boldsymbol{\alpha} + [\boldsymbol{\lambda} \odot (\mathbf{1} - \boldsymbol{\alpha})]^{\top} \widehat{\boldsymbol{\mu}} \\ & \underbrace{-(\boldsymbol{\lambda}_{EX} \odot \boldsymbol{\theta}_F)^{\top}(\widehat{\mathbf{P}}^{gap}_{EX} - \mathbf{1}\widehat{S}^{gap}) + (\boldsymbol{\lambda}_{EX} \odot \boldsymbol{\theta}_F)^{\top}(\widehat{\mathbf{P}}^{gap}_{EX} - \mathbf{1}\widehat{S}^{gap})}_{\text{export profit maximization through choices of export prices}} \\ & \underbrace{-\boldsymbol{\lambda}^{\top}_{EX}(\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap})}_{\text{export profit term linked to domestic sectoral prices}} + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^{\top} \widehat{S}^{gap},\end{aligned} \tag{C.9}$$

which is the key condition to derive the output gap and sectoral OG weights as we show in equation (27) of Section 3.[10] In equation (C.9), the terms linked to the planner's maximization of export profits (on the second row) correspond to the first terms in equations (C.8) and (C.7), and they cancel out as in those equations, which is consistent with the envelope theorem.[11] The fact that these terms relating to the planner's maximization of export profits cancel out in condition (C.9) —which is the key condition to derive the output gap and OG weights in Theorem 1— indicates that the sectoral OG weight, including the profit channel, *does not directly reflect the solution to the domestic planner's optimization problem.*

[8]As in the closed economy, the domestic sectoral prices are set to be equal to the marginal cost of production.

[9]The envelope theorem says that when the planner's first-order condition with respect to its choice variable holds at the steady state, changes in the objective function (i.e., export profit here) due to small deviations of the planner's choice variable from the flexible-price equilibrium (induced by markup wedges) are equal to zero up to a first-order approximation.

[10]Substituting the goods market clearing condition (K.25) into equation (C.9) yields condition (27) in Section 3.

[11]Notably, although the non-contingent export tax makes the export price proportional to the domestic sectoral price —and, therefore, $\widehat{\mathbf{P}}^{gap}_{EX} = \widehat{\mathbf{P}}^{gap}$— these two prices are different choice variables from the perspective of the social planner, of which only the export price is used by the social planner to maximize export profits. Therefore, when we apply the envelope theorem to the planner's problem of export profit maximization, only the terms of log-deviations of export prices (i.e., $\widehat{\mathbf{P}}^{gap}_{EX}$) —even though equal to domestic sectoral price gaps $\widehat{\mathbf{P}}^{gap}$— are included in the term linked to the export profit maximization, while the other terms of domestic sectoral price gaps are unrelated to export profit maximization.

In the remaining part of condition (C.9) (specifically, the first term on the third row), domestic sectoral price gaps —unlike the export prices $P_{EX,i}/S$ that are chosen by the planner to maximize the export profit— negatively relate to the export profit by raising the (opportunity) costs of exports, which is exactly what the export profit sub-channel in the profit channel captures. The other sub-channel of the imported factor in the profit channel relates to the cost of imported intermediate inputs, which is also unrelated to the planner's problem of maximizing export profits.

Because neither export profit nor imported factor sub-channel directly links to the planner's problem, the optimization of the planner does *not imply* that these two sub-channels cancel each other out to make the profit channel zero.

D. The slopes of the divine coincidence and sectoral Phillips curves

D.1. The slope of the DC Phillips curve

Expanding the inverse of the slope of the divine coincidence Phillips curve in equation (33) yields:

$$
\begin{aligned}
\frac{\kappa_{OG}}{\kappa_C} =& \underbrace{\frac{\frac{1}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}} + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}{\frac{(\sigma+\varphi/\Lambda_L)}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}} + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}(\sigma + \varphi/\Lambda_L) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})} \widetilde{\boldsymbol{\lambda}}_D^\top (\boldsymbol{I} - \boldsymbol{\Delta}) \boldsymbol{\Delta}^{-1} \mathbf{1}}_{\text{CPI channel}} \\
&+ \underbrace{\frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top \boldsymbol{L}_{vx}}{\kappa_C \kappa_{CPI}}}_{\text{expenditure-switching channel}} (\boldsymbol{I} - \boldsymbol{\Delta}) \boldsymbol{\Delta}^{-1} \mathbf{1} + \underbrace{\frac{\boldsymbol{\lambda}_{EX}^T \boldsymbol{L}_{vx} - [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top}{\kappa_C \kappa_{CPI}}}_{\text{profit channel}} (\boldsymbol{I} - \boldsymbol{\Delta}) \boldsymbol{\Delta}^{-1} \mathbf{1}. \qquad \text{(D.1)}
\end{aligned}
$$

where the expenditure-switching and the profit channels are both unique to the open economy. Because the expenditure-switching channel is strictly positive, this open-economy-specific channel increases the inverse of the divine coincidence Phillips curve slope, thereby *flattening* the slope in open economies relative to that in closed economies.[12]

Then, we focus on the CPI channel underlying the inverse of the slope and obtain the following three inequalities in equations (D.2), (D.3), and (D.4):

$$
\begin{aligned}
&\underbrace{\frac{\frac{1}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}} + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}{\frac{(\sigma+\varphi/\Lambda_L)}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}} + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}(\sigma + \varphi/\Lambda_L) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})} \widetilde{\boldsymbol{\lambda}}_D^\top}_{\text{CPI channel}} \\
&< \frac{\frac{1}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}} + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}{\frac{(\sigma+\varphi)}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}} + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}(\sigma + \varphi) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})} \widetilde{\boldsymbol{\lambda}}_D^\top \qquad \text{(D.2)}
\end{aligned}
$$

[12]The profit channel is close to zero and barely affects the slope of the divine coincidence Phillips curve.

$$\underbrace{\frac{\frac{1}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}{\frac{(\sigma+\varphi/\Lambda_L)}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}(\sigma+\varphi/\Lambda_L)+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}\widetilde{\boldsymbol{\lambda}}_D^\top}_{\text{CPI channel}}$$

$$> \frac{\frac{1}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}{\frac{(\sigma+\varphi/\Lambda_L)}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}(\sigma+\varphi/\Lambda_L)+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})(\sigma+\varphi/\Lambda_L)}\widetilde{\boldsymbol{\lambda}}_D^\top = \frac{\widetilde{\boldsymbol{\lambda}}_D^\top}{(\sigma+\varphi/\Lambda_L)} \tag{D.3}$$

$$\underbrace{\frac{\frac{1}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}{\frac{(\sigma+\varphi/\Lambda_L)}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}(\sigma+\varphi/\Lambda_L)+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}\widetilde{\boldsymbol{\lambda}}_D^\top}_{\text{CPI channel}}$$

$$< \frac{\frac{1}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}{\frac{(\sigma+\varphi/\Lambda_L)}{(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widetilde{\boldsymbol{\alpha}}+\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}(\sigma+\varphi/\Lambda_L)+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})}\boldsymbol{\lambda}^\top \tag{D.4}$$

The inequality in equation (D.4) shows that the inverse of the DC curve slope can be smaller than that in closed economies because, in the CPI channel, the domestic sectors' total contents in domestic consumption are smaller in open than in closed economies, i.e., $\widetilde{\boldsymbol{\lambda}}_D^\top < \boldsymbol{\lambda}^\top$. Therefore, the elasticities of the domestic output gap to domestic sectoral inflation through the CPI channel are smaller in open economies, thereby *steepening* the DC Phillips curve slope relative to that in closed economies.

The inequality in equation (D.2) shows that the inverse of the DC curve slope can be smaller than that in closed economies because, in the CPI channel, the total domestic labor income share in open economies is less than unity in closed economies, i.e., $\Lambda_L < 1$. Therefore, the elasticity of the output gap to domestic labor supply —and, therefore, the real wage gap— is smaller than in closed economies, as evinced by $1/(\sigma+\varphi/\Lambda_L) < 1/(\sigma+\varphi)$, thereby *steepening* the DC Phillips curve slope relative to that in closed economies.

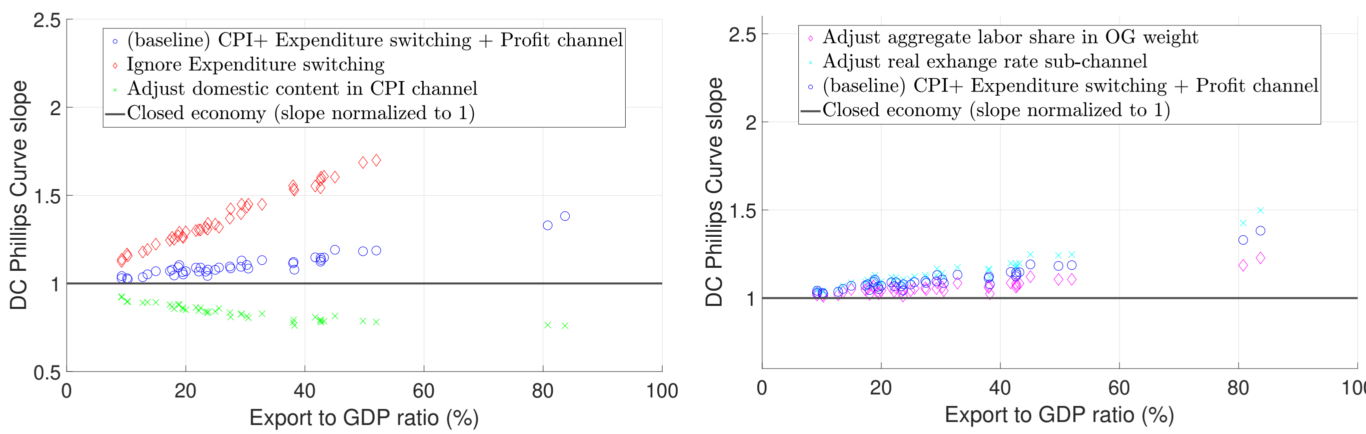
The inequality in equation (D.3) shows that the inverse of slope can be larger than that in closed economies because of the real exchange rate sub-channel that is unique to open economies. In this real exchange rate sub-channel, the elasticity of the output gap to the CPI gap is unity and larger than the counterpart elasticity through the real wage channel (i.e., $1 > 1/(\sigma+\varphi/\Lambda_L)$ in general) —which is the only channel in closed economies. As a result, the existence of the open-economy-specific sub-channel of real exchange rate increases the size of the CPI channel, hence *flattening* the DC Phillips curve slope in open economies relative to that in closed economies.

Combining the three changes of the open-economy CPI channel toward the closed-economy CPI channel on the RHS of equations (D.2), (D.3), and (D.4), yields the slope of the divine coincidence Phillips curve in the closed economies, $\boldsymbol{\lambda}/(\sigma+\varphi)$, which is consistent with Rubbo (2023).

In Section 6.1 and in Figure D.1 below, we calibrate our model using the WIOD and show that the first force through the total contents in domestic consumption (equation D.4) and the second force through the expenditure-switching channel (equation D.1) are the two quantitatively dominant forces. Moreover, the first force dominates in most economies, making the open-economy slope steeper in open economies than in closed economies.

Plotted in panel (b) in Figure D.1 below are two other counterfactual cases: (i) adjusting the aggregate share of domestic labor in domestic output to unity as in closed economies (i.e., change Λ_L to 1, indicated by the magenta diamonds, corresponding to equation D.2), and (ii) adjusting the size of the real exchange rate sub-channel upward to that of the real wage sub-channel in the CPI channel (i.e., change $(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})$ to $(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})(\sigma+\varphi/\Lambda_L)$ in the numerator of κ_C, indicated by the cyan crosses, corresponding to equation D.3). Consistent with equation (D.2), the less-than-unity total domestic labor income share in open economies makes the elasticity of the output gap to domestic inflation smaller in open than in closed economies, thereby *steepening* the DC Phillips curve slope relative to that in closed economies —as evinced by the blue circles higher than the magenta diamonds in panel (b) of Figure D.1. Comparing panels (b) to (a) shows that these alternative forces in the two counterfactual cases are quantitatively small relative to the two main countervailing forces as discussed above.

Figure D.1: Slopes of the DC Phillips curves (relative to a closed economy): more counterfactual cases



(a) CPI channel and ignoring Expenditure switching

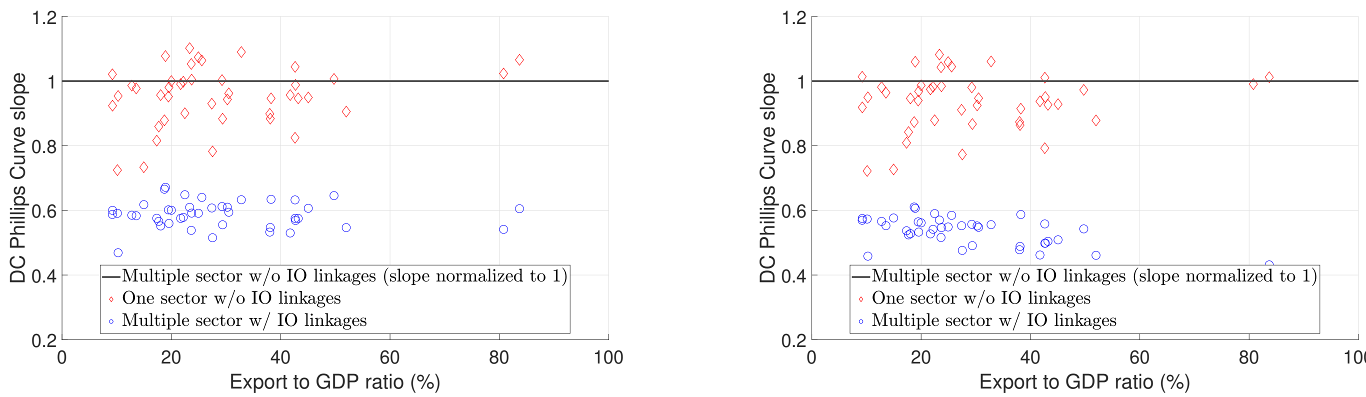
(b) More counterfactuals

Notes: Shown in both panels is the ratio of the slope of the DC Phillips curve in open economies to its counterpart in closed economies, for each country in the sample. The closed-economy slopes are normalized to one, corresponding to the bold black horizontal line at one. In both panels, the blue circles indicate the baseline open-economy Phillips curve slopes. In panel (a), the red diamonds indicate the counterfactual case that ignores the expenditure-switching channel and the green crosses indicate the counterfactual slope that replaces the open-economy sectoral content in domestic consumption with its closed-economy counterpart in the CPI channel (i.e., change $\widetilde{\boldsymbol{\lambda}}_D$ to $\boldsymbol{\lambda}$). In panel (b), the magenta diamonds indicate the counterfactual case that adjusts the aggregate share of domestic labor in domestic output to unity as in closed economies (i.e., change Λ_L to 1) and the cyan crosses indicate the counterfactual case that adjusts the size of the real exchange rate sub-channel upward to that of the real wage sub-channel in the CPI channel (i.e., change $(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})$ to $(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})(\sigma+\varphi/\Lambda_L)$ in the numerator of κ_C).

The DC Phillips curve slope with and without IO linkages. Plotted in Figure D.2 below are the ratios of the DC Phillips curve slopes in multi-sector economies with IO linkages and in one-sector economies relative to that in multi-sector economies without IO linkages (y-axis), for each country in the sample, against the country's export to GDP ratio (x-axis). Panels (a) and (b) correspond to the cases of the baseline open economy and the closed economy à la Rubbo (2023), respectively. In both panels, the slopes in multi-sector economies without IO linkages are normalized to one, corresponding to the bold black horizontal line at one.

In both panels, the DC Phillips curve slope in the baseline economy with IO linkages is flatter than in counterfactual one-sector economies and multi-sector economies without IO linkages, as evinced by the blue circles that are below both the red diamonds and the bold black horizontal line at one. Further comparing panels (b) to (a) shows that the introduction of IO linkages flattens the slope more in closed (relative to open) economies for those economies that are relatively open and have a large expenditure-switching channel.

Figure D.2: Ratios of the DC Phillips curve slopes relative to those in multi-sector economies without IO linkages



(a) Open economy

(b) Closed economy

Notes: Shown in the figure are the ratios of the DC Phillips curve slopes in multi-sector SOEs with IO linkages (blue circles) and in one-sector SOEs (red diamonds) relative to multi-sector economies without IO linkages, for each country in the sample, against the country's export to GDP ratio (x-axis). Panels (a) and (b) correspond to the cases of open and closed economies, respectively. In both panels, the slopes in multi-sector economies without IO linkages are normalized to one, corresponding to the bold black horizontal line at one. In one-sector SOEs, the domestic country's parameter of home bias is set to its average across sectors, i.e., $\beta^\top \mathbf{v}$.

D.2. The slope of sectoral Phillips curves

In this paragraph, we describe the alternative slopes of the sectoral Phillips curves used in the quantitative analysis in subsection 6.1, including the following three cases:

1. **Closed economy.** In the counterfactual closed-economy case, $\mathbf{v} = \mathbf{0}$, $\mathbf{V}_{1-x} = \mathbf{0}$, $\Lambda_L = 1$, and $\boldsymbol{\mathcal{M}}_p = \boldsymbol{\mathcal{M}}_\mu = \mathbf{0}$. As a result, $\Gamma_{CPI,C} = 0$ and $\mathbf{\Gamma}_{CPI,P} = \boldsymbol{\beta}$, and the slopes $\boldsymbol{\mathcal{B}}$ in equation (35) reduce to:

$$\boldsymbol{\mathcal{B}} = \left[\mathbf{\Delta}^{-1} - \mathbf{\Omega} - \boldsymbol{\alpha}\boldsymbol{\beta}^\top\right]^{-1} \boldsymbol{\alpha}\left(\sigma + \varphi\right),$$

which are exactly the same as those in Rubbo (2023).

2. **Ignoring the expenditure-switching channel.** Ignoring the expenditure-switching channel in the slopes of sectoral Phillips curves corresponds to the case (ii) in panel (b) of Figure 1, as indicated by red diamonds. In this case, the term corresponding to the expenditure-switching channel becomes zero (i.e., $\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} = \mathbf{0}$) and $\boldsymbol{\mathcal{M}}_p$ in equation (L.5) reduces to $\boldsymbol{\mathcal{M}}_p = (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1} \boldsymbol{\lambda}_{EX}$. The parameters $\Gamma_{CPI,C}$, $\boldsymbol{\Gamma}_{CPI,P}$, $\Gamma_{S,C}$, and $\boldsymbol{\Gamma}_{S,P}$ in the slopes $\boldsymbol{\mathcal{B}}$ in equation (35) adjust accordingly for their components of $\boldsymbol{\mathcal{M}}_p$.
3. **Changing domestic contents in domestic consumption in the CPI channel.** Changing the domestic contents in domestic consumption in the CPI channel of the slopes of sectoral Phillips curves corresponds to the case (iii) in panel (b) of Figure 1, as indicated by green crosses. In this case, we adjust the domestic sectoral content in domestic consumption in the CPI channel upward to the closed-economy case —i.e., $\boldsymbol{v} = \mathbf{1}$ and $\mathbf{V}_x = \mathbf{I}$ in $\boldsymbol{\Delta}_\Phi$— which leads to the following counterfactual values for the baseline parameters $\Gamma_{CPI,C}$, $\boldsymbol{\Gamma}_{CPI,P}$, $\Gamma_{S,C}$, $\boldsymbol{\Gamma}_{S,P}$, and $\boldsymbol{\Delta}_\Phi$ in equations (L.10), (L.11), (L.13), (L.14), and (36):

$$
\begin{aligned}
\Gamma_{CPI,C} &\equiv (\boldsymbol{\beta}^\top \mathbf{1} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1} (1 - \boldsymbol{\beta}^\top \mathbf{v}), \\
\boldsymbol{\Gamma}_{CPI,P} &\equiv (\boldsymbol{\beta}^\top \mathbf{1} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1} \left[(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})(\boldsymbol{\beta} \odot \mathbf{1}) + (1 - \boldsymbol{\beta}^\top \mathbf{v})(\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu) \right], \\
\Gamma_{S,C} &\equiv \left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1} (1 + \Gamma_{CPI,C}), \\
\boldsymbol{\Gamma}_{S,P} &\equiv \left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1} (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu + \boldsymbol{\Gamma}_{CPI,P}), \\
\boldsymbol{\Delta}_\Phi &\equiv \left[\boldsymbol{\Delta}^{-1} - \boldsymbol{\Omega} - \boldsymbol{\alpha} \boldsymbol{\Gamma}_{CPI,P}^\top \right]^{-1}.
\end{aligned}
$$

E. The DC Phillips curve under foreign-currency pricing

Our baseline model assumes producer-currency pricing (PCP) —in which domestic producers set exporting prices in producers' (i.e., domestic) currencies. In this Appendix, we follow Engel (2011) to extend our model to the alternative settings in the literature, i.e., the foreign-currency pricing (FCP) that includes both local-currency pricing (LCP) and dominant-currency pricing (DCP). Under local-currency and dominant-currency pricing, domestic producers set sectoral exporting prices in foreign and dominant (e.g., US dollars) currencies, respectively, and can price discriminate among domestic and foreign markets, facing different Calvo-pricing rigidities in these two markets. In particular, because our model summarizes the rest of the world using a single foreign country and treats the import prices of foreign products denominated in foreign currency as exogenous, local-currency pricing is equivalent to dominant-currency pricing in our setting.

E.1. Extension of baseline model to foreign-currency pricing

In this subsection, we describe the changes in our extended model with foreign-currency pricing, compared to the baseline model with producer-currency pricing. In each domestic sector i, we assume that there are two types of monopolistically competitive firms, each of which has a unit mass: the first type of firms only sell their products to domestic customers, which we denote by type DM; the second type of firms only export their products to foreign customers, which we denote by type EX. In comparison, our baseline model includes only one type of firms that sell products to both domestic and foreign customers in each sector. In each sector i, the two types of firms share the same production function and, therefore, the same marginal cost of production Φ_i. While the selling prices of type DM firms are denominated in the domestic currency and denoted by P_{if}, the exporting prices of type EX firms are denominated in the foreign currency and denoted by $P^*_{EX,if}$.

For the monopolistically competitive firms of type DM in each sector i, there are perfectly competitive and identical sectoral goods packers that transform their differentiated goods into a *sectoral domestic-market product*, which is sold only to domestic customers, using the following constant-elasticity-of-substitution technology:

$$Y_{DM,i} = \left(\int_0^1 Y_{DM,if}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}} \qquad \text{and} \qquad P_i = \left(\int_0^1 P_{if}^{1-\varepsilon_i} df \right)^{\frac{1}{1-\varepsilon_i}}, \tag{E.1}$$

where the within-sector elasticity of substitution between different firms' products is equal to $\varepsilon_i > 1$, and P_i denotes the price of the sectoral domestic-market product of sector i.

Similarly, for the monopolistically competitive firms of type EX in each sector i, there are perfectly competitive and identical sectoral export goods packers located in the foreign country that transform their differentiated goods into a *sectoral foreign-market product* that is only exported to foreign customers, using the following constant-elasticity-of-substitution technology:

$$Y_{EX,i} = \left(\int_0^1 Y_{EX,if}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}} \qquad \text{and} \qquad P^*_{EX,i} = \left(\int_0^1 \left(P^*_{EX,if} \right)^{1-\varepsilon_i} df \right)^{\frac{1}{1-\varepsilon_i}}, \tag{E.2}$$

where the within-sector elasticity of substitution between different firms' products is also equal to ε_i, and $P^*_{EX,i}$ denotes the price of the sectoral foreign-market product of sector i. We denote the total quantity of sector i's products by $Y_i \equiv Y_{DM,i} + Y_{EX,i}$.

The two types of firms face separate Calvo-pricing friction. Type DM firms face the same Calvo-pricing friction as in the baseline model, with domestic-market price rigidity of sector i equal to $(1-\delta_i)/\delta_i$. Among EX firms, only firms indexed by $f \leq \delta^*_{EX,i} \in [0,1]$ are allowed to choose their desired price $P^{*,\#}_{EX,if}$ and the remaining firms maintain their prices at the steady-state level $P^{*,SS}_{EX,i}$. We refer to $(1-\delta^*_{EX,i})/\delta^*_{EX,i}$ as the foreign-market price rigidity of sector i. Facing the sectoral sales tax rate τ_i, type DM firms that can adjust prices choose the desired price to maximize the following nominal profits:

$$\max_{P^*_{EX,if}} (1-\tau_i) P_{if} Y_{DM,if} - \Phi_i Y_{DM,if},$$

$$s.t. \quad Y_{DM,if} = \left(\frac{P_{if}}{P_i}\right)^{-\varepsilon_i} Y_{DM,i}.$$

Facing both the sectoral sales tax rate τ_i and the sectoral export tax rate $\tau_{EX,i}$, type EX firms that can adjust prices choose the desired price to maximize the following nominal profits:

$$\max_{P^*_{EX,if}} (1-\tau_i)(1-\tau_{EX,i})SP^*_{EX,if}Y_{EX,if} - \Phi_i Y_{EX,if},$$
$$s.t. \quad Y_{EX,if} = \left(\frac{P^*_{EX,if}}{P^*_{EX,i}}\right)^{-\varepsilon_i} Y_{EX,i},$$

where the foreign demand for sector i's foreign-market products is the same as in the baseline model and equal to:

$$Y_{EX,i} = \left(P^*_{EX,i}\right)^{-\theta_{F,i}} D^*_{EX,Fi}. \tag{E.3}$$

We keep Assumption 1 —i.e., $\tau_i = -1/(\varepsilon_i - 1)$ and $\tau_{EX,i} = 1/\theta_{Fi}$— from the baseline model such that in both the steady state and the flexible-price equilibrium,

$$P_i = \Phi_i \qquad \text{and} \qquad P_{EX,i} = \frac{\Phi_i}{S}\frac{\theta_{F,i}}{\theta_{F,i}-1},$$

which are the same as those in the baseline model. In this way, the within-sector distortion due to monopolistic competition is removed, and the monopoly power of exporting firms on the international market is retained —in both the steady state and the flexible-price equilibrium. Thus, the allocation in the flexible-price equilibrium is equivalent to the solution to the optimization problem of the domestic social planner, as in the baseline model.[13]

We define the sectoral markups of domestic-market and foreign-market products as $\mu_i \equiv P_i/\Phi_i$ and $\mu^*_{EX,i} \equiv SP^*_{EX,i}/\Phi_i$, respectively. Under Assumption 1, in the sticky-price equilibrium and outside the steady state, the desired prices of sectoral domestic-market and foreign-market products are equal to:

$$\mu_i^{\#} \equiv \frac{P_i^{\#}}{\Phi_i} = 1 \qquad \text{and} \qquad \mu^{*,\#}_{EX,i} \equiv \frac{SP^{*,\#}_{EX,i}}{\Phi_i} = \frac{\theta_{F,i}}{\theta_{F,i}-1},$$

respectively. We further define the sectoral markup wedges of domestic and export products as $\widehat{\mu}_i \equiv \ln(\mu_i) - \ln(\mu_i^{\#}) = \ln(\mu_i) - \ln(\mu_i^{ss})$ and $\widehat{\mu}^*_{EX,i} \equiv \ln(\mu^*_{EX,i}) - \ln(\mu^{*,\#}_{EX,i}) = \ln(\mu^*_{EX,i}) - \ln(\mu^{*,ss}_{EX,i})$, respectively, because

[13] Intuitively, one can think of type EX firms in each sector as constituting an extreme sector in the baseline model that only exports to foreign countries and supplies no goods to domestic customers. Therefore, the same Assumption 1 as in the baseline model is needed and sufficient to make the allocation in the flexible-price equilibrium equivalent to the solution to the optimization problem of the domestic social planner.

the steady-state markups are equal to the desired markups.

Finally, we follow the baseline model to assume the import prices of foreign products denominated in foreign currency to be exogenous, and sectoral markups of imported foreign products are completely embedded in such exogenous sectoral import prices. As a result, sectoral markups of imported foreign products —whether under our baseline PCP or under the FCP— will not emerge in the expressions of the domestic output gap and, therefore, not in the divine coincidence Phillips curve.

E.2. The DC Phillips curve under foreign-currency pricing

In the extended model with foreign-currency pricing, we show that the output gap under foreign-currency pricing is related to both domestic-market (i.e., $\widehat{\mu}_i$) and foreign-market sectoral markup wedges (i.e., $\widehat{\mu}^*_{EX,i}$) of domestic products, as outlined in the following corollary of Theorem 1:[14]

Corollary E.1 (Output gap and the DC Phillips curve under foreign-currency pricing)**.** *Under foreign-currency pricing, in a sticky-price equilibrium, negative sectoral markup wedges in both domestic market $\{\widehat{\mu}_i(\boldsymbol{\xi})\}_i$ and foreign market $\{\widehat{\mu}^*_{EX,i}(\boldsymbol{\xi})\}_i$ are related to a positive output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ as follows:*

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\sum_{i=1}^{N} \left[\mathcal{M}_{OG,i} \cdot \widehat{\mu}_i(\boldsymbol{\xi}) + \kappa_{CPI}^{-1} \cdot (\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i) \cdot \left(\widehat{\mu}^*_{EX,i}(\boldsymbol{\xi}) - \widehat{\mu}_i(\boldsymbol{\xi})\right)\right], \tag{E.4}$$

where the sectoral OG weight ($\mathcal{M}_{OG,i}$), κ_{CPI}, κ_C are the same as those in Theorem 1. Accordingly, the divine coincidence Phillips curve under foreign-currency pricing is:

$$\pi_{DC,FCP} = \frac{\kappa_C}{\kappa_{OG,FCP}} \widehat{C}^{gap}(\boldsymbol{\xi}), \tag{E.5}$$

where the divine coincidence index under foreign-currency pricing is defined as

$$\pi_{DC,FCP} \equiv \sum_{i=1}^{N} \widetilde{\mathcal{M}}_{OG,D,i} \widehat{P}_i + \sum_{i=1}^{N} \widetilde{\mathcal{M}}_{OG,F,i} \widehat{P}^*_{EX,i}, \tag{E.6}$$

and $\widetilde{\mathcal{M}}_{OG,D,i} \equiv \frac{\left(\mathcal{M}_{OG,i} - \kappa_{CPI}^{-1}\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i\right)\cdot(1-\delta_i)/\delta_i}{\kappa_{OG,FCP}}$, $\widetilde{\mathcal{M}}_{OG,F,i} \equiv \frac{\kappa_{CPI}^{-1}\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i\cdot(1-\delta_{EX,i})/\delta_{EX,i}}{\kappa_{OG,FCP}}$, *and* $\kappa_{OG,FCP} \equiv \sum_{i'=1}^{N} \left[\left(\mathcal{M}_{OG,i'} - \kappa_{CPI}^{-1}\theta_{F,i'}\lambda_{EX,i'}\widetilde{\alpha}_{i'}\right) \cdot (1-\delta_{i'})/\delta_{i'} + \kappa_{CPI}^{-1}\theta_{F,i'}\lambda_{EX,i'}\widetilde{\alpha}_{i'}(1-\delta_{EX,i'})/\delta_{EX,i'}\right]$.

Proof: See Appendix K.9.

Corollary E.1 shows that under foreign-currency pricing, sectoral markup wedges are linked to the output gap in a very similar fashion to that under producer-currency pricing as in equation (21). Under foreign-currency pricing, however, exports are determined by foreign-market sectoral inflation and markup

[14] Similar to equation (20), foreign-market sectoral markup wedges in exporting prices are linked to domestic producers' exporting prices as follows: $\widehat{\mu}^*_{EX,i}(\boldsymbol{\xi}) = -(1-\delta_{EX,i})/\delta_{EX,i} \cdot \widehat{P}^*_{EX,i}(\boldsymbol{\xi})$.

wedges instead of domestic ones, thus leading to the extra export-related term on the RHS of equation (E.4) that replaces domestic-market with foreign-market sectoral markup wedges. Specifically, negative foreign-market sectoral markup wedges —caused by price rigidities under foreign-market sectoral inflation— reduce domestic products' exporting prices relative to the foreign products' prices in the foreign market. As a result, the foreign expenditure switches from foreign to domestic products, increasing domestic labor income from international trade and connecting to a positive output gap —as summarized by the coefficient of the foreign-market sectoral markup wedge $\kappa_{CPI}^{-1}(\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i)$. The existence of both domestic- and foreign-market markup wedges in the output gap of equation (E.4) implies that, under foreign-currency pricing, the divine coincidence Phillips curve involves a divine coincidence index that is a weighted average of sectoral inflation of both domestic-market prices and export prices in the foreign market, as shown in equation (E.6). In particular, while the CPI and profit channels remain dependent on domestic sectoral inflation, the expenditure-switching channel relies on inflation in both domestic and export prices.

F. Role of input-output linkages in multi-sector open economies versus closed economies

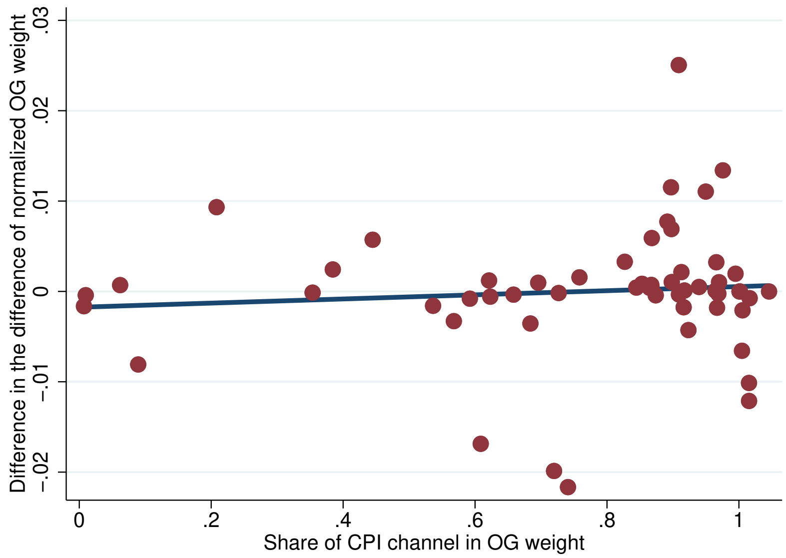
We analyze the impact of introducing input–output linkages into a multi-sector, horizontal SOE versus a multi-sector, horizontal closed economy on normalized sectoral OG weights. In multi-sector SOEs with production networks, the role of input-output linkages is primarily captured by the Leontief inverse $\mathbf{L}_{vx}$, which is smaller than its counterpart in closed economies $\mathbf{L} \equiv (\mathbf{I} - \boldsymbol{\Omega})^{-1}$, but larger than the identity matrix that corresponds to the horizontal economy without IO linkages. Therefore, for the CPI channel common to open and closed economies —i.e., $\widetilde{\boldsymbol{\lambda}}_D \equiv (\boldsymbol{\beta} \odot \boldsymbol{v})^\top \mathbf{L}_{vx}$ versus $\boldsymbol{\lambda} \equiv \boldsymbol{\beta}^\top \mathbf{L}$— the introduction of IO linkages increases sectoral OG weights in open economies, but by a *smaller* extent than the increase in closed economies. For the open-economy-specific expenditure-switching channel, however, the introduction of IO linkages increases sectoral OG weights in open economies, but not in closed economies. The compounding of these two countervailing forces generates an ambiguous effect of introducing IO linkages in open economies relative to closed economies on different sectors.

In Figure F.1, we plot the difference between multi-sector open and closed economies in the difference between the normalized OG weights with and without IO linkages for Mexico sectors. As shown in Figure F.1, the difference in the difference of the sectoral normalized OG weight —capturing the increase in relative sectoral weights due to the introduction of IO linkages in multi-sector open relative to closed economies— is weakly and positively correlated with the sectoral share of the CPI channel in the OG weight.

G. Import shares and OG weights

Our definitions of sectoral relevance metrics in equations (16), (17), and (19) include the Leontief inverse that depends on the import shares and input-output matrix. Therefore, by combining the equations of sectoral relevance metrics and the definition of the Leontief inverse matrix, we can determine how the

Figure F.1: Difference between multi-sector open and closed economies in the difference between the normalized OG weights with and without IO linkages for sectors (Mexico)



Notes: Shown in the figure is the difference between multi-sector open and closed economies in the difference between the normalized OG weights with and without IO linkages (y-axis) for sectors in Mexico, against the sector's share of CPI channel in the OG weight (x-axis). The solid black line is a linear fit of the difference in difference on the x-axis.

import structure of the economy influences our sectoral relevance metrics and the sectoral OG weights, as summarized by the following proposition.

Proposition G.1. *The total content in domestic consumption of domestic sector i (i.e., $\widetilde{\lambda}_{D,i}$) strictly decreases in its import share of consumption $(1 - v_i)$ if and only if $\beta_i > 0$; $\widetilde{\lambda}_{D,i}$ strictly decreases in its direct downstream sector r's import share of sector i goods (i.e., $\omega_{r,i} v_{x,r,i} > 0$) if and only if $\widetilde{\lambda}_{D,r} > 0$; $\widetilde{\lambda}_{D,i}$ strictly decreases in its indirect downstream sector s' import share of sector r goods if and only if $\widetilde{\lambda}_{D,s} > 0$, $\omega_{s,r} > 0$, and $\ell_{vx,r,i} > 0$.*

Proof: See Appendix K.10.

Proposition G.1 shows that the total content in domestic consumption of a domestic sector i decreases in sector i's import share of foreign goods as consumption, as well as sector i's direct and indirect downstream sectors' import shares (of intermediate inputs). Intuitively, more direct and indirect imports reduce the sector's contribution to the domestic aggregate output, thereby reducing the size of the CPI channel and resulting in a smaller OG weight. This implies that monetary policy *should* assign higher weights to inflation in domestic sectors with small direct and indirect (via downstream sectors) import shares.[15]

[15] Our model with a fully-fledged production network and analytical solutions allows us to identify three channels determining the sectoral weights in the monetary policy. The total content in exports in our analysis encompasses the export share of upstream sector that Wei and Xie (2020) outline by numerical simulations in the special case of a vertical network.

H. Additional results of welfare and the optimal monetary policy

Welfare loss as a function of the output gap. We substitute the sectoral Phillips curves (equation 34) in Proposition 2 into the welfare loss (equation 39) in Proposition 4 to re-write the welfare loss as a function of the output gap and exogenous shocks, yielding the following:

$$\begin{aligned} u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = & \underbrace{-\frac{1}{2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{C}^{gap}(\boldsymbol{\xi})^2}_{\textit{output gap misallocation}} \\ & \underbrace{\underbrace{-\frac{1}{2}\boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\cdot\widehat{C}^{gap}(\boldsymbol{\xi})^2 - (\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\cdot\widehat{C}^{gap}(\boldsymbol{\xi})}_{\text{output-gap-related}} \underbrace{-\frac{1}{2}(\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})^\top\boldsymbol{\mathcal{L}}(\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})}_{\text{policy-irrelevant}}}_{\textit{within- and across-sector, and cross-border misallocation}} + o(\|\widehat{\boldsymbol{\xi}}\|^2), \end{aligned} \quad \text{(H.1)}$$

where $\boldsymbol{\mathcal{L}} \equiv \boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb}$.

Equation (H.1) shows that the welfare loss depends on the *output gap misallocation* (the first line on the RHS of equation H.1, as already shown in equation 39), as well as the *within- and across-sector, and cross-border misallocation* (the second line of equation H.1). This second component is further decomposed into two sub-components: (i) the output-gap-related component, and (ii) the policy-irrelevant component of exogenous shocks that cannot be influenced by monetary policy.

Equation (H.1) shows that closing the output gap (i.e., $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$) eliminates the output gap misallocation and the output-gap-related component of the within- and across-sector, and cross-border misallocation, but it is unable to eliminate the misallocation arising from the policy-irrelevant sectoral shocks.

Optimal monetary policy as a function of the output gap. To further study the difference between the optimal and the output gap targeting policies, we relate the optimal monetary policy to the output gap by noticing that the optimal monetary policy is equivalent to choosing the output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ that minimizes welfare loss in equation (H.1).

Proposition H.1 (Output gap in the optimal monetary policy)**.** *The optimal monetary policy satisfies the first-order condition of equation (H.1) with respect to the output gap* $\widehat{C}^{gap}(\boldsymbol{\xi})$*, viz.:*

$$\left[\sigma - 1 + (\varphi+1)/\Lambda_L + \boldsymbol{\mathcal{B}}^\top\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\right]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}} = 0. \quad \text{(H.2)}$$

Proof: See Appendix M.2.

Proposition H.1 highlights that the output gap targeting —which closes the output gap (i.e., $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$)— does not satisfy condition (H.2) for the optimal monetary policy. In multi-sector economies, those sector-specific cost-push components in sectoral Phillips curves do not comove with the one-dimensional output gap (i.e., $\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}} \neq \mathbf{0}$ in equation 34), thus making the output gap targeting unable to simultaneously minimize the within- and across-sector, and cross-border misallocation (as captured by $\mathcal{B}^\top \mathcal{L}\mathcal{V}\widehat{\boldsymbol{\xi}}$ in equation H.2). Proposition H.1 shows that the "divine coincidence" in multi-sector open economies breaks down as in multi-sector closed economies: the output gap targeting that closes the output gap does not simultaneously minimize the within- and across-sector, and cross-border misallocation and is therefore suboptimal.

I. Quantitative analysis

I.1. Data and calibration

We calibrate our model of a small open economy with production networks using the World Input-Output Database. The WIOD covers 28 EU countries and 15 other major countries in the world from 2000 to 2014 and provides information for 56 major sectors.[16] Specifically, we calibrate our model using the National Input-Output Tables from the WIOD in 2014 for each country. The NIOTS provides each country's sector-level imports from the Rest of the World (RoW) and exports to the RoW, which are aggregates of the country's imports from and exports to all other countries, respectively, including those countries that are not listed in the WIOD. For each sector in each country, the NIOTS reports the following sectoral values: (i) intermediate goods expenditures on goods from different domestic and foreign sectors, (ii) labor compensation, (iii) gross output, (iv) value-added, and (v) exports to foreign countries. Using the NIOTS data, we calibrate each country one at a time as a small open economy against the RoW, instead of simultaneously calibrating all countries at once in a global equilibrium.

For each country, we calibrate the parameters as follows: (i) the (i, j) element of the input-output matrix $\boldsymbol{\Omega}$ is calibrated using the share of customer sector i's intermediate goods expenditure on the supplier sector j (the sum of expenditures on the domestic and foreign sector j) in the customer i's gross output; (ii) the (i, j) element of the home bias in intermediate inputs $\mathbf{V}_x$ is calibrated using the ratio of customer sector i's intermediate goods expenditure on the domestic supplier sector j to the sum of expenditures on the domestic and foreign sector j's goods; (iii) the sectoral labor share of $\boldsymbol{\alpha}$ is calibrated using the share of sectoral labor compensation in sectoral gross output for each sector; (iv) the steady-state values of sectoral demand from foreign countries ($D^*_{EX,Fi}$) are calibrated such that the sectoral export-to-GDP ratios in the model matches the sector's export-to-GDP ratios in the data; (v) the i-th element of the consumption shares $\boldsymbol{\beta}$ is calibrated using the ratio of the sum of domestic households' and government's consumption expenditures on sector i goods to the value added of sector i; and (vi) the i-th element of the home bias in consumption $\mathbf{v}$ is calibrated using the ratio of the sum of domestic household's and government's consumption expenditures on the domestic sector i's goods to the sum of expenditures on the domestic and foreign sector i's goods.

We calibrate the values of other parameters to their standard levels as per the literature. The risk aversion parameter and the inverse of the labor supply elasticity of the households are calibrated to $\sigma = 2$ and $\varphi = 1$, respectively, following the business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019). We

[16] We use the version of Release 2016 of the World Input-Output Database. Shown in Table I.1 is the list of sectors.

follow Atkeson and Burstein (2008) and calibrate the within-sector elasticity of substitution to $\varepsilon_i = 8$ for all sectors i. We calibrate the elasticity of substitution between domestic and foreign goods to 5 for both domestic and foreign households and firms —*viz.*, $\theta_i = \theta_{Fi} = 5$ for all sectors i, following Head and Mayer (2014). We calibrate the sector-level parameters of price rigidity δ_i using the sector-level price rigidities from Pasten et al. (2024). With the calibrated sector-level price rigidities, the average quarterly frequency of price adjustment across all sectors equals 0.49. We follow Rubbo (2023) and La'O and Tahbaz-Salehi (2022) to introduce wage stickiness by adding a labor sector 0; it uses domestic labor to produce the product of "labor" that is supplied to all other sectors as inputs. We follow Beraja et al. (2019) and Barattieri et al. (2014) to calibrate the parameter of wage rigidity δ_0 such that the quarterly frequency of wage adjustment equals 0.25. Summarized in Table 2 in Section 6 is the calibration of different parameters.

Last, we calibrate the exogenous shocks as now described. We calculate the growth rates of sectoral import prices and productivity using the social economic accounts in the WIOD. We compute the covariance matrix between different sectors' import price series and use it to calibrate the covariance matrix of import prices used in the simulation of the model. We use the same method to calibrate the covariance matrix for the sectoral productivity.

Table I.1: Industry classifications in the World Input-Output Database

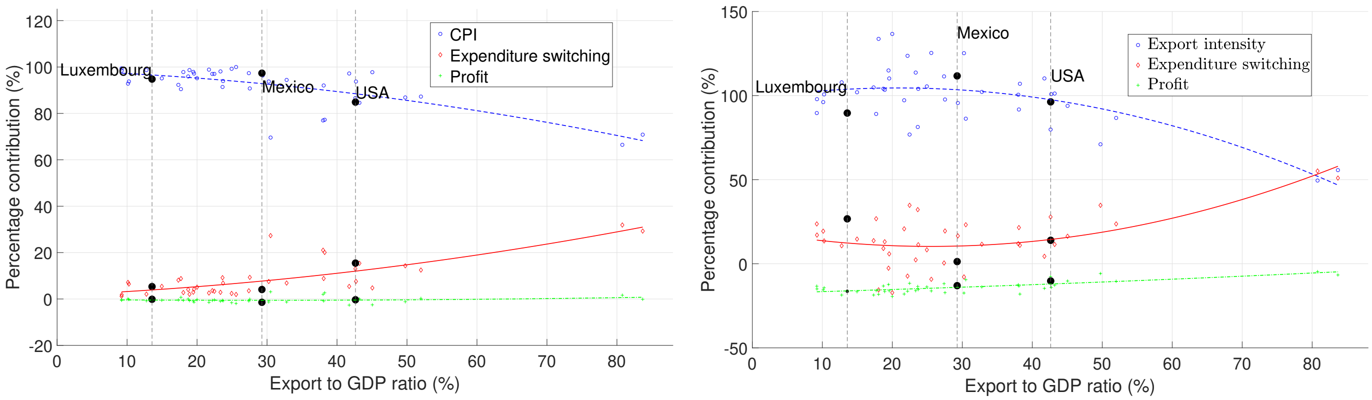
ID	Industry code	Description	ID	Industry code	Description
1	A01	Crop and animal production, hunting and related service	29	G46	Wholesale trade, except of motor vehicles and motorcycles
2	A02	Forestry and logging	30	G47	Retail trade, except of motor vehicles and motorcycles
3	A03	Fishing and aquaculture	31	H49	Land transport and transport via pipelines
4	B	Mining and quarrying	32	H50	Water transport
5	C10-C12	Manufacture of food products, beverages and tobacco products	33	H51	Air transport
6	C13-C15	Manufacture of textiles, wearing apparel and leather products	34	H52	Warehousing and support activities for transportation
7	C16	Manufacture of wood products, plaiting materials	35	H53	Postal and courier activities
8	C17	Manufacture of paper and paper products	36	I	Accommodation and food service activities
9	C18	Printing and reproduction of recorded media	37	J58	Publishing activities
10	C19	Manufacture of coke and refined petroleum products	38	J59_J60	Motion picture, video, and television
11	C20	Manufacture of chemicals and chemical products	39	J61	Telecommunications
12	C21	Manufacture of basic pharmaceutical products	40	J62_J63	Computer programming, consultancy and related activities; information service activities
13	C22	Manufacture of rubber and plastic products	41	K64	Financial service activities, except insurance and pension funding
14	C23	Manufacture of other non-metallic mineral products	42	K65	Insurance, reinsurance and pension funding, except compulsory social security
15	C24	Manufacture of basic metals	43	K66	Activities auxiliary to financial services and insurance activities
16	C25	Manufacture of fabricated metal products	44	L68	Real estate activities
17	C26	Manufacture of computer, electronic and optical products	45	M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities
18	C27	Manufacture of electrical equipment	46	M71	Architectural and engineering activities; technical testing and analysis
19	C28	Manufacture of machinery and equipment n.e.c.	47	M72	Scientific research and development
20	C29	Manufacture of motor vehicles, trailers and semi-trailers	48	M73	Advertising and market research
21	C30	Manufacture of other transport equipment	49	M74_M75	Other professional, scientific and technical activities; veterinary activities
22	C31_C32	Manufacture of furniture; other manufacturing	50	N	Administrative and support service activities
23	C33	Repair and installation of machinery and equipment	51	O84	Public administration and defence; compulsory social security
24	D35	Electricity, gas, steam and air conditioning supply	52	P85	Education
25	E36	Water collection, treatment and supply	53	Q	Human health and social work activities
26	E37-E39	Sewerage; waste management services	54	R_S	Other service activities
27	F	Construction	55	T	Activities of households as employers
28	G45	Wholesale and retail trade,repair motor vehicles	56	U	Activities of extraterritorial organizations and bodies

I.2. Variance decomposition of normalized OG weights and their differences from misspecified weights

For each country, we compute the percentage contribution of each of the three components in the normalized OG weights —namely, the *CPI*, the *expenditure-switching*, and the *profit channels*, respectively— to the variance of the normalized OG weight in equation (31) using the following variance decomposition:

$$100\% = \frac{cov\big(\widetilde{\lambda}_{D,i}\frac{1-\delta_i}{\delta_i\kappa_{OG}}, \widetilde{\mathcal{M}}_{OG,i}\big)}{var(\widetilde{\mathcal{M}}_{OG,i})} + \frac{cov\big(\kappa_{CPI}^{-1}\widetilde{\rho}_{ES,i}\frac{1-\delta_i}{\delta_i\kappa_{OG}}, \widetilde{\mathcal{M}}_{OG,i}\big)}{var(\widetilde{\mathcal{M}}_{OG,i})} + \frac{cov\big(\kappa_{CPI}^{-1}(\widetilde{\lambda}_{F,i} - \lambda_i(1-\widetilde{\alpha}_i))\frac{1-\delta_i}{\delta_i\kappa_{OG}}, \widetilde{\mathcal{M}}_{OG,i}\big)}{var(\widetilde{\mathcal{M}}_{OG,i})}. \quad \text{(I.1)}$$

Figure I.1: Variance decomposition of normalized OG weights and their deviations from normalized Domar weights



(a) Normalized OG weights

(b) Difference between normalized Domar and OG weights

Notes: Shown in the scatter plot in panels (a) and (b) are the percentage contribution of each of the three channels (or components) to the normalized OG weight and the percentage deviation of normalized Domar from OG weights, respectively, for each country (y-axis) against the country's economy-wise export-to-GDP ratio (x-axis). In panel (a), the CPI, the expenditure-switching, and the profit channels are denoted by blue circles, red dots, and green stars, respectively. In panel (b), the export intensity, expenditure-switching, and profit components are denoted by blue circles, red dots, and green stars, respectively. The dashed-blue, solid-red, and dash-dotted-green lines are the fitted curves for the three channels (or components), respectively.

Shown in panel (a) of Figure I.1 is the percentage contribution of each of the three channels to the total variation in normalized OG weights for each country in the sample. Each set of the vertically aligned markers in blue circles, red dots, and green stars represents the contributions of the *CPI*, the *expenditure-switching*, and the *profit channels*, respectively, for a specific country. The vertical dashed lines indicate the cases for the U.S., Mexico, and Luxembourg, as representative economies with polar degrees of openness (relatively closed or fully open). The dashed-blue, solid-red, and dash-dotted-green lines reflect the fitted curves for each of the three channels across countries.

As shown in the figure, the *CPI channel* (blue circle) and *expenditure-switching channel* (red dot) explain the bulk of the variation in the normalized sectoral OG weights across sectors for all countries. In contrast, the contribution of the *profit channel* (green star) is marginal, as evinced by the near-zero dashed-dotted green line. Moreover, the percentage contribution of the *expenditure-switching channel* (*CPI channel*) increases (declines) with the openness of the country measured by the economy-wise export-to-GDP ratio, as shown by the rising solid-red line (the declining dashed-blue line).[17] For example, in Luxembourg —the most open economy in our sample with an economy-wise export-to-GDP ratio of 83%— both the *CPI* and the *expenditure-switching channels* contribute significantly to the sectoral variance in the normalized OG weights, with contributions of 70.8% and 29.3%, respectively. In contrast, for the U.S. —a nearly closed economy with an export-to-GDP ratio of 9%— the *CPI channel* contributes to almost the entire variation in normalized OG weights (99.4%) while the contribution of the *expenditure-switching channel* is minimal

[17]The patterns are robust to the alternative measurement of the degree of openness using the economy-wise import-to-GDP ratio and the ratio of total trade volume to GDP.

(1.1%).

Variance decomposition of the pitfalls in output gap targeting that disregards cross-border linkages. To quantify the pitfalls of output gap targeting that disregards cross-border linkages, panel (b) of Figure I.1 further shows the percentage contribution of each of the three components —namely, the *export intensity*, *expenditure-switching*, and *profit components*, respectively— to the variance of the difference between normalized Domar and OG weights using the following variance decomposition:[18]

$$100\% = \frac{cov\Big(\frac{\kappa_\lambda}{\kappa_{OG}}\frac{\widetilde{\lambda}_{F,i}}{\lambda_i}, \frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i}\Big)}{var\big(\frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i}\big)} + \frac{cov\Big(-\frac{\kappa_\lambda \kappa_{CPI}^{-1}}{\kappa_{OG}}\frac{\widetilde{\rho}_{ES,i}}{\lambda_i}, \frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i}\Big)}{var\big(\frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i}\big)} + \frac{cov\Big(\frac{\kappa_\lambda \kappa_{CPI}^{-1}}{\kappa_{OG}}\frac{\lambda_i(1-\widetilde{\alpha}_i)-\widetilde{\lambda}_{F,i}}{\lambda_i}, \frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i}\Big)}{var\big(\frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i}\big)}. \tag{I.2}$$

As shown in the figure, the normalized sectoral Domar-OG difference is predominantly driven by the export intensity (blue circle) component with an average percentage contribution of 99.6%. Therefore, the pitfalls of the output gap targeting that disregards cross-border linkages arise from overlooking the sector's contribution to exports as an input supplier, particularly for economies with a medium degree of openness (i.e., 20% to 30%). In contrast, the contributions of the *expenditure-switching* (red dot) and *profit components* (green star) are small, except in economies with extremely large openness like Luxembourg, where the *expenditure-switching component* contributes to a substantial percentage of the normalized Domar-OG difference that is even greater than the *export intensity component*. These results imply that, to correct for the pitfalls of closed-economy output gap targeting —which coincides with the PPI targeting used in one-sector SOE literature— the monetary authority needs to adjust the weights of sectors downward by their direct and indirect exports, with larger downward adjustments on those sectors with large export intensity.

As shown in the figure, the normalized sectoral Domar-OG difference is mostly driven by the export intensity (blue circle) and the expenditure switching (red dot) components, with average percentage contributions of 99% and 14%, respectively. Moreover, the percentage contribution of the export intensity (expenditure switching) component increases (declines) with the openness of the country measured by the economy-wise export-to-GDP ratio, as shown by the rising solid-red line (the declining dashed-blue line).

Therefore, in economies with small and medium openness, the pitfalls of output gap targeting that disregards cross-border linkages —which uses closed-economy normalized Domar weights and coincides with the PPI targeting policy used in one-sector SOE literature— arise from overlooking the sector's contribution to exports as an input supplier. To correct for such pitfalls, the monetary authority mainly needs to adjust the weights of sectors downward by their direct and indirect exports, with larger downward adjustments on those sectors with large export intensity.

In contrast, in economies with extremely large openness like Luxembourg, the pitfalls of output gap targeting that disregards openness (or PPI targeting) also arise from overlooking the sector's direct and indirect contribution to the relative domestic-to-foreign prices and demand and, in turn, the demand for

[18]Equation (I.2) is derived using the bilinearity of covariance together with equation (38).

domestic labor. To correct for such a pitfall, the monetary authority needs to adjust the weights of sectors upward, with larger adjustments on those sectors with large generalized expenditure-switching elasticity.

I.3. Sectoral weights under alternative monetary policies

All of the alternative monetary policies we study in Section 6.3 are implemented by setting the following aggregate inflation index to zero:

$$\boldsymbol{\chi}^{\top}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = 0, \tag{I.3}$$

where the sectoral weights $\boldsymbol{\chi} \equiv \{\chi_i\}_i$ sum to be one, and for each monetary policy $\boldsymbol{\chi}$, are specified as follows:

$$\begin{aligned}
\text{optimal monetary policy:}\quad & \chi_i = \frac{\mathcal{M}_{OP,i}}{\sum_{i'} \mathcal{M}_{OP,i'}};\\
\text{output gap targeting:}\quad & \chi_i = \widetilde{\mathcal{M}}_{OG,i} \equiv \frac{\mathcal{M}_{OG,i}(1-\delta_i)/\delta_i}{\sum_{i'} \mathcal{M}_{OG,i'}(1-\delta_{i'})/\delta_{i'}};\\
\text{PPI targeting policy:}\quad & \chi_i = \widetilde{\lambda}_i \equiv \frac{\lambda_i(1-\delta_i)/\delta_i}{\sum_{i'} \lambda_{i'}(1-\delta_{i'})/\delta_{i'}};\\
\text{CPI-weight policy:}\quad & \chi_i = \widetilde{\beta}_i \equiv \frac{\beta_i(1-\delta_i)/\delta_i}{\sum_{i'} \beta_{i'}(1-\delta_{i'})/\delta_{i'}};\\
\text{output gap targeting w/o IO linkages:}\quad & \chi_i = \frac{\mathcal{M}^{NoIO}_{OG,i}(1-\delta_i)/\delta_i}{\sum_{i'} \mathcal{M}^{NoIO}_{OG,i'}(1-\delta_{i'})/\delta_{i'}},
\end{aligned}$$

where $\{\mathcal{M}_{OP,i}\}_i$ in the optimal monetary policy is given by

$$\{\mathcal{M}_{OP,i}\}_i \equiv \boldsymbol{\mathcal{M}}_{OP} = \left\{[\sigma - 1 + (\varphi+1)/\Lambda_L]\,\kappa_C^{-1}\boldsymbol{\mathcal{M}}_{OG}^{\top}(\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \boldsymbol{\mathcal{B}}^{\top}\boldsymbol{\mathcal{L}}\right\}^{\top},$$

as in Proposition 5. Note that $\boldsymbol{\mathcal{M}}_{OG}$ and $\boldsymbol{\mathcal{M}}_{OG}^{NoIO}$ are the vectors of the OG weights with and without IO linkages (by setting $\boldsymbol{\Omega} = \mathbf{0}$ and $\boldsymbol{\alpha} = \mathbf{1}$), respectively. $\boldsymbol{\Delta}$ is the diagonal matrix for the frequency of price adjustment (see Table 1). Combining the monetary policy rule in equation (I.3) with the sectoral Phillips curves in equation (34) yields the output gap as a function of the specific policy weights $\boldsymbol{\chi}$ and the parameters of the sectoral Phillips curves, *viz.*:

$$\widehat{C}^{gap}(\widehat{\boldsymbol{\xi}}) = -\frac{\boldsymbol{\chi}^{\top}\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}}}{\boldsymbol{\chi}^{\top}\boldsymbol{\mathcal{B}}}. \tag{I.4}$$

Substituting equation (I.4) into the welfare loss function in equation (H.1) of Appendix H, we obtain the welfare loss under the alternative monetary policy with policy weights $\boldsymbol{\chi}$ and any realized state $\boldsymbol{\xi}$.

I.4. *Welfare loss under alternative shocks and alternative monetary policies*

Table I.2: Welfare loss under different monetary policies: Shocks to import prices of only manufacturing sectors

	(1) Optimal	(2) Output gap targeting	(3) PPI targeting	(4) Output gap targeting w/o IO	(5) CPI targeting
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-3.334	-3.357	-3.428	-6.669	-6.620
Improvement by OG targeting towards optimal			75.8%	99.3%	99.3%
Output gap misallocation	-0.003	0.000	-0.004	-0.415	-0.408
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.026	0.000	-0.068	-2.898	-2.855
— policy-irrelevant	-3.357	-3.357	-3.357	-3.357	-3.357
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-1.595	-1.604	-1.678	-3.012	-4.545
Improvement by OG targeting towards optimal			89.2%	99.4%	99.7%
Output gap misallocation	-0.002	0.000	-0.007	-0.220	-0.481
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.011	0.000	-0.067	-1.189	-2.460
— policy-irrelevant	-1.604	-1.604	-1.604	-1.604	-1.604
U.S. Export-to-GDP ratio: 9.2%					
Total welfare loss	-2.634	-2.734	-2.740	-9.248	-9.816
Improvement by OG targeting towards optimal			5.5%	98.5%	98.6%
Output gap misallocation	-0.015	0.000	0.000	-0.758	-0.832
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.115	0.000	-0.006	-5.757	-6.250
— policy-irrelevant	-2.734	-2.734	-2.734	-2.734	-2.734

Notes: Reported in this table is the welfare loss —expressed in units of percent of steady-state consumption— under different monetary policy designs. Shown in columns (1) to (5) are the welfare losses under the optimal policy, output gap targeting, PPI targeting, output gap targeting without IO linkages, and CPI targeting policy, respectively.

Table I.3: Welfare loss under different monetary policies: Shocks to sectoral productivity

	(1) Optimal	(2) Output gap targeting	(3) PPI targeting	(4) Output gap targeting w/o IO	(5) CPI targeting
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-0.744	-0.754	-0.755	-1.527	-1.529
Improvement by OG targeting towards optimal			6.6%	98.7%	98.7%
Output gap misallocation	-0.001	0.000	0.000	-0.092	-0.093
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.011	0.000	-0.001	-0.681	-0.682
— policy-irrelevant	-0.754	-0.754	-0.754	-0.754	-0.754
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-3.057	-3.061	-3.213	-3.833	-3.459
Improvement by OG targeting towards optimal			97.4%	99.5%	99.0%
Output gap misallocation	-0.001	0.000	-0.022	-0.123	-0.061
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.005	0.000	-0.130	-0.648	-0.337
— policy-irrelevant	-3.061	-3.061	-3.061	-3.061	-3.061
U.S. Export-to-GDP ratio: 9.2%					
Total welfare loss	-1.208	-1.216	-1.216	-2.047	-2.056
Improvement by OG targeting towards optimal			2.3%	99.1%	99.1%
Output gap misallocation	-0.001	0.000	0.000	-0.103	-0.104
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.009	0.000	0.000	-0.729	-0.737
— policy-irrelevant	-1.216	-1.216	-1.216	-1.216	-1.216

Notes: Reported in this table is the welfare loss —expressed in units of percent of steady-state consumption— under different monetary policy designs. Shown in columns (1) to (5) show the welfare losses under the optimal policy, output gap targeting, PPI targeting, output gap targeting without IO linkages, and CPI targeting policy, respectively.

Table I.4: Welfare loss under alternative optimal monetary policies in open versus closed economies

	(1)	(2)	(3)	(4)
	Open-economy policies		Closed-economy policies	
	w/ IO Policy	w/o IO Policy	w/ IO Policy	w/o IO Policy
Mexico Export-to-GDP ratio: 19%				
Total welfare loss	-0.751	-0.754	-0.979	-0.983
Welfare difference		-0.42%		-0.36%
Output gap misallocation	-0.001	-0.004	-0.002	-0.004
Within- and across-sector, and cross-border misallocation				
— output-gap-related	0.011	0.010	0.016	0.015
— policy-irrelevant	-0.760	-0.760	-0.993	-0.993
Luxembourg Export-to-GDP ratio: 83%				
Total welfare loss	-3.067	-3.209	-2.398	-2.409
Welfare difference		-4.64%		-0.45%
Output gap misallocation	-0.001	-0.034	-0.007	-0.016
Within- and across-sector, and cross-border misallocation				
— output-gap-related	0.005	-0.105	0.056	0.053
— policy-irrelevant	-3.071	-3.071	-2.447	-2.447
U.S. Export-to-GDP ratio: 9.2%				
Total welfare loss	-1.189	-1.189	-1.228	-1.229
Welfare difference		-0.07%		-0.06%
Output gap misallocation	-0.001	-0.002	-0.001	-0.002
Within- and across-sector, and cross-border misallocation				
— output-gap-related	0.009	0.009	0.009	0.010
— policy-irrelevant	-1.196	-1.196	-1.236	-1.236

Notes: Shown in the table is the welfare loss (in units of percent of steady-state consumption) and its associated welfare components under optimal and alternative monetary policies. In the open economy, we compute the welfare losses under two monetary policies: column (1) is for the correct optimal policy considering IO linkages, and column (2) uses an alternative policy, which is the optimal monetary policy but for the counterfactual multi-sector SOE without IO linkages. Similarly, in the closed economy, we compute the welfare losses under two monetary policies: column (3) shows the correct optimal policy considering IO linkages, and column (4) shows the results for an alternative policy, which is the optimal monetary policy for the counterfactual multi-sector closed economies without IO linkages. For each case, we assume the economy has productivity shocks with the mean of 1% for all sectors, and the variance-covariance matrix of these shocks is calibrated based on Mexico data; and then we simulate the shocks 100,000 times for each case to compute the expected welfare loss.

J. Basic results of the model

This section derives some basic results of the model, thus preparing for the proofs of our main theoretical results in Sections 2, 3, 4, and 5.

J.1. Feasible allocation

The feasible allocation of the economy can be defined at the sector level with the help of an additional variable ι_i that captures the within-sector output dispersion in each sector i, as stated in the following definition:

Definition J.1 (Feasible allocation)**.** *Denote the use of labor and intermediate inputs of each sector i and j by*

$$\big(L_i, X_{i,j}, X_{Hi,Hj}, X_{Hi,Fj}\big) \equiv \int_0^1 \big(L_{if}, X_{if,j}, X_{Hif,Hj}, X_{Hif,Fj}\big) df.$$

A feasible allocation is a state-contingent allocation of C, $\{C_i\}_i$, $\{Y_i\}_i$, $\{L_i\}_i$, $\{X_{i,j}\}_{i,j}$, $\{C_{Hi}\}_i$, $\{C_{Fi}\}_i$, $\{X_{Hi,Hj}\}_{i,j}$, $\{X_{Hi,Fj}\}_{i,j}$, L, $\{Y_{EX,i}\}_i$, and $\{\iota_i\}_i$ that satisfies the following equations (J.1)-(J.8) for each $i, j \in \{1, 2, \cdots, N\}$ and any realized state $\boldsymbol{\xi} \equiv \big\{A_i, D^*_{EX,Fi}, P^*_{IM,Fi}\big\}_i \in \boldsymbol{\Xi}$:

$$\text{(consumption basket)} \qquad C = \mathcal{C}\big(\{C_i\}_i\big), \tag{J.1}$$

$$\text{(production function)} \qquad Y_i = A_i \cdot \iota_i \cdot F_i\big(L_i, \{X_{i,j}\}_j\big), \tag{J.2}$$

$$\text{(consumption with import)} \qquad C_i = \mathcal{C}_i\big(C_{Hi}, C_{Fi}\big), \tag{J.3}$$

$$\text{(intermediate inputs with import)} \qquad X_{i,j} = \mathcal{X}_{i,j}\big(X_{Hi,Hj}, X_{Hi,Fj}\big), \tag{J.4}$$

$$\text{(labor market clearing)} \qquad L = \sum_i L_i, \tag{J.5}$$

$$\text{(goods market clearing)} \qquad Y_i = C_{Hi} + \sum_j X_{Hj,Hi} + Y_{EX,i}, \tag{J.6}$$

$$\text{(balance of trade)} \qquad EX \equiv \sum_i \big(D^*_{EX,Fi}\big)^{\frac{1}{\theta_{F,i}}} Y_{EX,i}^{\frac{\theta_{F,i}-1}{\theta_{F,i}}} = \sum_i P^*_{IM,Fi}\Big(C_{Fi} + \sum_j X_{Hj,Fi}\Big), \tag{J.7}$$

$$\text{(within-sector output dispersion)} \qquad \iota_i \equiv Y_i \Big/ \Big(\int_0^1 Y_{if} df\Big), \tag{J.8}$$

where the aggregators $F_i = (L_{if}/\alpha_i)^{\alpha_i} \prod_{j=1}^N (X_{i,j}/\omega_{i,j})^{\omega_{i,j}}$ following equation (1), $\{\mathcal{X}_{i,j}\}_{i,j}$ is defined in equation (2), and $\mathcal{C}$ and $\{\mathcal{C}_i\}_i$ are defined in equation (5).

For sector-level conditions in equations (J.1) to (J.8) to summarize the feasible allocation of the economy at the firm level, all firms within each sector must share the same marginal product of inputs, which happens to hold in the *first-best allocation*, the *sticky-price equilibrium*, and the *flexible-price equilibrium* under our model setup.

J.2. Proof of Lemma 1: Efficient flexible-price equilibrium

To prove Lemma 1, we define the *first-best allocation* (Definition J.2), present the conditions for it (Lemma J.1), and show that these conditions coincide with those for the *flexible-price equilibrium* when Assumption 1 holds.

The *first-best allocation* is the feasible allocation that solves the social planner's problem, as outlined in the following definition.

Definition J.2 (First-best allocation)**.** *The first-best allocation is a feasible allocation that maximizes the representative household's utility* $u(C, L)$ *—i.e., it solves the following social planner's problem:*

$$\max_{\{\iota_i, L_i, \{X_{Hi,Hj}, X_{Hi,Fj}\}_j, C_{Hi}, C_{Fi}\}_i} u(C, L)$$
$$\text{s.t.} \quad \text{equations (J.1) to (J.7) and } \iota_i \in [0,1] \text{ for all } i.$$

Substituting equations (J.1), (J.3), and (J.5) into the utility function $u(C, L)$ yields the following:

$$u(C, L) = u\Big(\mathcal{C}\big(\{\mathcal{C}_i(C_{Hi}, C_{Fi})\}_i\big), \sum_i L_i\Big). \tag{J.9}$$

Substituting equations (J.2), (J.4), and (J.6) into equation (J.7) yields the consolidated constraint of the social planner's problem in the following:

$$\sum_i \left(D^*_{EX,Fi}\right)^{\frac{1}{\theta_{F,i}}} \Big[A_i F_i\big(\{L_i, \mathcal{X}_{i,j}(X_{Hi,Hj}, X_{Hi,Fj})\}_j\big) - C_{Hi} - \sum_j X_{Hj,Hi}\Big]^{\frac{\theta_{F,i}-1}{\theta_{F,i}}} = \sum_i P^*_{IM,Fi}\Big(C_{Fi} + \sum_j X_{Hj,Fi}\Big). \tag{J.10}$$

As a result, the first-best allocation is the feasible allocation that maximizes the utility function in equation (J.9) —subject to the constraint in equation (J.10)— which, in turn, satisfies the optimality conditions outlined in Lemma J.1.

Lemma J.1 (First-best allocation)**.** *The first-best allocation satisfies the following optimality conditions:*

$$\iota_i = 1, \tag{J.11}$$

$$-\frac{\partial u/\partial L}{\frac{\partial u}{\partial C}\frac{\partial \mathcal{C}}{\partial \mathcal{C}_i}\frac{\partial \mathcal{C}_i}{\partial C_{Hi}}} = A_i \frac{\partial F_i}{\partial L_i}, \tag{J.12}$$

$$\frac{\partial \mathcal{C}/\partial \mathcal{C}_j}{\partial \mathcal{C}/\partial \mathcal{C}_i}\frac{\partial \mathcal{C}_j/\partial C_{Hj}}{\partial \mathcal{C}_i/\partial C_{Hi}} = A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}\frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}}, \tag{J.13}$$

$$\frac{\partial \mathcal{C}_i/\partial C_{Fi}}{\partial \mathcal{C}_i/\partial C_{Hi}} = P^*_{IM,Fi} \cdot \frac{\theta_{F,i}}{\theta_{F,i}-1}\left(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\right)^{\frac{1}{\theta_{F,i}}}, \tag{J.14}$$

$$\frac{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Fj}}{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Hj}} = P^*_{IM,Fj} \cdot \frac{\theta_{F,j}}{\theta_{F,j}-1}\Big(\frac{Y_{EX,j}}{D^*_{EX,Fj}}\Big)^{\frac{1}{\theta_{F,j}}}. \tag{J.15}$$

Proof of Lemma J.1. To eliminate distortions and maximize welfare, the social planner would close the within-sector dispersion in output —i.e., choosing $\iota_i = 1$. Furthermore, denote κ the multiplier for the constraint (J.10) of the social planner's problem, and the first-order conditions w.r.t. L_i, $X_{Hi,Hj}$, $X_{Hi,Fj}$, C_{Hi}, and C_{Fi} are as follows:

$$\begin{aligned}
0 &= \frac{\partial u}{\partial L} + \kappa \cdot \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial L_i},\\
0 &= \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}\frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}} - \frac{\theta_{F,j}-1}{\theta_{F,j}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fj}}\Big)^{-\frac{1}{\theta_{F,j}}},\\
0 &= \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}\frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Fj}} - P^*_{IM,Fj},\\
0 &= \frac{\partial u}{\partial C}\frac{\partial \mathcal{C}}{\partial C_i}\frac{\partial \mathcal{C}_i}{\partial C_{Hi}} - \kappa \cdot \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}},\\
0 &= \frac{\partial u}{\partial C}\frac{\partial \mathcal{C}}{\partial C_i}\frac{\partial \mathcal{C}_i}{\partial C_{Fi}} - \kappa \cdot P^*_{IM,Fi}.
\end{aligned}$$

Rearranging the above first-order conditions and eliminating the multiplier κ yields equations (J.12)-(J.15) of Lemma J.1. □

Proof of Lemma 1. Under $\tau_i = -1/(\varepsilon_i - 1)$ of Assumption 1, in the *flexible-price equilibrium*, combining the optimal pricing conditions of the firms that maximize profits in equation (A.1) —subject to the demand function in equation (A.3)— with the cost minimization conditions that minimize the total costs in equation (3) —subject to the production technology in equations (1) and (2) —yields the following two conditions:

$$A_i \frac{\partial F_i}{\partial L_i}(\boldsymbol{\xi}) = \frac{W^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}, \tag{J.16}$$

$$A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}(\boldsymbol{\xi})\frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}}(\boldsymbol{\xi}) = \frac{P_j^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{J.17}$$

Under $\tau_{EX,i} = 1/\theta_{F,i}$ of Assumption 1, combining the export demand $Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}$ with the no-arbitrage condition $(1-\tau_{EX,i})P_{EX,i} = P_i$, yields the following equation:

$$\frac{\theta_{F,i}}{\theta_{F,i}-1}\Big(\frac{Y^{flex}_{EX,i}(\boldsymbol{\xi})}{D^*_{EX,Fi}}\Big)^{\frac{1}{\theta_{F,i}}} = \frac{S^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{J.18}$$

Furthermore, for the households' problem that maximizes utility function (4) —subject to the consumption aggregator (5) and budget constraint (6)— combining the first-order conditions with respect to L and C_{Hi} yields condition (J.19), combining the first-order conditions with respect to C_{Hj} and C_{Hi} yields condition

(J.20), and combining the first-order conditions with respect to C_{Fi} and C_{Hi} yields condition (J.21). For the firm's cost minimization problem that minimizes the total costs in equation (3) subject to the production technology in equations (1) and (2), combining the first-order conditions with respect to $X_{Hi,Fj}$ and $X_{Hi,Hj}$, yields condition (J.22).

$$-\frac{\partial u/\partial L}{\frac{\partial u}{\partial \mathcal{C}}\frac{\partial \mathcal{C}}{\partial C_i}\frac{\partial \mathcal{C}_i}{\partial C_{Hi}}}(\boldsymbol{\xi}) = \frac{W^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}, \tag{J.19}$$

$$\frac{\partial \mathcal{C}/\partial C_j}{\partial \mathcal{C}/\partial C_i}(\boldsymbol{\xi})\frac{\partial \mathcal{C}_j/\partial C_{Hj}}{\partial \mathcal{C}_i/\partial C_{Hi}}(\boldsymbol{\xi}) = \frac{P_j^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})} \tag{J.20}$$

$$\frac{\partial \mathcal{C}_i/\partial C_{Fi}}{\partial \mathcal{C}_i/\partial C_{Hi}}(\boldsymbol{\xi}) = \frac{P^*_{IM,Fi}S^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{J.21}$$

$$\frac{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Fj}}{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Hj}}(\boldsymbol{\xi}) = \frac{P^*_{IM,Fj}S^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{J.22}$$

Substituting equations (J.16)-(J.18) into equations (J.19)-(J.22) to eliminate all of the equilibrium prices $W^{flex}(\boldsymbol{\xi})$, $S^{flex}(\boldsymbol{\xi})$, and $\{P_i^{flex}(\boldsymbol{\xi})\}_i$, yields *exactly the same* conditions for the *flexible-price equilibrium as* the conditions (J.12)-(J.15) for the first-best allocation in Lemma J.1, thereby proving the efficiency of the *flexible-price equilibrium*. □

The role of export taxes $\{\tau_{EX,i}\}_i$. In closed economies à la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), non-contingent sector-specific subsidies $\tau_i = -1/(\varepsilon_i - 1)$ eliminate sectoral distortions due to monopolistic competition and, therefore, are sufficient to make the *flexible-price equilibrium* efficient. In open economies, however, it is welfare-enhancing for the social planner of the small open economy to exploit fully the monopoly powers of the domestic producers in the international market. As a result, the non-contingent sector-specific subsidies that eliminate the sectoral distortions due to monopolistic competition *alone* are no longer optimal in small open economies, and an additional non-contingent export tax $\tau_{EX,i} = 1/\theta_{F,i}$ is required to retain the monopoly powers of the domestic producers in the international market and make the *flexible-price equilibrium* efficient. Under such export taxes, the sectoral export prices become:

$$P_{EX,i} = \frac{1}{1-\tau_{EX,i}}P_i = \frac{\theta_{F,i}}{\theta_{F,i}-1}P_i, \quad \forall i \in \{1, 2, \cdots, N\}.$$

J.3. Steady-state Domar weights and sectoral export-to-GDP ratios

Lemma J.2 (Steady-state Domar weights and sectoral export-to-GDP ratios)**.** *The steady-state Domar weights* $\boldsymbol{\lambda}$ *and sectoral export-to-GDP ratios* $\boldsymbol{\lambda}_{EX}$ *are functions of parameters as in the following equations:*

$$\boldsymbol{\lambda}^\top = \left\{\boldsymbol{\beta} \odot \mathbf{v} + (1 - \boldsymbol{\beta}^\top \mathbf{v})[(\boldsymbol{\theta}_F - 1) \oslash \boldsymbol{\theta}_F \odot \mathbf{v}^*_H]\right\}^\top$$

$$\cdot \left\{ \mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x - (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x}) \mathbf{1} [(\boldsymbol{\theta}_F - 1) \oslash \boldsymbol{\theta}_F \odot \mathbf{v}_H^*]^\top \right\}^{-1}, \tag{J.23}$$

$$\boldsymbol{\lambda}_{EX}^\top = \boldsymbol{\lambda}^\top (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x) - (\boldsymbol{\beta} \odot \mathbf{v})^\top, \tag{J.24}$$

where $\mathbf{v}_H^*$ is the vector of the steady-state shares of sectoral exports in the value of the aggregate exports, with the i-th element v_{Hi}^* equal to:

$$v_{Hi}^* \equiv \frac{\left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{1-\theta_{F,i}} D_{EX,Fi}^{*,ss}}{\sum_{i'} \left(\frac{\theta_{F,i'}}{\theta_{F,i'}-1}\right)^{1-\theta_{F,i'}} D_{EX,Fi'}^{*,ss}}.$$

Proof of Lemma J.2. In the steady state, the nominal exchange rate S^{ss} and the sectoral prices P_i^{ss} are both normalized to 1. As a result, for each sector i, the export price $P_{EX,i}^{ss}$ is equal to $\theta_{F,i}/(\theta_{F,i} - 1)$, and the foreign demand for domestic sector i's product in terms of quantity and value are equal to

$$Y_{EX,i}^{ss} = \left(\frac{P_{EX,i}^{ss}}{S^{ss}}\right)^{-\theta_{F,i}} D_{EX,Fi}^{*,ss} = \left(\frac{\theta_{F,i}}{\theta_{F,i} - 1}\right)^{-\theta_{F,i}} D_{EX,Fi}^{*,ss}, \tag{J.25}$$

and

$$\frac{\theta_{F,i}}{\theta_{F,i} - 1} Y_{EX,i}^{ss} = v_{Hi}^* \cdot \sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'} - 1} Y_{EX,i'}^{ss}, \tag{J.26}$$

respectively. In the steady state, the import price $P_{IM,Fi}^{*,ss}$ is also normalized to 1, which yields the steady-state balance of trade condition $\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'}-1} Y_{EX,i'}^{ss} = \sum_{i'} (C_{Fi'}^{ss} + \sum_j X_{Hj,Fi'}^{ss})$. Combining this steady-state balance of trade condition with equation (J.26), yields the following equation of the quantity of foreign demand:

$$Y_{EX,i}^{ss} = \frac{\theta_{F,i} - 1}{\theta_{F,i}} v_{Hi}^* \sum_{i'} \left(C_{Fi'}^{ss} + \sum_j X_{Hj,Fi'}^{ss} \right). \tag{J.27}$$

Substituting equation (J.27) into the steady-state goods market clearing condition $Y_i^{ss} = C_{Hi}^{ss} + \sum_j X_{Hj,Hi}^{ss} + Y_{EX,i}^{ss}$ and dividing both sides by the steady-state aggregate output C^{ss} yields:

$$\lambda_i = \beta_i v_i + \sum_j \lambda_j \omega_{j,i} v_{x,j,i} + \frac{\theta_{F,i} - 1}{\theta_{F,i}} v_{Hi}^* \sum_{i'} \left[\beta_{i'} (1 - v_{i'}) + \sum_j \lambda_j \omega_{j,i'} (1 - v_{x,j,i'}) \right],$$

which has equation (J.23) as its matrix form.

Dividing both sides of the steady-state goods market clearing condition $Y_i^{ss} = C_{Hi}^{ss} + \sum_j X_{Hj,Hi}^{ss} + Y_{EX,i}^{ss}$ by the steady-state aggregate output C^{ss} and substituting in the definition of the sectoral export-to-GDP ratio

$\lambda_{EX,i} \equiv (P_i^{ss} Y_{EX,i}^{ss})/(P_C^{ss} C^{ss})$ with normalized $P_i^{ss} = P_C^{ss} = 1$ yields the following equation:

$$\lambda_{EX,i} = \lambda_i - \Big(\beta_i \upsilon_i + \sum_j \lambda_j \omega_{j,i} \upsilon_{x,j,i}\Big), \tag{J.28}$$

which has equation (J.24) as its matrix form. □

J.4. Goods market clearing condition up to the first-order approximation

Lemma J.3 (Goods market clearing condition)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*.*

$$\begin{aligned}\big[\boldsymbol{\lambda} \odot \big(\widehat{\mathbf{P}}(\boldsymbol{\xi}) + \widehat{\mathbf{Y}}(\boldsymbol{\xi})\big)\big]^\top &= \widetilde{\boldsymbol{\lambda}}_D^\top \big(\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi})\big) \\ &- \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{L}_{vx} - \mathbf{I}) + \big[\boldsymbol{\lambda}_{EX} \widehat{S}(\boldsymbol{\xi}) - \boldsymbol{\rho}_{ES} \odot \big(\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})\big)\big]^\top \mathbf{L}_{vx} \\ &+ \big\{\boldsymbol{\lambda}_{EX} \odot \widehat{\mathbf{D}}_{EX,F}^* + \big[\boldsymbol{\rho}_{ES} - (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}\big] \odot \widehat{\mathbf{P}}_{IM,F}^*\big\}^\top \mathbf{L}_{vx} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{J.29}$$

Proof of Lemma J.3. The goods market clearing condition (J.6) multiplied by the sectoral price P_i is

$$P_i Y_i = P_i C_{Hi} + P_i \sum_j X_{Hj,Hi} + P_i Y_{EX,i}. \tag{J.30}$$

Denote $P_{c,i}$ as the price index of the sectoral consumption goods from sector i and $P_{x,j,i}$ as the price index of the intermediate inputs purchased by sector j from sector i —both of which are weighted averages of domestic price P_i and import price $S \cdot P_{IM,Fi}^*$. Minimizing the costs of purchasing $\mathcal{C}, \{F_i\}_i, \{\mathcal{C}_i\}_i, \{\mathcal{X}_{i,j}\}_{i,j}$ yields the following quantity of the demand for consumption and intermediate inputs as functions of prices:

$$\begin{aligned}C_{Hi} &= \left(\frac{P_i}{P_{c,i}}\right)^{-\theta_i} \upsilon_i C_i = \left(\frac{P_i}{P_{c,i}}\right)^{-\theta_i} \frac{\upsilon_i \beta_i P_C C}{P_{c,i}}, \\ X_{Hj,Hi} &= \left(\frac{P_i}{P_{x,j,i}}\right)^{-\theta_i} \upsilon_{x,j,i} X_{j,i} = \left(\frac{P_i}{P_{x,j,i}}\right)^{-\theta_i} \frac{\upsilon_{x,j,i} \omega_{j,i} P_j Y_j}{P_{x,j,i} \mu_j}.\end{aligned}$$

Substituting the export tax rate $\tau_{EX,i} = 1/\theta_{F,i}$ and the export price $P_{EX,i} = P_i/(1 - \tau_{EX,i})$ into equation (8) yields the export demand as follows:

$$Y_{EX,i} = \left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D_{EX,Fi}^* = \left(\frac{\theta_{F,i}}{\theta_{F,i} - 1}\right)^{-\theta_{F,i}} \left(\frac{P_i}{S}\right)^{-\theta_{F,i}} D_{EX,Fi}^*.$$

Substituting the quantity of consumption, intermediate inputs, and export demand above back to the goods market-clearing condition in equation (J.30) yields:

$$P_i Y_i = \left(\frac{P_i}{P_{c,i}}\right)^{1-\theta_i} \upsilon_i \beta_i P_C C + \sum_j \left(\frac{P_i}{P_{x,j,i}}\right)^{1-\theta_i} \frac{\upsilon_{x,j,i} \omega_{j,i} P_j Y_j}{\mu_j}$$

$$
+\left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{-\theta_{F,i}}\left(\frac{P_i}{S}\right)^{1-\theta_{F,i}} S\cdot D^*_{EX,Fi}. \tag{J.31}
$$

Log-linearizing equation (J.31) yields:

$$
\begin{aligned}
\lambda_i(\widehat{P}_i+\widehat{Y}_i) &= \beta_i v_i\Big[(\theta_i-1)(\widehat{P}_{c,i}-\widehat{P}_i)+\widehat{P}_C+\widehat{C}\Big] \\
&\quad + \sum_j \lambda_j\omega_{j,i}v_{x,j,i}\Big[(\theta_i-1)(\widehat{P}_{x,j,i}-\widehat{P}_i)+\widehat{P}_j+\widehat{Y}_j-\widehat{\mu}_j\Big] \\
&\quad + \lambda_{EX,i}\Big[(\theta_{F,i}-1)(\widehat{S}-\widehat{P}_i)+\widehat{S}+\widehat{D}^*_{EX,Fi}\Big]+o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned} \tag{J.32}
$$

Log-linearizing the price indices $P_{c,i}$ and $P_{x,j,i}$ yields:

$$
\widehat{P}_{c,i} = v_i\widehat{P}_i+(1-v_i)\big(\widehat{S}+\widehat{P}^*_{IM,Fi}\big)+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{J.33}
$$

$$
\widehat{P}_{x,j,i} = v_{x,j,i}\widehat{P}_i+(1-v_{x,j,i})\big(\widehat{S}+\widehat{P}^*_{IM,Fi}\big)+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{J.34}
$$

which implies the following relative prices:

$$
\begin{aligned}
\widehat{P}_{c,i}-\widehat{P}_i &= (1-v_i)(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+o(\|\widehat{\boldsymbol{\xi}}\|), \\
\widehat{P}_{x,j,i}-\widehat{P}_i &= (1-v_{x,j,i})(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned}
$$

Substituting these relative prices into equation (J.32) yields:

$$
\begin{aligned}
\lambda_i(\widehat{P}_i+\widehat{Y}_i) &= \beta_i v_i\Big[(\theta_i-1)(1-v_i)(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+\widehat{P}_C+\widehat{C}\Big] \\
&\quad + \sum_j \lambda_j\omega_{j,i}v_{x,j,i}\Big[(\theta_i-1)(1-v_{x,j,i})(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+\widehat{P}_j+\widehat{Y}_j-\widehat{\mu}_j\Big] \\
&\quad + \lambda_{EX,i}\Big[(\theta_{F,i}-1)(\widehat{S}-\widehat{P}_i)+\widehat{S}+\widehat{D}^*_{EX,Fi}\Big]+o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned}
$$

Rearranging the above equation and substituting in the definition of the expenditure-switching elasticity $\rho_{ES,i}$ in equation (18) yield the following:

$$
\begin{aligned}
\lambda_i(\widehat{P}_i+\widehat{Y}_i)-\sum_j\lambda_j\omega_{j,i}v_{x,j,i}(\widehat{P}_j+\widehat{Y}_j) &= \beta_i v_i(\widehat{P}_C+\widehat{C})-\sum_j\lambda_j\omega_{j,i}v_{x,j,i}\widehat{\mu}_j+\lambda_{EX,i}\widehat{S}-\rho_{ES,i}(\widehat{P}_i-\widehat{S}) \\
&\quad + \lambda_{EX,i}\widehat{D}^*_{EX,Fi}+[\rho_{ES,i}-(\theta_{F,i}-1)\lambda_{EX,i}]\widehat{P}^*_{IM,Fi}+o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}
$$

which has the following matrix form as in equation (J.29) in Lemma J.3:

$$
\begin{aligned}
\big[\boldsymbol{\lambda}\odot(\widehat{\mathbf{P}}+\widehat{\mathbf{Y}})\big]^\top &= \widetilde{\boldsymbol{\lambda}}_D^\top(\widehat{P}_C+\widehat{C})-(\boldsymbol{\lambda}\odot\widehat{\boldsymbol{\mu}})^\top(\mathbf{L}_{vx}-\mathbf{I})+\big[\boldsymbol{\lambda}_{EX}\widehat{S}-\boldsymbol{\rho}_{ES}\odot(\widehat{\mathbf{P}}-\mathbf{1}\widehat{S})\big]^\top\mathbf{L}_{vx} \\
&\quad + \big\{\boldsymbol{\lambda}_{EX}\odot\widehat{\mathbf{D}}^*_{EX,F}+\big[\boldsymbol{\rho}_{ES}-(\boldsymbol{\theta}_F-\mathbf{1})\odot\boldsymbol{\lambda}_{EX}\big]\odot\widehat{\mathbf{P}}^*_{IM,F}\big\}^\top\mathbf{L}_{vx}+o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned}
$$

□

J.5. Household's budget constraint up to first-order approximation

Lemma J.4 (Household's budget constraint)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*:*

$$\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi}) = \left[\boldsymbol{\lambda} \odot \left(\widehat{\mathbf{P}}(\boldsymbol{\xi}) + \widehat{\mathbf{Y}}(\boldsymbol{\xi})\right)\right]^\top \boldsymbol{\alpha} + \left(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\right)^\top (\mathbf{1} - \boldsymbol{\alpha}) + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S}(\boldsymbol{\xi})$$
$$- \boldsymbol{\lambda}_{EX}^\top \left(\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})\right) + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{J.35}$$

Proof of Lemma J.4. Substituting the profit, total cost of inputs, and lump-sum transfer in equations (A.1), (3), and (11) into the household budget constraint in equation (6) yields:

$$\begin{aligned} P_C C &= WL + \sum_i \int_0^1 \Pi_{if} df + T \\ &= WL + \sum_i \left[(1-\tau_i)P_i Y_i - W L_i - \sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right)\right] \\ &\quad + \sum_i \left(\tau_i P_i Y_i + \tau_{EX,i} P_{EX,i} Y_{EX,i}\right) \\ &= \sum_i \left[P_i Y_i - \sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right)\right] + \sum_i \tau_{EX,i} P_{EX,i} Y_{EX,i}. \end{aligned} \tag{J.36}$$

Under the Cobb-Douglas production functions, $\sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right) = P_i Y_i (1-\alpha_i)/\mu_i$. Therefore, substituting the export tax rate $\tau_{EX,i} = 1/\theta_{F,i}$, the export price $P_{EX,i} = P_i/(1-\tau_{EX,i})$, and the export demand $Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}$ into equation (J.36) yields:

$$P_C C = \sum_i P_i Y_i \left(1 - \frac{1-\alpha_i}{\mu_i}\right) + \sum_i \left(\frac{S}{\theta_{F,i}}\right)^{\theta_{F,i}} \left(\frac{P_i}{\theta_{F,i}-1}\right)^{1-\theta_{F,i}} D^*_{EX,Fi}. \tag{J.37}$$

In the steady state, the sectoral markups, prices, and nominal exchange rate are normalized to $\mu_i^{ss} = P_i^{ss} = S^{ss} = 1$. As a result, equation (J.37) becomes:

$$1 = \sum_i \lambda_i \alpha_i + \sum_i \frac{\lambda_{EX,i}}{\theta_{F,i}-1}. \tag{J.38}$$

Log-linearizing equation (J.37) around the steady state yields:

$$\widehat{P}_C + \widehat{C} = \sum_i \lambda_i \alpha_i \left(\frac{1-\alpha_i}{\alpha_i}\widehat{\mu}_i + \widehat{P}_i + \widehat{Y}_i\right) + \sum_i \frac{\lambda_{EX,i}}{\theta_{F,i}-1}\left[\widehat{S} - (\theta_{F,i}-1)(\widehat{P}_i - \widehat{S}) + \widehat{D}^*_{EX,Fi}\right] + o(\|\widehat{\boldsymbol{\xi}}\|),$$

which has the following matrix form as in equation (J.35) in Lemma J.4:

$$\widehat{P}_C + \widehat{C} = \left[\boldsymbol{\lambda} \odot (\widehat{\mathbf{P}} + \widehat{\mathbf{Y}})\right]^\top \boldsymbol{\alpha} + (\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}})^\top (\mathbf{1} - \boldsymbol{\alpha}) \\ + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S} - \boldsymbol{\lambda}_{EX}^\top (\widehat{\mathbf{P}} - \mathbf{1}\widehat{S}) + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} + o(\|\widehat{\boldsymbol{\xi}}\|).$$

□

J.6. Sectoral markup wedges and sectoral inflation

Lemma J.5 (Sectoral markup wedges and sectoral inflation)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*:*

$$\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = -(\boldsymbol{\Delta}^{-1} - \mathbf{I})\widehat{\mathbf{P}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{J.39}$$

Proof of Lemma J.5. Under static Calvo-pricing, the vector of sectoral inflation is a function of the sectoral frequency of price adjustment $\boldsymbol{\Delta}$ and the vector of sectoral nominal marginal costs $\boldsymbol{\Phi}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\Delta}\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{J.40}$$

On the other hand, the definition of the sectoral markup wedges $\widehat{\boldsymbol{\mu}}$ yields:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}). \tag{J.41}$$

Combining the above two conditions to eliminate $\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi})$ yields equation (J.39). □

K. Proofs of the theoretical results in Sections 3

This appendix derives the theoretical results that relate sectoral markup wedges to the output gap in Section 3. These theoretical results are all up to the first-order approximation around the efficient steady state under Assumption 1.

K.1. Proof of Lemma B.1: The open economy version of Hulten's theorem

Hulten's theorem in Hulten (1978) characterizes the first-order impact of disaggregated productivity shocks on the aggregate TFP in an efficient closed economy (e.g., Baqaee and Farhi, 2019). Our paper extends the closed-economy version of Hulten's theorem into a small open economy with international trade, exchange rate adjustments, and sector-specific shocks to import prices and export demand besides sectoral productivity.

Under $\tau_i = -1/(\varepsilon_i - 1)$ and $\tau_{EX,i} = 1/\theta_{F,i}$ of Assumption 1 and with all of the prices but $P^{ss}_{EX,i}$ and W^{ss} normalized to 1, the first-order approximation of the conditions in Lemma J.1 around the efficient steady state yields the following:

$$C^{ss}\widehat{C}(\boldsymbol{\xi}) = \sum_i C_i^{ss}\widehat{C}_i(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.1}$$

$$Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) = Y_i^{ss}\widehat{A}_i + W^{ss}L_i^{ss}\widehat{L}_i(\boldsymbol{\xi}) + \sum_j X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.2}$$

$$C_i^{ss}\widehat{C}_i(\boldsymbol{\xi}) = C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) + C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.3}$$

$$X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi}) = X_{Hi,Hj}^{ss}\widehat{X}_{Hi,Hj}(\boldsymbol{\xi}) + X_{Hi,Fj}^{ss}\widehat{X}_{Hi,Fj}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.4}$$

$$L^{ss}\widehat{L}(\boldsymbol{\xi}) = \sum_i L_i^{ss}\widehat{L}_i(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.5}$$

$$Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) = C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) + \sum_j X_{Hj,Hi}^{ss}\widehat{X}_{Hj,Hi}(\boldsymbol{\xi}) + Y_{EX,i}^{ss}\widehat{Y}_{EX,i}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.6}$$

$$\begin{aligned}\sum_i EX_i^{ss}\widehat{EX}_i(\boldsymbol{\xi}) &= \sum_i Y_{EX,i}^{ss}\Big[(\theta_{F,i}-1)^{-1}\widehat{D}_{EX,Fi}^{*} + \widehat{Y}_{EX,i}(\boldsymbol{\xi})\Big] \\ &= \sum_i \Big[C_{Fi}^{ss}\big(\widehat{P}_{IM,Fi}^{*} + \widehat{C}_{Fi}(\boldsymbol{\xi})\big) + \sum_j X_{Hj,Fi}^{ss}\big(\widehat{P}_{IM,Fi}^{*} + \widehat{X}_{Hj,Fi}(\boldsymbol{\xi})\big)\Big] + o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{K.7}$$

Then, we combine equations (K.1)-(K.7) to prove Lemma B.1. Rearranging the balance of trade condition (K.7) to move all endogenous terms to the LHS and all exogenous ones to the RHS yields the following:

$$\begin{aligned} LHS &\equiv \sum_i \Big(Y_{EX,i}^{ss}\widehat{Y}_{EX,i}(\boldsymbol{\xi}) - C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) - \sum_j X_{Hi,Fj}^{ss}\widehat{X}_{Hi,Fj}(\boldsymbol{\xi})\Big) \\ &= \sum_i \Big(C_{Fi}^{ss}\widehat{P}_{IM,Fi}^{*} + \sum_j X_{Hj,Fi}^{ss}\widehat{P}_{IM,Fi}^{*} - \frac{Y_{EX,i}^{ss}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^{*}\Big) + o(\|\widehat{\boldsymbol{\xi}}\|) \equiv RHS. \end{aligned} \tag{K.8}$$

Combined with the goods market clearing condition in equation (K.6), the LHS of equation (K.8) becomes:

$$LHS = \sum_i \Big(Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) - C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) - \sum_j X_{Hj,Hi}^{ss}\widehat{X}_{Hj,Hi}(\boldsymbol{\xi}) - C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) - \sum_j X_{Hj,Fi}^{ss}\widehat{X}_{Hj,Fi}(\boldsymbol{\xi})\Big).$$

Further combined with the aggregators in equations (K.1), (K.3), and (K.4), the LHS becomes:

$$LHS = \sum_i \Big(Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) - \sum_j X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi})\Big) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Combined with the production function in equation (K.2):

$$LHS = \sum_i \Big(Y_i^{ss}\widehat{A}_i + W^{ss}L_i^{ss}\widehat{L}_i(\boldsymbol{\xi})\Big) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Combined with the labor market clearing condition in equation (K.5):

$$LHS = \sum_i Y_i^{ss} \widehat{A}_i + W^{ss} L^{ss} \widehat{L}(\boldsymbol{\xi}) - C^{ss} \widehat{C}(\boldsymbol{\xi}).$$

Substituting LHS back into equation (K.8) yields:

$$\begin{aligned} & C^{ss}\widehat{C}(\boldsymbol{\xi}) - W^{ss}L^{ss}\widehat{L}(\boldsymbol{\xi}) \\ &= \sum_i \left(Y_i^{ss}\widehat{A}_i - C_{Fi}^{ss}\widehat{P}_{IM,Fi}^* - \sum_j X_{Hj,Fi}^{ss}\widehat{P}_{IM,Fi}^* + \frac{Y_{EX,i}^{ss}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^* \right) + o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{K.9}$$

In the steady state, the sectoral output prices and the CPI are normalized to 1. Therefore, dividing both sides of equation (K.9) by the steady-state aggregate output C^{ss} yields the following:

$$\begin{aligned} \widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi}) = \sum_i \Bigg\{ & \lambda_i\widehat{A}_i + \frac{\lambda_{EX,i}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^* \\ & - \Big[\beta_i(1-v_i) + \sum_j \lambda_j\omega_{j,i}(1-v_{x,j,i})\Big]\widehat{P}_{IM,Fi}^* \Bigg\} + o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{K.10}$$

K.2. Proof of Proposition B.1: Efficiency and labor wedges

Efficiency wedge. Log-linearizing the efficiency wedge $A_{agg}(\boldsymbol{\xi})$ in Definition B.1 around the steady state yields

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L^{flex}(\boldsymbol{\xi})\widehat{L}(\boldsymbol{\xi}).$$

Substituting $\Lambda_L^{flex}(\boldsymbol{\xi}) = \Lambda_L + O(\|\widehat{\boldsymbol{\xi}}\|)$ into the above equation yields

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),$$

where $\widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi})$ are functions of only exogenous shocks up to the first-order approximation, as shown in equation (K.10) of Appendix K.1. Therefore, taking the difference of equation (K.10) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields the following:

$$\begin{aligned} \widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi}) &= \left(\widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|)\right) - \left(\widehat{C}^{flex}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}^{flex}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|)\right) \\ &= \widehat{C}^{gap}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) = o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{K.11}$$

In sum, sectoral markup wedges under price rigidities have no first-order impact on the efficiency wedge.

Labor wedge. Consider a prototype economy similar to the closed economy à la Chari et al. (2007), except that the aggregate production function defined on domestic labor inputs has state-contingent aggregate TFP and returns-to-scale, as in the following equation:

$$F(L,\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \cdot L^{\Lambda_L^{flex}(\boldsymbol{\xi})},$$

where $\Lambda_L^{flex}(\boldsymbol{\xi})$ is the economy-wise labor share in the *flexible-price equilibrium* of the small open economy that is contingent on the states of exogenous shocks. According to Definition B.1, $C(\boldsymbol{\xi}) = F(L(\boldsymbol{\xi}),\boldsymbol{\xi})$ and, therefore, the labor wedge $\Gamma_L(\boldsymbol{\xi})$ satisfies:

$$-\frac{\partial u/\partial L}{\partial u/\partial C}(C(\boldsymbol{\xi}), L(\boldsymbol{\xi})) = \Gamma_L(\boldsymbol{\xi}) \cdot \frac{\partial F}{\partial L}(L(\boldsymbol{\xi}),\boldsymbol{\xi}), \tag{K.12}$$

where the marginal product of labor in the *sticky-price equilibrium* is equal to:

$$\frac{\partial F}{\partial L}(L(\boldsymbol{\xi}),\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \cdot \Lambda_L^{flex}(\boldsymbol{\xi}) \cdot L^{\Lambda_L^{flex}(\boldsymbol{\xi})-1} \equiv \frac{\partial C}{\partial L}(\boldsymbol{\xi}).$$

Therefore, substituting the utility function in equation (4) into equation (K.12) and log-linearizing it around the steady state yields:

$$\widehat{\Gamma}_L(\boldsymbol{\xi}) = \sigma \widehat{C}(\boldsymbol{\xi}) + \varphi \widehat{L}(\boldsymbol{\xi}) - \widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{\Lambda}_L^{flex}(\boldsymbol{\xi}) - \big(\Lambda_L^{flex}(\boldsymbol{\xi}) - 1\big)\widehat{L}(\boldsymbol{\xi}). \tag{K.13}$$

Taking the difference of equation (K.13) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields:

$$\begin{aligned}\widehat{\Gamma}_L(\boldsymbol{\xi}) - \widehat{\Gamma}_L^{flex}(\boldsymbol{\xi}) &= \sigma \widehat{C}^{gap}(\boldsymbol{\xi}) + \varphi \widehat{L}^{gap}(\boldsymbol{\xi}) \\ &\quad - \big(\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi})\big) - (\Lambda_L - 1)\widehat{L}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|)\end{aligned} \tag{K.14}$$

Combining equation (K.14) with equation (K.11) yields the labor wedge as follows:

$$\widehat{\Gamma}_L(\boldsymbol{\xi}) - \widehat{\Gamma}_L^{flex}(\boldsymbol{\xi}) = \left(\sigma - 1 + \frac{\varphi + 1}{\Lambda_L}\right) \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).$$

□

K.3. Relation of sectoral markup wedges to gaps in domestic sectoral prices and CPI

Under the production technology and the total cost of inputs in equations (1), (2), and (3), deriving the sectoral nominal marginal costs $\boldsymbol{\Phi}(\boldsymbol{\xi})$ from the producers' cost minimization problem and log-linearizing it around the steady state, yields the following:

$$\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}) = \boldsymbol{\alpha}\widehat{W}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^*_{IM,F}\big) - \widehat{\mathbf{A}} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.15}$$

which, substituted into equation (J.41), yields:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\alpha}\widehat{W}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^*_{IM,F}\big) - \widehat{\mathbf{A}} + \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{K.16}$$

Taking the difference of equation (K.16) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* to eliminate the exogenous shocks, yields:

$$\begin{aligned}\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) &= \boldsymbol{\alpha}\widehat{W}^{gap}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \\ &= \mathbf{L}_{vx}\big(\boldsymbol{\alpha}\widehat{W}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\alpha}\widehat{S}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big) + \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned} \tag{K.17}$$

where the second equality is derived using the Leontief inverse matrix $\mathbf{L}_{vx} \equiv (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1}$ and the identity $\boldsymbol{\alpha} = \mathbf{1} - \boldsymbol{\Omega}\mathbf{1}$. Re-arranging equation (K.17) yields:

$$\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{\alpha}}\big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) + \boldsymbol{L}_{vx}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.18}$$

which is equation (28) in Section 3.

Log-linearizing the CPI in equation (7) around the steady state yields:

$$\widehat{P}_C(\boldsymbol{\xi}) = (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^*_{IM,F}\big) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{K.19}$$

Taking the difference of equation (K.19) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* to eliminate the exogenous shocks yields:

$$\widehat{P}^{gap}_C(\boldsymbol{\xi}) = (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{K.20}$$

Substituting equation (K.17) into equation (K.20) and using the identity $(\boldsymbol{\beta} \odot \mathbf{v})^\top \mathbf{L}_{vx} = \widetilde{\boldsymbol{\lambda}}_D^\top$ yields:

$$\widehat{P}^{gap}_C(\boldsymbol{\xi}) = \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\widehat{W}^{gap}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\widehat{S}^{gap}(\boldsymbol{\xi}) + \widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),$$

which can be rearranged to:

$$\widehat{P}^{gap}_C(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) + \widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.21}$$

which is equation (29) in Section 3. □

K.4. Real wage gap and output gap

For the households' problem that maximizes utility function (4) subject to the budget constraint (6), combining the first-order conditions with respect to L and C and log-linearizing it yield:

$$\widehat{W}(\boldsymbol{\xi}) - \widehat{P}_C(\boldsymbol{\xi}) = \sigma \widehat{C}(\boldsymbol{\xi}) + \varphi \widehat{L}(\boldsymbol{\xi}). \tag{K.22}$$

Taking the difference of equation (K.22) in the *sticky-price equilibrium* and in the *flexible-price equilibrium*, and substituting in equation (K.11) from Section K.2 yields:

$$\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi}) = \big(\sigma + \varphi/\Lambda_L\big)\widehat{C}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.23}$$

which is equation (25) in Section 3. Substituting equation (K.21) into equation (K.23) yields:

$$\big(\sigma + \varphi/\Lambda_L\big)\widehat{C}^{gap}(\boldsymbol{\xi}) = (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) - \widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{K.24}$$

which is equation (26) in Section 3.

K.5. Log-linearized open-economy budget constraint

Taking the difference of equation (J.29) from Lemma J.3 in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields:

$$\begin{aligned}\big[\boldsymbol{\lambda} \odot \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) + \widehat{\mathbf{Y}}^{gap}(\boldsymbol{\xi})\big)\big]^\top &= \widetilde{\boldsymbol{\lambda}}_D^\top \big(\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi})\big) - \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{L}_{vx} - \mathbf{I}) \\ &\quad + \big[\boldsymbol{\lambda}_{EX} \widehat{S}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\rho}_{ES} \odot \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big)\big]^\top \mathbf{L}_{vx} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{K.25}$$

Taking the difference of equation (J.35) from Lemma J.4 in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields:

$$\begin{aligned}\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi}) &= \big[\boldsymbol{\lambda} \odot \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) + \widehat{\mathbf{Y}}^{gap}(\boldsymbol{\xi})\big)\big]^\top \boldsymbol{\alpha} + \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{1} - \boldsymbol{\alpha}) \\ &\quad + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\lambda}_{EX}^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{K.26}$$

Substituting equation (K.25) into equation (K.26) and using the identity equations $\widetilde{\boldsymbol{\alpha}} = \mathbf{L}_{vx}\boldsymbol{\alpha}$ and $\widetilde{\boldsymbol{\lambda}}_F = \boldsymbol{\lambda}_{EX}^\top \mathbf{L}_{vx}$ yield

$$\begin{aligned}\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi}) &= \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\big(\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi})\big) + [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha} + \widetilde{\boldsymbol{\lambda}}_F^\top \boldsymbol{\alpha})\widehat{S}^{gap}(\boldsymbol{\xi}) \\ &\quad - (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) - \boldsymbol{\lambda}_{EX}^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{K.27}$$

Rearranging it and using $\boldsymbol{\lambda} = \widetilde{\boldsymbol{\lambda}}_D + \widetilde{\boldsymbol{\lambda}}_F$ from equation (37) in Lemma 2 yields

$$\begin{aligned}
&(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\widehat{C}^{gap}(\boldsymbol{\xi}) \\
&= -(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) - \boldsymbol{\lambda}_{EX}^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) \\
&+ (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big(\widehat{S}^{gap}(\boldsymbol{\xi} - \widehat{P}_C^{gap}(\boldsymbol{\xi}))\big), \qquad \text{(K.28)}
\end{aligned}$$

which is exactly equation (27) in Section 3.

K.6. *Proof of Theorem 1: output gap and sectoral markup wedges*

Substituting equations (K.18) and (K.21) from Appendix K.3, and equation (K.24) from Appendix K.4 into equation (K.28) from Appendix K.5, yields:

$$\begin{aligned}
&\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}(\sigma + \varphi/\Lambda_L)\widehat{C}^{gap}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big[\kappa_{CPI}^{-1} + (1 - \kappa_{CPI}^{-1})(\sigma + \varphi/\Lambda_L)\big]\widehat{C}^{gap}(\boldsymbol{\xi}) \\
&= -\widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) - \kappa_{CPI}^{-1} \cdot \widetilde{\boldsymbol{\rho}}_{ES}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \kappa_{CPI}^{-1} \cdot \big[- \widetilde{\boldsymbol{\lambda}}_F + \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\big]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}$$

where we have used the definition of κ_{CPI} in equation (23):

$$\kappa_{CPI} \equiv \frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}} + 1.$$

Further using the definition of κ_C in equation (24):

$$\kappa_C \equiv \Big[\frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}}\big(\sigma + \varphi/\Lambda_L\big) + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\big(\sigma + \varphi/\Lambda_L\big) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\Big]/\kappa_{CPI},$$

we obtain the following matrix form of equation (21) of Theorem 1:

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\big\{\widetilde{\boldsymbol{\lambda}}_D + \kappa_{CPI}^{-1} \cdot \widetilde{\boldsymbol{\rho}}_{ES} + \kappa_{CPI}^{-1} \cdot \big[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\big]\big\}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) = -\boldsymbol{\mathcal{M}}_{OG}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).$$

□

K.7. *Negative markup wedges and the gap in the nominal exchange rate relative to domestic wage*

We show that negative markup wedges ($\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) < \mathbf{0}$) are linked to a negative gap in the nominal exchange rate relative to domestic wage —i.e., $\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{W}^{gap}(\boldsymbol{\xi}) < 0$. To illustrate this, we first show the following:

$$(\sigma + \varphi/\Lambda_L)\frac{1}{\kappa_C} = \frac{\big(\sigma + \varphi/\Lambda_L\big)\left(\frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}} + 1\right)}{\frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}}\big(\sigma + \varphi/\Lambda_L\big) + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\big(\sigma + \varphi/\Lambda_L\big) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})} > 1,$$

where the inequality comes from the facts that $\sigma > 1$, $\varphi/\Lambda_L > 0$, and $\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} \in (0, 1)$. Accordingly, $\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{W}^{gap}(\boldsymbol{\xi})$ satisfies:

$$
\begin{aligned}
(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big(\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{W}^{gap}(\boldsymbol{\xi})\big) &= -\big(\sigma + \varphi/\Lambda_L\big)\widehat{C}^{gap}(\boldsymbol{\xi}) - \widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) \\
&= \Big[\big(\sigma + \varphi/\Lambda_L\big)\frac{1}{\kappa_C}\boldsymbol{\mathcal{M}}_{OG}^\top - \widetilde{\boldsymbol{\lambda}}_D^\top\Big]\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) < 0, \qquad \text{(K.29)}
\end{aligned}
$$

where the first and the second equality come from equations (26) and (21), respectively. The third inequality comes from the facts that $\widehat{\boldsymbol{\mu}} < \mathbf{0}$, $\big(\sigma + \varphi/\Lambda_L\big)\frac{1}{\kappa_C} > 1$, and $\boldsymbol{\mathcal{M}}_{OG} > \widetilde{\boldsymbol{\lambda}}_D$ (because $\widetilde{\boldsymbol{\lambda}}_D$ is a sub-component of $\boldsymbol{\mathcal{M}}_{OG}$).

K.8. Deriving the terms of trade gap

Due to the trade balance, we can denote the steady-state share of sector i's exports in total exports and the share of sector i's imports in total imports by:

$$
\begin{aligned}
&EX\_Share_i \equiv \frac{\theta_{F,i}}{\theta_{F,i} - 1}\lambda_{EX,i} \Big/ \Big(\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'} - 1}\lambda_{EX,i'}\Big), \quad \text{and} \\
&IM\_Share_i \equiv \Big[\big(\sum_j \lambda_j \omega_{j,i}\left(1 - v_{x,j,i}\right)\big) + \beta_i\left(1 - v_i\right)\Big] \Big/ \Big(\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'} - 1}\lambda_{EX,i'}\Big),
\end{aligned}
$$

respectively. Then, we can define the terms of trade as:

$$
ToT \equiv \frac{\prod_i P_{EX,i}^{EX\_Share_i}}{\prod_i \left(SP_{IM,Fi}^*\right)^{IM\_Share_i}}.
$$

The terms of trade gap is equal to:

$$
\begin{aligned}
\widehat{ToT}^{gap} &= \sum_i EX\_Share_i \widehat{P}_i^{gap} - \sum_i IM\_Share_i \widehat{S}^{gap} \\
&= \sum_i EX\_Share_i \widehat{P}_i^{gap} - \sum_i EX\_Share_i \widehat{S}^{gap} \\
&= \Big[\sum_i \frac{\theta_{F,i}}{\theta_{F,i} - 1}\lambda_{EX,i}\left(\widehat{P}_i^{gap} - \widehat{S}^{gap}\right)\Big] \Big/ \Big(\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'} - 1}\lambda_{EX,i'}\Big) \\
&= \big[(\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}))^\top \boldsymbol{\lambda}_{EX}\big]^{-1} (\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX})^\top (\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap}),
\end{aligned}
$$

where the second quality comes from the trade balance.

K.9. Proof of Corollary E.1 under foreign-currency pricing

In this proof, for the sake of space we only show equilibrium equations under foreign-currency pricing that differ from those in the baseline model under producer-currency pricing.

With the definition of sectoral markup wedges of foreign-market products, we have the following pricing equation of sectoral foreign-market products:

$$\widehat{P}^*_{EX,i} = (\widehat{\Phi}_i - \widehat{S}) + \widehat{\mu}^*_{EX,i}. \tag{K.30}$$

Thus, the log deviation of the export demand function in equation (E.3) is equal to:

$$\widehat{Y}_{EX,i} = -\theta_{F,i}(\widehat{\Phi}_i - \widehat{S} + \widehat{\mu}^*_{EX,i}) + \widehat{D}^*_{EX,Fi}. \tag{K.31}$$

Under our Calvo-pricing friction, the price of sectoral foreign-market product satisfies:

$$\widehat{P}^*_{EX,i} = \delta^*_{EX,i}(\widehat{\Phi}_i - \widehat{S}),$$

which, combined with equation (K.30), yields:

$$\widehat{\mu}^*_{EX,i} = -\frac{1-\delta^*_{EX,i}}{\delta^*_{EX,i}}\widehat{P}^*_{EX,i}. \tag{K.32}$$

Under Assumption 1, the sectoral goods market clearing condition under foreign-currency pricing is the same as in the baseline model under PCP as follows:

$$P_iY_i = \left(\frac{P_i}{P_{c,i}}\right)^{1-\theta_i} v_i\beta_iP_CC + \sum_j \left(\frac{P_i}{P_{x,j,i}}\right)^{1-\theta_i} \frac{v_{x,j,i}\omega_{j,i}P_jY_j}{\mu_j} + P_iY^*_{EX,i}. \tag{K.33}$$

However, log-linearizing equation (K.33) and combining it with the log linearization of the demand and pricing equations (K.30) and (K.31) of sectoral foreign-market products yields the following condition:

$$\begin{aligned}\lambda_i(\widehat{P}_i + \widehat{Y}_i) = {} & \beta_iv_i\big[(\theta_i-1)(1-v_i)(\widehat{S} + \widehat{P}^*_{IM,Fi} - \widehat{P}_i) + \widehat{P}_C + \widehat{C}\big] \\ & + \sum_j \lambda_j\omega_{j,i}v_{x,j,i}\big[(\theta_i-1)(1-v_{x,j,i})(\widehat{S} + \widehat{P}^*_{IM,Fi} - \widehat{P}_i) + \widehat{P}_j + \widehat{Y}_j - \widehat{\mu}_j\big] \\ & + \lambda_{EX,i}\big[\widehat{P}_i - \theta_{F,i}(\widehat{P}_i - \widehat{\mu}_i - \widehat{S} + \widehat{\mu}^*_{EX,i}) + \widehat{D}^*_{EX,Fi}\big] + o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned}$$

which can be re-arranged and stacked into the following matrix form:

$$\begin{aligned}\big[\boldsymbol{\lambda}\odot(\widehat{\mathbf{P}}+\widehat{\mathbf{Y}})\big]^\top = {} & \widetilde{\boldsymbol{\lambda}}_D^\top(\widehat{P}_C + \widehat{C}) - (\boldsymbol{\lambda}\odot\widehat{\boldsymbol{\mu}})^\top(\mathbf{L}_{vx} - \mathbf{I}) + \big\{\boldsymbol{\lambda}_{EX}\widehat{S} - \big[\boldsymbol{\rho}_{ES}\odot(\widehat{\mathbf{P}} - \mathbf{1}\widehat{S})\big]\big\}^\top\mathbf{L}_{vx} \\ & + \big\{\boldsymbol{\lambda}_{EX}\odot\widehat{\mathbf{D}}^*_{EX,F} + \big[\boldsymbol{\rho}_{ES} - (\boldsymbol{\theta}_F - \mathbf{1})\odot\boldsymbol{\lambda}_{EX}\big]\odot\widehat{\mathbf{P}}^*_{IM,F}\big\}^\top\mathbf{L}_{vx} \\ & - \big[\boldsymbol{\theta}_F\odot\boldsymbol{\lambda}_{EX}\odot(\widehat{\boldsymbol{\mu}}^*_{EX} - \widehat{\boldsymbol{\mu}})\big]^\top\mathbf{L}_{vx} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{K.34}$$

Equation (K.34) differs from its counterpart in the model under PCP (equation J.29 in Appendix J.4) in the last term $-\big[\boldsymbol{\theta}_F \odot \boldsymbol{\lambda}_{EX} \odot (\widehat{\boldsymbol{\mu}}^*_{EX} - \widehat{\boldsymbol{\mu}})\big]^\top \mathbf{L}_{vx}$ that represents exports.

Under foreign-currency pricing, the household's budget constraint is equal to:

$$\begin{aligned}
P_C C &= WL + \Pi + T \\
&= \sum_i \Big\{ [P_i Y_{DM,i} - \Phi_i Y_{DM,i}(1-\alpha_i)] + \big[S P^*_{EX,i} Y^*_{EX,i} - \Phi_i Y^*_{EX,i}(1-\alpha_i)\big] \Big\} \\
&= \sum_i \Big\{ [P_i Y_{DM,i} - P_i Y_{DM,i}(1-\alpha_i)/\mu_i] + \big[S P^*_{EX,i} Y^*_{EX,i} - P_i Y^*_{EX,i}\,(1-\alpha_i)\,/\mu_i\big] \Big\} \\
&= \sum_i \Big[P_i Y_i \Big(1 - \frac{1-\alpha_i}{\mu_i}\Big) + S P^*_{EX,i} Y^*_{EX,i} - P_i Y^*_{EX,i} \Big],
\end{aligned}$$

the log-linearization of which —combined with equations (K.30) and (K.31)— yields:

$$\begin{aligned}
\widehat{P}_C + \widehat{C} &= \sum_i \lambda_i \alpha_i \Big(\frac{1-\alpha_i}{\alpha_i} \widehat{\mu}_i + \widehat{P}_i + \widehat{Y}_i \Big) + \sum_i \lambda_{EX,i} \frac{\theta_{F,i}}{\theta_{F,i}-1} \big[\widehat{S} + (1-\theta_{F,i})(\widehat{\Phi}_i - \widehat{S} + \widehat{\mu}^*_{EX,i}) + \widehat{D}^*_{EX,Fi}\big] \\
&\quad - \sum \lambda_{EX,i} \big[\widehat{P}_i - \theta_{F,i}(\widehat{\Phi}_i - \widehat{S} + \widehat{\mu}^*_{EX,i}) + \widehat{D}^*_{EX,Fi}\big] + o(\|\widehat{\boldsymbol{\xi}}\|) \\
&= \sum_i \lambda_i \alpha_i \Big(\frac{1-\alpha_i}{\alpha_i} \widehat{\mu}_i + \widehat{P}_i + \widehat{Y}_i \Big) + \sum_i \lambda_{EX,i} \Big[\frac{1}{\theta_{F,i}-1} \widehat{S} + \big(\widehat{S} - \widehat{P}_i\big) + \frac{1}{\theta_{F,i}-1} \widehat{D}^*_{EX,Fi} \Big] + o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}$$

which is the same as its counterpart in the baseline model (equation J.35 in Appendix J.5). Taking its matrix form, combining it with equation (K.34), and taking the difference of it between the sticky-price and flexible-price equilibria, yields the following log-linearization of the trade balance condition under foreign-currency pricing:

$$\begin{aligned}
\big(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\big) \widehat{C}^{gap} &= -\left(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX}\right)^\top \left(\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap}\right) + [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}} \\
&\quad + \big(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\big)\big(\widehat{S}^{gap} - \widehat{P}_C^{gap}\big) - \left(\boldsymbol{\theta}_F \odot \boldsymbol{\lambda}_{EX} \odot \widetilde{\boldsymbol{\alpha}}\right)^\top \left(\widehat{\boldsymbol{\mu}}^*_{EX} - \widehat{\boldsymbol{\mu}}\right) + o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned} \tag{K.35}$$

Substitute equation (K.18) for $\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})$ into equation (K.35) to express the domestic-to-foreign price gaps as functions of real wage gaps $\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})$ and $\widehat{\boldsymbol{\mu}}$; similarly, substitute equation (K.21) into equation (K.35) to express $\widehat{P}_C^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})$ as functions of real wage gaps and $\widehat{\boldsymbol{\mu}}$.[19] Further, using equation (K.24) for the real wage gaps $\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})$, we can express $\widehat{C}^{gap}$ only as a function of $\widehat{\boldsymbol{\mu}}$ as follows:

$$\begin{aligned}
&\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} (\sigma + \varphi/\Lambda_L) \widehat{C}^{gap} + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) \big[\kappa_{CPI}^{-1} + (1 - \kappa_{CPI}^{-1})(\sigma + \varphi/\Lambda_L)\big] \widehat{C}^{gap} \\
&= -\big\{\widetilde{\boldsymbol{\lambda}}_D + \kappa_{CPI}^{-1} \cdot \widetilde{\boldsymbol{\rho}}_{ES} + \kappa_{CPI}^{-1} \cdot \big[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\big]\big\}^\top \widehat{\boldsymbol{\mu}} - \kappa_{CPI}^{-1} \cdot \left(\boldsymbol{\theta}_F \odot \boldsymbol{\lambda}_{EX} \odot \widetilde{\boldsymbol{\alpha}}\right)^\top \left(\widehat{\boldsymbol{\mu}}^*_{EX} - \widehat{\boldsymbol{\mu}}\right) + o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}$$

[19] As in the baseline, in the derivations we have used the definition $\widetilde{\boldsymbol{\rho}}_{ES} \equiv (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top \mathbf{L}_{vx}$ to simplify the coefficients of sectoral markup wedges of domestic products $\widehat{\boldsymbol{\mu}}$.

which is exactly equation (E.4) in Corollary E.1. Lastly, substituting equations (J.39) and (K.32) which link markups and inflation into equation (E.4), we obtain equation (E.5) in Corollary E.1.

Throughout the above proof of Corollary E.1, only $-\left(\boldsymbol{\theta}_F \odot \boldsymbol{\lambda}_{EX} \odot \widetilde{\boldsymbol{\alpha}}\right)^{\top}\left(\widehat{\boldsymbol{\mu}}^*_{EX} - \widehat{\boldsymbol{\mu}}\right)$ in equation (K.34) deviates from the baseline model under PCP —i.e., the quantity of demand for sectoral exports depends on the markup wedge of sectoral foreign-market rather than domestic-market products in the expenditure-switching channel. Therefore, compared to the three channels in the baseline OG weight in equation (22), only the expenditure-switching channel changes under foreign-currency pricing, while the CPI and the profit channels remain dependent on sectoral domestic-market markup wedges and inflation as in the baseline model under PCP.

K.10. Proof of Proposition G.1: Sectoral relevance metrics and import shares

As preparation, we derive the partial derivatives of the Leontief inverse matrix $\mathbf{L}_{vx}$ with respect to the home bias in intermediate inputs, as in the following equation:

$$\frac{\partial \mathbf{L}_{vx}}{\partial v_{x,r,s}} = -\mathbf{L}_{vx}\frac{\partial \mathbf{L}_{vx}^{-1}}{\partial v_{x,r,s}}\mathbf{L}_{vx} = -\mathbf{L}_{vx}\frac{\partial(\mathbf{I}-\boldsymbol{\Omega}\odot\mathbf{V}_x)}{\partial v_{x,r,s}}\mathbf{L}_{vx} = \left\{\ell_{vx,j,r}\omega_{r,s}\ell_{vx,s,i}\right\}_{j,i},$$

where $\left\{\ell_{vx,j,r}\omega_{r,s}\ell_{vx,s,i}\right\}_{j,i}$ is the (j,i)-th element of the partial derivative matrix.

Because $\mathbf{L}_{vx} = (\mathbf{I}-\boldsymbol{\Omega}\odot\mathbf{V}_x)^{-1} = \mathbf{I}+\sum_{n=1}^{+\infty}(\boldsymbol{\Omega}\odot\mathbf{V}_x)^n$, $\omega_{j,i}\geq 0$ and $v_{x,j,i}\geq 0$ for all j and i, we have:

$$\ell_{vx,j,i}\begin{cases} > 0 & \forall\, j = i, \\ \geq 0 & \forall\, j \neq i. \end{cases}$$

Proof of Proposition G.1. According to $\widetilde{\boldsymbol{\lambda}}_D^{\top} \equiv (\boldsymbol{\beta}\odot\mathbf{v})^{\top}\mathbf{L}_{vx}$ in equation (16) of Definition 3, the partial derivatives of the total content in domestic consumption in sector i ($\widetilde{\lambda}_{D,i}$) with respect to the import shares of consumption goods and intermediate inputs are as follows:

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_j)} = -\beta_j \ell_{vx,j,i}, \qquad \forall\, j, \tag{K.36}$$

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,r,s})} = -\Big(\sum_j \beta_j v_j \ell_{vx,j,r}\Big)\omega_{r,s}\ell_{vx,s,i} = -\widetilde{\lambda}_{D,r}\omega_{r,s}\ell_{vx,s,i}, \qquad \forall\, r,s. \tag{K.37}$$

Equation (K.36) implies that the total content in domestic consumption of sector i strictly decreases in its own import share of consumption —*viz.*, $\frac{\partial\widetilde{\lambda}_{D,i}}{\partial(1-v_i)} < 0$— if and only if $\beta_i > 0$, because $\ell_{vx,i,i} > 0$.

Equation (K.37) implies that the total content in domestic consumption of sector i strictly decreases in its direct downstream sector r's import share of sector i's goods (i.e., $\omega_{r,i} > 0$ and $v_{x,r,i} > 0$), if and only if sector r, directly and indirectly, supplies to domestic aggregate output (i.e., $\sum_j \beta_j v_j \ell_{vx,j,r} > 0$), that is:

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,r,i})} = -\widetilde{\lambda}_{D,r}\omega_{r,i}\ell_{vx,i,i} < 0.$$

Equation (K.37) also implies that, the total content in domestic consumption of sector i strictly decreases in its indirect downstream sector s's import share of sector r goods if and only if both of the following two conditions hold: (i) sector s, directly and indirectly, supplies to domestic aggregate output (i.e., $\sum_j \beta_j v_j \ell_{vx,j,s} > 0$); and (ii) sector i indirectly supplies inputs to sector s via sector r (i.e., $\omega_{s,r} > 0$ and $\ell_{vx,r,i} > 0$), that is:

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,s,r})} = -\widetilde{\lambda}_{D,s}\omega_{s,r}\ell_{vx,r,i} < 0.$$

□

L. Proofs of the theoretical results in Section 4

This appendix derives the theoretical results associated with the divine coincidence and sectoral Phillips curves, and the normalized sectoral OG weights in Section 4.

L.1. Proof of Proposition 2: Sectoral Phillips curves

Step 1: Derive $\widehat{S}$ and $\widehat{P}_C$ as functions of $\{\widehat{C}, \widehat{\mathbf{P}}, \widehat{\boldsymbol{\xi}}\}$. Following every step in the proof of equation (K.28) in Appendix K.5 —except that now we do not need to take the difference between the *sticky-price* and *flexible-price equilibria* but just need to look at the *sticky-price equilibrium*— yields:

$$\begin{aligned}
&\left[1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}+(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\mathbf{1}\right]\widehat{S}(\boldsymbol{\xi})\\
&=(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\widehat{\mathbf{P}}(\boldsymbol{\xi})+[\boldsymbol{\lambda}\odot(\mathbf{1}-\widetilde{\boldsymbol{\alpha}})]^\top\boldsymbol{\Delta}^{-1}(\mathbf{I}-\boldsymbol{\Delta})\widehat{\mathbf{P}}(\boldsymbol{\xi})+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})(\widehat{P}_C(\boldsymbol{\xi})+\widehat{C}(\boldsymbol{\xi}))\\
&-[\widetilde{\boldsymbol{\lambda}}_F\odot\boldsymbol{\alpha}+\boldsymbol{\lambda}_{EX}\oslash(\boldsymbol{\theta}_F-\mathbf{1})]^\top\widehat{\mathbf{D}}^*_{EX,F}-(\boldsymbol{\rho}_{IM}\odot\widetilde{\boldsymbol{\alpha}})^\top\widehat{\mathbf{P}}^*_{IM,F}+o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}\tag{L.1}$$

where the shorthand notation $\boldsymbol{\rho}_{IM} \equiv \boldsymbol{\rho}_{ES} - (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}$ is the elasticity of sectoral imports to import price shocks, which is equal to the expenditure switching elasticity $\boldsymbol{\rho}_{EX}$ diminished by the export component $(\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}$.

Rearranging equation (L.1) and introducing shorthand notations yield:

$$\begin{aligned}
(\boldsymbol{\mathcal{M}}_{EX}+\boldsymbol{\mathcal{M}}_{IM})^\top\mathbf{1}\widehat{S}(\boldsymbol{\xi}) &= \widehat{P}_C(\boldsymbol{\xi})+\widehat{C}(\boldsymbol{\xi})+(\boldsymbol{\mathcal{M}}_p+\boldsymbol{\mathcal{M}}_\mu)^\top\widehat{\mathbf{P}}(\boldsymbol{\xi})\\
&-(\boldsymbol{\mathcal{M}}_{EX}\oslash\boldsymbol{\theta}_F)^\top\widehat{\mathbf{D}}^*_{EX,F}-\boldsymbol{\mathcal{M}}_{IM}^\top\widehat{\mathbf{P}}^*_{IM,F}+o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}\tag{L.2}$$

where the shorthand notations are as follows:

$$\boldsymbol{\mathcal{M}}_{EX} \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}[\boldsymbol{\lambda}_{EX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})] \odot \boldsymbol{\theta}_F, \tag{L.3}$$

$$\boldsymbol{\mathcal{M}}_{IM} \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{IM} \odot \widetilde{\boldsymbol{\alpha}}), \tag{L.4}$$

$$\boldsymbol{\mathcal{M}}_{p} \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX}), \tag{L.5}$$

$$\boldsymbol{\mathcal{M}}_{\mu} \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\Delta}^{-1} - \mathbf{I})[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]. \tag{L.6}$$

According to equation (J.38) in Appendix J.5, it is clear that $1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1} = (\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}$. That is, we have:

$$\begin{aligned}\widehat{S}(\boldsymbol{\xi}) &= \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}\left[\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi})\right] + \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu)^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) \\ &\quad - \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top \widehat{\mathbf{D}}^*_{EX,F} - \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}\boldsymbol{\mathcal{M}}_{IM}^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{L.7}$$

Recall that the CPI pricing equation (K.19) in Appendix K.3 is given by:

$$\widehat{P}_C(\boldsymbol{\xi}) = (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \left(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^*_{IM,F}\right) + o(\|\widehat{\boldsymbol{\xi}}\|),$$

and substituting equation (L.7) into the CPI equation above, we have:

$$\begin{aligned}\widehat{P}_C(\boldsymbol{\xi}) &= (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) \\ &\quad + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\left[\left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}\left(\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi})\right) + \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu)^\top \widehat{\mathbf{P}}(\boldsymbol{\xi})\right] \\ &\quad + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\left[-\left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top \widehat{\mathbf{D}}^*_{EX,F} - \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}\boldsymbol{\mathcal{M}}_{IM}^\top \widehat{\mathbf{P}}^*_{IM,F}\right] \\ &\quad + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned} \tag{L.8}$$

or, equivalently,

$$\begin{aligned}&\widehat{P}_C(\boldsymbol{\xi})\left\{1 - \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\right\} \\ &= (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\left[\widehat{C}(\boldsymbol{\xi}) + (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu)^\top \widehat{\mathbf{P}}(\boldsymbol{\xi})\right] \\ &+ [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\left[-\left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}(\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top \widehat{\mathbf{D}}^*_{EX,F} - \left(1+\boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1}\boldsymbol{\mathcal{M}}_{IM}^\top \widehat{\mathbf{P}}^*_{IM,F}\right] \\ &+ [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned}$$

which can be further simplified as:

$$\widehat{P}_C(\boldsymbol{\xi}) = \Gamma_{CPI,C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{CPI,P}^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{CPI,EX}^\top \widehat{\mathbf{D}}^*_{EX} + \boldsymbol{\Gamma}_{CPI,IM}^\top \widehat{\mathbf{P}}^*_{IM} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{L.9}$$

with shorthand notations defined as:

$$
\begin{aligned}
\Gamma_{CPI,C} &\equiv (\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1}(1 - \boldsymbol{\beta}^\top \mathbf{v}), && \text{(L.10)}\\
\boldsymbol{\Gamma}_{CPI,P} &\equiv (\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1}\left[(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})(\boldsymbol{\beta} \odot \mathbf{v}) + (1 - \boldsymbol{\beta}^\top \mathbf{v})(\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu)\right], && \text{(L.11)}\\
\boldsymbol{\Gamma}_{CPI,EX} &\equiv -\Gamma_{CPI,C} \cdot (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F),\\
\boldsymbol{\Gamma}_{CPI,IM} &\equiv -\Gamma_{CPI,C} \cdot \boldsymbol{\mathcal{M}}_{IM} + (\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1}(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}) \cdot [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})].
\end{aligned}
$$

Similarly, we can also express $\widehat{S}(\boldsymbol{\xi})$ as functions of $\widehat{C}(\boldsymbol{\xi})$, $\widehat{\mathbf{P}}(\boldsymbol{\xi})$ and shocks only:

$$
\widehat{S}(\boldsymbol{\xi}) = \Gamma_{S,C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{S,P}^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{S,EX}^\top \widehat{\mathbf{D}}_{EX,F}^* + \boldsymbol{\Gamma}_{S,IM}^\top \widehat{\mathbf{P}}_{IM,F}^* + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{L.12}
$$

with shorthand notations defined as:

$$
\begin{aligned}
\Gamma_{S,C} &\equiv \left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1} (1 + \Gamma_{CPI,C}), && \text{(L.13)}\\
\boldsymbol{\Gamma}_{S,P} &\equiv \left(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}\right)^{-1} (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu + \boldsymbol{\Gamma}_{CPI,P}), && \text{(L.14)}\\
\boldsymbol{\Gamma}_{S,EX} &\equiv -\Gamma_{S,C} \cdot (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F),\\
\boldsymbol{\Gamma}_{S,IM} &\equiv -\Gamma_{S,C} \cdot \boldsymbol{\mathcal{M}}_{IM} + \Gamma_{S,C} \cdot [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})].
\end{aligned}
$$

Step 2: Derive $\widehat{W}$ as a function of $\{\widehat{\mathbf{P}}, \widehat{C}, \widehat{\boldsymbol{\xi}}\}$. Substituting $\widehat{P}_C$ in equation (L.9) and $\widehat{L}$ in equation (K.10) into the labor supply equation (K.22), yields:

$$
\widehat{W}(\boldsymbol{\xi}) = \Gamma_{W,C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{CPI,P}^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{W,A}^\top \widehat{\mathbf{A}} + \boldsymbol{\Gamma}_{W,EX}^\top \widehat{\mathbf{D}}_{EX,F}^* + \boldsymbol{\Gamma}_{W,IM}^\top \widehat{\mathbf{P}}_{IM,F}^* + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{L.15}
$$

where the shorthand notations are

$$
\begin{aligned}
\Gamma_{W,C} &\equiv \sigma + \frac{\varphi}{\Lambda_L} + \Gamma_{CPI,C},\\
\boldsymbol{\Gamma}_{W,A} &\equiv -\frac{\varphi}{\Lambda_L}\boldsymbol{\lambda},\\
\boldsymbol{\Gamma}_{W,EX} &\equiv \boldsymbol{\Gamma}_{EX} - \frac{\varphi}{\Lambda_L}[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})],\\
\boldsymbol{\Gamma}_{W,IM} &\equiv \boldsymbol{\Gamma}_{IM} + \frac{\varphi}{\Lambda_L}\left[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})^\top \boldsymbol{\lambda}\right].
\end{aligned}
$$

Step 3: Substitute $\widehat{W}$ and $\widehat{S}$ into sectoral pricing equation. Substituting the sectoral marginal costs in equation (K.15) into the sectoral inflation in equation (J.40) yields the following pricing equation:

$$
\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}\widehat{W}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\left(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}_{IM,F}^*\right) - \widehat{\mathbf{A}}\right] + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{L.16}
$$

Substituting $\widehat{W}$ and $\widehat{S}$ in equations (L.15) and (L.12) into the pricing equation (L.16) yields the following sectoral Phillips curves in terms of $\widehat{C}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}_{C,A}\widehat{\mathbf{A}} + \boldsymbol{\mathcal{V}}_{C,EX}\widehat{\mathbf{D}}^{*}_{EX,F} + \boldsymbol{\mathcal{V}}_{C,IM}\widehat{\mathbf{P}}^{*}_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{L.17}$$

where the shorthand notations are as follows:

$$\begin{aligned}
\boldsymbol{\mathcal{B}} &\equiv \boldsymbol{\Delta}_{\Phi}\Big[\boldsymbol{\alpha}\left(\sigma + \frac{\varphi}{\Lambda_L} + \Gamma_{CPI,C}\right) + (\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\mathbf{1}\Gamma_{S,C}\Big],\\
\boldsymbol{\Delta}_{\Phi} &\equiv \Big[\boldsymbol{\Delta}^{-1} - \boldsymbol{\Omega}\odot\mathbf{V}_{x} - \boldsymbol{\alpha}\boldsymbol{\Gamma}^{\top}_{CPI,P} - (\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}^{\top}_{S,P}\Big]^{-1}.\\
\boldsymbol{\mathcal{V}}_{C,A} &\equiv \boldsymbol{\Delta}_{\Phi}\big(\boldsymbol{\alpha}\boldsymbol{\Gamma}^{\top}_{W,A} - \mathbf{1}\big),\\
\boldsymbol{\mathcal{V}}_{C,EX} &\equiv \boldsymbol{\Delta}_{\Phi}\big[\boldsymbol{\alpha}\boldsymbol{\Gamma}^{\top}_{W,EX} + (\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}^{\top}_{S,EX}\big],\\
\boldsymbol{\mathcal{V}}_{C,IM} &\equiv \boldsymbol{\Delta}_{\Phi}\big[\boldsymbol{\alpha}\boldsymbol{\Gamma}^{\top}_{W,IM} + (\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}^{\top}_{S,IM}\big].
\end{aligned}$$

To derive further the sectoral Phillips curves in terms of the aggregate output gap $\widehat{C}^{gap}$, we need to solve for the log deviation of the aggregate output in the *flexible-price equilibrium* from the steady state, denoted by $\widehat{C}^{flex}(\boldsymbol{\xi})$. To do so, we derive the flexible-price version of equations (L.16), (K.22), (L.2), (K.10), and (K.19) by setting $\boldsymbol{\Delta} = \mathbf{I}$, which yields the following equations, respectively:

$$\begin{aligned}
\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi}) &= \widetilde{\boldsymbol{\alpha}}(\widehat{W}^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi})) - \mathbf{L}_{vx}\widehat{\mathbf{A}} + \mathbf{L}_{vx}(\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\widehat{\mathbf{P}}^{*}_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\\
\widehat{W}^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) &= \widehat{P}^{flex}_{C}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) + \sigma\widehat{C}^{flex}(\boldsymbol{\xi}) + \varphi\widehat{L}^{flex}(\boldsymbol{\xi}),\\
\widehat{P}^{flex}_{C}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) + \widehat{C}^{flex}(\boldsymbol{\xi}) &= -\boldsymbol{\mathcal{M}}^{\top}_{P}(\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi})) + (\boldsymbol{\mathcal{M}}_{EX}\oslash\boldsymbol{\theta}_{F})^{\top}\widehat{\mathbf{D}}^{*}_{EX,F} + \boldsymbol{\mathcal{M}}^{\top}_{IM}\widehat{\mathbf{P}}^{*}_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\\
\widehat{C}^{flex}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}^{flex}(\boldsymbol{\xi}) &= \boldsymbol{\lambda}^{\top}\widehat{A} + [\boldsymbol{\lambda}_{EX}\oslash(\boldsymbol{\theta}_{F} - \mathbf{1})]^{\top}\widehat{\mathbf{D}}^{*}_{EX,F} - \big[\boldsymbol{\beta}^{\top}\odot(\mathbf{1}-\mathbf{v})^{\top} + \boldsymbol{\lambda}^{\top}(\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\big]\widehat{\mathbf{P}}^{*}_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\\
\widehat{P}^{flex}_{C}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) &= (\boldsymbol{\beta}\odot\mathbf{v})^{\top}(\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi})) + [\boldsymbol{\beta}\odot(\mathbf{1}-\mathbf{v})]^{\top}\widehat{\mathbf{P}}^{*}_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned}$$

Combining the above five equations yields:

$$\widehat{C}^{flex}(\boldsymbol{\xi}) = \boldsymbol{\Gamma}^{flex}_{C,A}\widehat{\mathbf{A}} + \boldsymbol{\Gamma}^{flex}_{C,EX}\widehat{\mathbf{D}}^{*}_{EX,F} + \boldsymbol{\Gamma}^{flex}_{C,IM}\widehat{\mathbf{P}}^{*}_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{L.18}$$

where the shorthand notations are as follows:

$$\begin{aligned}
\boldsymbol{\Gamma}^{flex}_{C,A} &\equiv (\Delta^{flex}_{C})^{-1}\big(\boldsymbol{\mathcal{M}}^{\top}_{L}\widetilde{\boldsymbol{\alpha}}\boldsymbol{\lambda}^{\top}\varphi/\Lambda_L + \boldsymbol{\mathcal{M}}^{\top}_{L}\mathbf{L}_{vx}\big),\\
\boldsymbol{\Gamma}^{flex}_{C,IM} &\equiv -(\Delta^{flex}_{C})^{-1}\Big\{\boldsymbol{\mathcal{M}}^{\top}_{L}\widetilde{\boldsymbol{\alpha}}\big[(\Lambda_L+\varphi)\boldsymbol{\beta}^{\top}\odot(\mathbf{1}-\mathbf{v})^{\top} + \boldsymbol{\lambda}^{\top}(\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\big]/\Lambda_L\\
&\qquad + \boldsymbol{\mathcal{M}}^{\top}_{L}\mathbf{L}_{vx}(\boldsymbol{\Omega}\odot\mathbf{V}_{1-x}) - \boldsymbol{\mathcal{M}}^{\top}_{IM} + \boldsymbol{\beta}^{\top}\odot(\mathbf{1}-\mathbf{v})^{\top}\Big\},\\
\boldsymbol{\Gamma}^{flex}_{C,EX} &\equiv (\Delta^{flex}_{C})^{-1}\big\{\boldsymbol{\mathcal{M}}^{\top}_{L}\widetilde{\boldsymbol{\alpha}}[\boldsymbol{\lambda}_{EX}\oslash(\boldsymbol{\theta}_{F}-\mathbf{1})]^{\top}\varphi/\Lambda_L + (\boldsymbol{\mathcal{M}}_{EX}\oslash\boldsymbol{\theta}_{F})^{\top}\big\},\\
\boldsymbol{\mathcal{M}}^{\top}_{L} &\equiv (\boldsymbol{\mathcal{M}}_{P} + \boldsymbol{\beta}\odot\mathbf{v})^{\top}\big[\mathbf{I} - \widetilde{\boldsymbol{\alpha}}(\boldsymbol{\beta}\odot\mathbf{v})^{\top}\big]^{-1},\\
\Delta^{flex}_{C} &\equiv 1 + \boldsymbol{\mathcal{M}}^{\top}_{L}\widetilde{\boldsymbol{\alpha}}(\sigma + \varphi/\Lambda_L).
\end{aligned}$$

Combining equations (L.17) and (L.18), yields the following sectoral Phillips curves in terms of the output gap $\widehat{C}^{gap}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}_A\widehat{\mathbf{A}} + \boldsymbol{\mathcal{V}}_{EX}\widehat{\mathbf{D}}^*_{EX,F} + \boldsymbol{\mathcal{V}}_{IM}\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{L.19}$$

where the matrices of coefficients of exogenous shocks are as follows:

$$\begin{aligned} \boldsymbol{\mathcal{V}}_A &\equiv \boldsymbol{\mathcal{V}}_{C,A} + \boldsymbol{\mathcal{B}} \cdot \boldsymbol{\Gamma}^{flex}_{C,A}, \\ \boldsymbol{\mathcal{V}}_{EX} &\equiv \boldsymbol{\mathcal{V}}_{C,EX} + \boldsymbol{\mathcal{B}} \cdot \boldsymbol{\Gamma}^{flex}_{C,EX}, \\ \boldsymbol{\mathcal{V}}_{IM} &\equiv \boldsymbol{\mathcal{V}}_{C,IM} + \boldsymbol{\mathcal{B}} \cdot \boldsymbol{\Gamma}^{flex}_{C,IM}. \end{aligned}$$

L.2. Proof of Lemma 2: OG reduces to Domar weight in closed economies

Recall the expression of OG weights (21) in Theorem 1 in the following:

$$\boldsymbol{\mathcal{M}}_{OG} = \widetilde{\boldsymbol{\lambda}}_D + \kappa^{-1}_{CPI} \cdot \widetilde{\boldsymbol{\rho}}_{ES} + \kappa^{-1}_{CPI} \cdot \left[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right].$$

The sectoral relevance metrics reduce to the following values in closed economies:

$$\widetilde{\boldsymbol{\lambda}}_D = \boldsymbol{\lambda}, \qquad \widetilde{\boldsymbol{\rho}}_{ES} = \mathbf{0}, \qquad \widetilde{\boldsymbol{\lambda}}_F = \mathbf{0}, \qquad \widetilde{\boldsymbol{\alpha}} = \mathbf{1},$$

which, substituted into the OG weights in equation (21) of Theorem 1, yields $\boldsymbol{\mathcal{M}}_{OG} = \boldsymbol{\lambda}$.

Multiplying both sides of equation (J.24) in Lemma J.2 by the Leontief inverse matrix $\mathbf{L}_{vx} \equiv (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1}$, yields the following:

$$\boldsymbol{\lambda}^\top_{EX}\mathbf{L}_{vx} = \boldsymbol{\lambda}^\top - (\boldsymbol{\beta} \odot \mathbf{v})^\top\mathbf{L}_{vx} \qquad \Longleftrightarrow \qquad \widetilde{\boldsymbol{\lambda}}^\top_F = \boldsymbol{\lambda}^\top - \widetilde{\boldsymbol{\lambda}}^\top_D,$$

where the last equality holds due to definitions of $\widetilde{\boldsymbol{\lambda}}_D$ and $\widetilde{\boldsymbol{\lambda}}_F$ in equation (16). □

L.3. Output strictly increases in money supply

Lemma L.1 (aggregate output increases in money supply)**.** *In the* sticky-price equilibrium *where $\delta_i > 0$ for all $i \in \{1, 2, \cdots, N\}$, for any realized state $\boldsymbol{\xi} \in \boldsymbol{\Xi}$, a rise in $\widehat{M}$ strictly increases $\widehat{C}(\boldsymbol{\xi})$ up to the first-order approximation.*

Proof of Lemma L.1. Up to the first-order approximation, given the shock to the money supply $\widehat{M}$, we have the following five conditions: (i) decomposition of CPI in equation (K.19):

$$\widehat{P}_C = (\boldsymbol{\beta} \odot \mathbf{v})^\top\widehat{\mathbf{P}} + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top\mathbf{1}\widehat{S} + \boldsymbol{\Upsilon}^\top_1\widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(ii) the determination of the exchange rate in equation (K.27):

$$(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})(\widehat{P}_C-\widehat{S}+\widehat{C}) = \left[\boldsymbol{\lambda}\odot(\mathbf{1}-\widetilde{\boldsymbol{\alpha}})\right]^\top\widehat{\boldsymbol{\mu}} - (\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top(\widehat{\mathbf{P}}-\mathbf{1}\widehat{S}) + \boldsymbol{\Upsilon}_2^\top\widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(iii) the sectoral Phillips curves in equation (L.17):

$$\widehat{\mathbf{P}} = \boldsymbol{\mathcal{B}}\widehat{C} + \boldsymbol{\Upsilon}_3\widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(iv) the relationship of sectoral markup wedges and inflation in equation (J.39):

$$\widehat{\boldsymbol{\mu}} = -(\boldsymbol{\Delta}^{-1}-\mathbf{I})\widehat{\mathbf{P}} + o(\|\widehat{M}\|);$$

(v) the money demand equation (2.3):

$$\widehat{M} = \widehat{P}_C + \widehat{C}.$$

Combining the above five equations yields the following:

$$\widehat{C} = \frac{\boldsymbol{\beta}^\top\mathbf{v} + \boldsymbol{\mathcal{M}}_P^\top\mathbf{1}}{(1+\boldsymbol{\mathcal{M}}_P^\top\mathbf{1})[1+(\boldsymbol{\beta}\odot\mathbf{v})^\top\boldsymbol{\mathcal{B}}] + (1-\boldsymbol{\beta}^\top\mathbf{v})\left[(\boldsymbol{\Delta}^{-1}-\mathbf{I})\frac{\boldsymbol{\lambda}\odot(\mathbf{1}-\widetilde{\boldsymbol{\alpha}})}{1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}} + \boldsymbol{\mathcal{M}}_P\right]^\top\boldsymbol{\mathcal{B}}}\widehat{M} + \boldsymbol{\Upsilon}^\top\widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|), \tag{L.20}$$

where vector $\boldsymbol{\Upsilon}$ is a linear combination of $\{\boldsymbol{\Upsilon}_i\}_{i=1,2,3}$ and $\boldsymbol{\mathcal{M}}_p \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})$. In particular, we need $\delta_i > 0$ for all $i \in \{1,2,\cdots,N\}$ to ensure that the slopes $\boldsymbol{\mathcal{B}}$ of the sectoral Phillips curves will be finite. □

M. Proofs of the theoretical results in Section 5

This appendix derives the welfare loss up to the second-order approximation, from which we derive the analytical solution of the optimal monetary policy by solving a linear-quadratic programming problem.

M.1. Proof of Proposition 4: welfare loss up to the second-order approximation

***Step 1**: Decompose the welfare loss into labor wedge and efficiency wedge components.* Approximating the utility function around the *flexible-price equilibrium* up to the second-order approximation yields:

$$u - u^{flex} = u_C^{flex}C^{flex}\left[\widehat{C}^{gap} - \frac{\sigma-1}{2}(\widehat{C}^{gap})^2\right] + u_L^{flex}L^{flex}\left[\widehat{L}^{gap} + \frac{\varphi+1}{2}(\widehat{L}^{gap})^2\right] + o(\|\widehat{\boldsymbol{\xi}}\|^2). \tag{M.1}$$

Substituting into equation (M.1) the optimality condition of labor supply $-u_L^{flex}/u_C^{flex} = W^{flex}/P_C^{flex}$, the approximation of labor share $\Lambda_L^{flex} \equiv (W^{flex}L^{flex})/(P_C^{flex}C^{flex}) = \Lambda_L + O(\|\widehat{\boldsymbol{\xi}}\|)$, and the approximation of the coefficient $u_C^{flex} C^{flex} = \left(C^{flex}\right)^{1-\sigma} = 1 + O(\|\widehat{\boldsymbol{\xi}}\|)$ under normalization $C^{ss} = 1$, yields:

Combined with Definition B.1 and Proposition B.1 on efficiency and labor wedges, equation (M.1) becomes

$$u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = \underbrace{\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi})}_{\text{efficiency wedge component}} \underbrace{-\frac{1}{2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]^{-1}\widehat{\Gamma}_L(\boldsymbol{\xi})^2}_{\text{labor wedge component}} + o(\|\widehat{\boldsymbol{\xi}}\|^2).$$

Step 2*: Derive the second-order approximation of the labor wedge and introduce the equivalent economy.* Combining equation (B.5) in Proposition B.1 and equation (21) in Theorem 1 yields a quadratic form of the labor wedge component in terms of markup wedge $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$:

$$\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]^{-1}\widehat{\Gamma}_L(\boldsymbol{\xi})^2 = \kappa_C^{-2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{M}}_{OG}^\top \boldsymbol{\mathcal{M}}_{OG}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|^2). \quad \text{(M.2)}$$

To facilitate the derivation of the efficiency wedge component, we construct an *equivalent economy* with sectoral markup wedges. For the *sticky-price equilibrium* under realized shocks $\widehat{\boldsymbol{\xi}}$, the *equivalent economy* satisfies all of the equilibrium conditions in Definition 1 except that in condition (ii), the markups of sticky-price firms, μ_{if}, are derived from $1 - \delta_i + \delta_i \mu_{if}^{1-\theta_i} = \mu_i(\boldsymbol{\xi})^{1-\theta_i}$, where $\widehat{\mu}_i(\boldsymbol{\xi})$ is the markup wedge of sector i in the *sticky-price equilibrium*. Therefore, the constructed economy has *identical allocations, prices, and welfare loss* as the *sticky-price equilibirum* for any realized shock $\widehat{\boldsymbol{\xi}}$, and thus we refer to it as the *equivalent economy*. With slight abuse of notation, in the remainder of this subsection, we express the utility and other sector-level allocations and prices in the *equivalent economy* as functions of $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ and $\widehat{\boldsymbol{\xi}}$, using the same function names as in the *sticky-price equilibirum* (e.g., $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}})$ and $C(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}})$).

The *equivalent economy* enables us to express the welfare loss of the original economy as a function of only sectoral markup wedges, using the following lemma.

Lemma M.1. *Let $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ be the sectoral markup wedges in the* sticky-price equilibrium *under realized shocks $\widehat{\boldsymbol{\xi}}$. Up to the second-order approximation, the welfare loss in the* sticky-price equilibrium *under any shock $\widehat{\boldsymbol{\xi}}$ is equal to the welfare loss in the equivalent economy under the same sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ but absent of all shocks,* viz,

$$u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}}) = u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - u(\mathbf{0}, \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}_{\mu\mu}^u \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(||\widehat{\boldsymbol{\xi}}||^2). \quad \text{(M.3)}$$

which is, therefore, a function of only sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$.

To prove Lemma M.1, consider the following second-order approximation of the welfare loss:

$$u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}}) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}_{\mu\mu}^u \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}_{\xi\mu}^u \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}_{\xi\xi}^u \widehat{\boldsymbol{\xi}} + o(||\widehat{\boldsymbol{\xi}}||^2). \quad \text{(M.4)}$$

Because the allocation in the *flexible-price equilibrium* is the solution to the domestic social planner's problem, the welfare is maximized at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$ and $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) \leq u(\mathbf{0}, \widehat{\boldsymbol{\xi}})$ for any realized shocks $\widehat{\boldsymbol{\xi}}$. First, because the welfare is maximized at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$, the derivative of the RHS of equation (M.4) with respect to $\widehat{\boldsymbol{\mu}}$ equals $\mathbf{0}$ at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = 0$ for any realized shocks $\widehat{\boldsymbol{\xi}}$, requiring $\boldsymbol{\mathcal{L}}^u_{\xi\mu} = \mathbf{0}$. Second, we also have $\boldsymbol{\mathcal{L}}^u_{\xi\xi} = \mathbf{0}$. Otherwise, there exists some realized shocks $\widehat{\boldsymbol{\xi}}$ such that the RHS of equation (M.4) is strictly positive or negative at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$ (i.e., $|\widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}^u_{\xi\xi} \widehat{\boldsymbol{\xi}}| > 0$), which contradicts $u(\mathbf{0}, \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}}) = 0$. Therefore, we conclude that $\boldsymbol{\mathcal{L}}^u_{\xi\mu} = \mathbf{0}$ and $\boldsymbol{\mathcal{L}}^u_{\xi\xi} = \mathbf{0}$, and the RHS of equation (M.4) degenerates to $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^u_{\mu\mu} \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$, which proves the second equality in equation (M.4) of Lemma M.1.

Based on Lemma M.1, we derive the original welfare loss $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}})$ by deriving the equivalent $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - u(\mathbf{0}, \mathbf{0})$ with the sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ resulting from shocks $\widehat{\boldsymbol{\xi}}$ in the *sticky-price equilibrium*. In particular, because both the welfare loss in equation (M.3) and the labor wedge component in equation (M.2) are quadratic functions of only sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$, the efficiency wedge —as the remaining component of the welfare loss— is also a quadratic form of only sectoral markup wedges. Therefore, we arrive at the following:

$$\begin{aligned}\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}^{flex}_{agg}(\boldsymbol{\xi}) &= \widehat{A}_{agg}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - \widehat{A}_{agg}(\mathbf{0}, \widehat{\boldsymbol{\xi}}) = \widehat{A}_{agg}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \widehat{A}_{agg}(\mathbf{0}, \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2) \\ &= \widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \Lambda^{flex}_L(\mathbf{0}) \widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2) = \widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2),\end{aligned}$$

where the first equality holds because the allocation in the sticky-price equilibrium that is free of markup wedges is equivalent to those in the flexible-price equilibrium, under the same exogenous shocks —i.e., $\widehat{A}^{flex}_{agg}(\boldsymbol{\xi}) = \widehat{A}_{agg}(\mathbf{0}, \widehat{\boldsymbol{\xi}})$.

For simplicity of notation, in the remainder of this subsection, we denote $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ by $\widehat{\boldsymbol{\mu}}$ and ignore the entry of $\mathbf{0}$ for any function in the *equivalent economy* with sectoral markup wedges $\widehat{\boldsymbol{\mu}}$ but no realized shocks — e.g., $\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}) \equiv \widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0})$.

With the above simplifying notation, for any variable x, we have $\widehat{x}(\mathbf{0}) = \mathbf{0}$ when all sectoral markup wedges are set to zero to represent both the *flexible-price equilibrium* and the steady state, leading to $\widehat{x}(\widehat{\boldsymbol{\mu}}) = \widehat{x}(\widehat{\boldsymbol{\mu}}) - \widehat{x}(\mathbf{0}) \equiv \widehat{x}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \widehat{x}(\mathbf{0}, \mathbf{0})$. Up to the first-order approximation, $\widehat{x}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \widehat{x}(\mathbf{0}, \mathbf{0}) = \widehat{x}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \boldsymbol{\xi}) - \widehat{x}(\mathbf{0}, \boldsymbol{\xi}) + o(||\widehat{\boldsymbol{\xi}}||) = \widehat{x}^{gap}(\boldsymbol{\xi}) + o(||\widehat{\boldsymbol{\xi}}||)$. Thus, in the remaining proof of this section, we replace $\widehat{x}^{gap}(\boldsymbol{\xi})$ with $\widehat{x}(\widehat{\boldsymbol{\mu}})$ whenever only first-order approximation is used.

We also introduce, for any variable x, the notation of $\widehat{\Delta} x$ that denotes the percentage deviation of x from its steady state, compared to the log deviation of x from its steady state $\widehat{x}$.

Step 3*: Derive the second-order approximation of the efficiency wedge component.* In the *equivalent economy* without realized shocks, we express the efficiency wedge component in terms of the percentage deviations of different variables from their steady states as follows:

$$\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}) = \widehat{\Delta} C(\widehat{\boldsymbol{\mu}}) - \Lambda_L \widehat{\Delta} L(\widehat{\boldsymbol{\mu}}) + \frac{1}{2} \Lambda_L (1 - \Lambda_L) \widehat{L}(\widehat{\boldsymbol{\mu}})^2 + o(||\widehat{\boldsymbol{\mu}}||^2). \tag{M.5}$$

The equivalent economy satisfies the conditions (J.1)-(J.7) of the *feasible allocation* in Definition J.1. Therefore, the terms in equation (M.5) satisfy the following equations up to the second-order approximation:

$$\widehat{\Delta}C(\widehat{\boldsymbol{\mu}}) = \sum_{i=1}^{n} \beta_i \widehat{\Delta}C_i(\widehat{\boldsymbol{\mu}}) - \frac{1}{2}\sum_{i=1}^{n} \beta_i \big(\widehat{\Delta}C_i(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}C(\widehat{\boldsymbol{\mu}})\big)^2 + o(||\widehat{\boldsymbol{\mu}}||^2), \tag{M.6}$$

$$\begin{aligned}\widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}}) = {} & \widehat{\iota}_i(\widehat{\boldsymbol{\mu}}) + \alpha_i \widehat{\Delta}L_i(\widehat{\boldsymbol{\mu}}) + \sum_{j=1}^{n} \omega_{i,j} \widehat{\Delta}X_{i,j}(\widehat{\boldsymbol{\mu}}) \\ & - \frac{1}{2}\Big[\alpha_i \big(\widehat{\Delta}L_i(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}})\big)^2 + \sum_{j=1}^{n} \omega_{i,j}\big(\widehat{\Delta}X_{i,j}(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}})\big)^2\Big] + o(||\widehat{\boldsymbol{\mu}}||^2),\end{aligned} \tag{M.7}$$

$$\begin{aligned}\widehat{\Delta}C_i(\widehat{\boldsymbol{\mu}}) = {} & v_i \widehat{\Delta}C_{Hi}(\widehat{\boldsymbol{\mu}}) + (1 - v_i)\widehat{\Delta}C_{Fi}(\widehat{\boldsymbol{\mu}}) \\ & - \frac{v_i(1-v_i)}{2\theta_i}\big(\widehat{\Delta}C_{Hi}(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}C_{Fi}(\widehat{\boldsymbol{\mu}})\big)^2 + o(||\widehat{\boldsymbol{\mu}}||^2),\end{aligned} \tag{M.8}$$

$$\begin{aligned}\widehat{\Delta}X_{i,j}(\widehat{\boldsymbol{\mu}}) = {} & v_{x,i,j} \widehat{\Delta}X_{Hi,Hj}(\widehat{\boldsymbol{\mu}}) + (1 - v_{x,i,j})\widehat{\Delta}X_{Hi,Fj}(\widehat{\boldsymbol{\mu}}) \\ & - \frac{v_{x,i,j}(1-v_{x,i,j})}{2\theta_j}\big(\widehat{\Delta}X_{Hi,Hj}(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}X_{Hi,Fj}(\widehat{\boldsymbol{\mu}})\big)^2 + o(||\widehat{\boldsymbol{\mu}}||^2),\end{aligned} \tag{M.9}$$

$$\Lambda_L \widehat{\Delta}L(\widehat{\boldsymbol{\mu}}) = \sum_{i=1}^{n} \lambda_i \alpha_i \widehat{\Delta}L_i(\widehat{\boldsymbol{\mu}}), \tag{M.10}$$

$$\lambda_i \widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}}) = \beta_i v_i \widehat{\Delta}C_{Hi}(\widehat{\boldsymbol{\mu}}) + \sum_{j=1}^{n} \lambda_j \omega_{j,i} v_{x,j,i} \widehat{\Delta}X_{Hj,Hi}(\widehat{\boldsymbol{\mu}}) + \lambda_{EX,i}\widehat{\Delta}Y_{EX,i}(\widehat{\boldsymbol{\mu}}), \tag{M.11}$$

$$\begin{aligned}\lambda_{EX}\widehat{\Delta}EX(\widehat{\boldsymbol{\mu}}) &= \sum_{i=1}^{n} \lambda_{EX,i}\widehat{\Delta}Y_{EX,i}(\widehat{\boldsymbol{\mu}}) - \frac{1}{2}\sum_{i=1}^{n} \frac{\lambda_{EX,i}}{\theta_{F,i}} \widehat{\Delta}Y_{EX,i}(\widehat{\boldsymbol{\mu}})^2 + o(||\widehat{\boldsymbol{\mu}}||^2) \\ &= \sum_{i=1}^{n} \Big[\beta_i(1-v_i)\widehat{\Delta}C_{Fi}(\widehat{\boldsymbol{\mu}}) + \sum_{j=1}^{n} \lambda_j \omega_{j,i}(1 - v_{x,j,i})\widehat{\Delta}X_{Hj,Fi}(\widehat{\boldsymbol{\mu}})\Big].\end{aligned} \tag{M.12}$$

Combining equations (M.5)-(M.12) eliminates all first-order terms following the same proof of Proposition B.1, and further applying equality $\widehat{\Delta}x(\widehat{\boldsymbol{\mu}}) = \widehat{x}(\widehat{\boldsymbol{\mu}}) + o(||\widehat{\boldsymbol{\mu}}||)$ to all square terms yields:

$$\begin{aligned}\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}) = {} & \\ & \left. - \sum_{i=1}^{n} \lambda_i \widehat{\iota}_i(\widehat{\boldsymbol{\mu}}) \right\} \text{within-sector misallocation} \\ & \left.\begin{aligned} & - \frac{1}{2}\sum_{i=1}^{n} \beta_i \big[\widehat{C}_i(\widehat{\boldsymbol{\mu}}) - \widehat{C}(\widehat{\boldsymbol{\mu}})\big]^2 \\ & - \frac{1}{2}\sum_{i=1}^{n} \lambda_i \alpha_i \big[\widehat{L}_i(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 \\ & - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \omega_{i,j} \big[\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 \end{aligned}\right\} \text{across-sector misallocation}\end{aligned} \tag{M.13}$$

$$
\left.
\begin{aligned}
&-\frac{1}{2}\sum_{i=1}^{n}\frac{\beta_i}{\theta_i}v_i(1-v_i)\big[\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}})-\widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}})\big]^2\\
&-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\lambda_i\omega_{i,j}}{\theta_j}v_{x,i,j}(1-v_{x,i,j})\big[\widehat{X}_{Hi,Hj}(\widehat{\boldsymbol{\mu}})-\widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}})\big]^2\\
&-\frac{1}{2}\sum_{i=1}^{n}\frac{\lambda_{EX,i}}{\theta_{F,i}}\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}})^2\\
&+\frac{1}{2}\Lambda_L(1-\Lambda_L)\widehat{L}(\widehat{\boldsymbol{\mu}})^2
\end{aligned}
\right\}\text{cross-border misallocation}
$$

$$
+\,o(\|\widehat{\boldsymbol{\mu}}\|^2).
$$

The within-sector misallocation has the same expression as in Rubbo (2023) *viz.*:

$$
-\sum_{i=1}^{n}\lambda_i\widehat{\iota}_i(\widehat{\boldsymbol{\mu}})=-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2+o(\|\widehat{\boldsymbol{\mu}}\|^2). \tag{M.14}
$$

Replacing $\widehat{x}(\widehat{\boldsymbol{\mu}})$ with $\widehat{x}^{gap}(\boldsymbol{\xi})$ for all variables x in equation (M.13) and combining it with equations (M.14), (J.38), and (J.39) yield the RHS of equations (40), (41), and (42) in Proposition 4, which completes the main part of the proof.

Step 4*: Express the efficiency wedge component in square terms of sectoral inflation.* Rearranging equation (K.24) in Appendix K.4 yields:

$$
\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})=\frac{(\sigma+\varphi/\Lambda_L)\widehat{C}^{gap}(\boldsymbol{\xi})+\sum_{k=1}^{n}\widetilde{\lambda}_{D,k}\widehat{\mu}_k(\boldsymbol{\xi})}{1-\sum_k\widetilde{\lambda}_{D,k}\alpha_k}+o(\|\widehat{\boldsymbol{\xi}}\|).
$$

The scalar form of equation (K.18) implies that:

$$
\widehat{P}_i^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})=\widetilde{\alpha}_i\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+\sum_{k=1}^{n}\ell_{vx,i,k}\widehat{\mu}_k(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{M.15}
$$

$$
\widehat{P}_i^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})=-(1-\widetilde{\alpha}_i)\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+\sum_{k=1}^{n}\ell_{vx,i,k}\widehat{\mu}_k(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{M.16}
$$

$$
\widehat{P}_i^{gap}(\boldsymbol{\xi})-\widehat{P}_j^{gap}(\boldsymbol{\xi})=(\widetilde{\alpha}_i-\widetilde{\alpha}_j)\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+\sum_{k=1}^{n}(\ell_{vx,i,k}-\ell_{vx,j,k})\widehat{\mu}_k(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|). \tag{M.17}
$$

Denote the consumer price of sector i goods by

$$
P_{ci}\equiv\Big(v_iP_i^{1-\theta_i}+(1-v_i)(S\cdot P_{IM,Fi}^{*})^{1-\theta_i}\Big)^{\frac{1}{1-\theta_i}}.
$$

Using equation (K.23) in Appendix K.4, the difference between the gaps of the consumer price of sector i goods and the CPI is equal to:

$$\begin{aligned}&\widehat{P}_{ci}^{gap}(\boldsymbol{\xi})-\widehat{P}_{C}^{gap}(\boldsymbol{\xi})\\&=v_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(1-v_i)\big(\widehat{S}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{P}_{C}^{gap}(\boldsymbol{\xi}))+o(\|\widehat{\boldsymbol{\xi}}\|)\\&=v_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(1-v_i)\big(\widehat{S}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(\sigma+\varphi/\Lambda_L)\widehat{C}^{gap}(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned}\tag{M.18}$$

We can replace the terms of allocation gaps on the RHS of equations (41) and (42) in Proposition 4 with the relative price gaps on the LHS of equations (M.15)-(M.18) according to the following equations:

$$\begin{aligned}\widehat{C}_{i}^{gap}(\boldsymbol{\xi})-\widehat{C}^{gap}(\boldsymbol{\xi})&=-\big(\widehat{P}_{ci}^{gap}(\boldsymbol{\xi})-\widehat{P}_{C}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{L}_{i}^{gap}(\boldsymbol{\xi})-\widehat{Y}_{i}^{gap}(\boldsymbol{\xi})&=-\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{P}_{i}^{gap}(\boldsymbol{\xi})\big)-\widehat{\mu}_i(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{X}_{i,j}^{gap}(\boldsymbol{\xi})-\widehat{Y}_{i}^{gap}(\boldsymbol{\xi})&=-\big(\widehat{P}_{j}^{gap}(\boldsymbol{\xi})-\widehat{P}_{i}^{gap}(\boldsymbol{\xi})\big)-\widehat{\mu}_i(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{C}_{Hi}^{gap}(\boldsymbol{\xi})-\widehat{C}_{Fi}^{gap}(\boldsymbol{\xi})&=-\theta_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{X}_{Hi,Hj}^{gap}(\boldsymbol{\xi})-\widehat{X}_{Hi,Fj}^{gap}(\boldsymbol{\xi})&=-\theta_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{Y}_{EX,i}^{gap}(\boldsymbol{\xi})&=-\theta_{Fi}\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned}$$

Further combining the above equation with equation (B.4) in Proposition B.1, equation (21) in Theorem 1, and equation (J.39), we can express each of the RHS of equations (40), (41), and (42) in Proposition 4 as a square term of sectoral inflation, *viz.*, $-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{within}\widehat{\mathbf{P}}(\boldsymbol{\xi})$, $-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{across}\widehat{\mathbf{P}}(\boldsymbol{\xi})$, and $-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{cb}\widehat{\mathbf{P}}(\boldsymbol{\xi})$, respectively, which are the LHS of equations (40), (41), and (42) in Proposition 4. □

Efficiency wedge component of welfare loss in closed economies. In closed economies à la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), $v_i=v_{x,i,j}=\Lambda_L=1$, $\lambda_{EX,i}=0$, $\mathcal{M}_{OG,i}=\widetilde{\lambda}_{D,i}=\lambda_i$, and $\ell_{vx,i,j}$ reduces to $\ell_{i,j}$. The cross-border misallocation disappears, and equation (M.13) reduces to the following expression:

$$\begin{aligned}&\widehat{C}(\widehat{\boldsymbol{\mu}})-\widehat{L}(\widehat{\boldsymbol{\mu}})\\&=-\frac{1}{2}\sum_{i=1}^n\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2-\frac{1}{2}\sum_{i=1}^n\beta_i\big[\widehat{C}_i(\widehat{\boldsymbol{\mu}})-\widehat{C}(\widehat{\boldsymbol{\mu}})\big]^2\\&-\frac{1}{2}\sum_{i=1}^n\lambda_i\alpha_i\big[\widehat{L}_i(\widehat{\boldsymbol{\mu}})-\widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2-\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n\lambda_i\omega_{i,j}\big[\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}})-\widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2+o(\|\widehat{\boldsymbol{\mu}}\|^2).\end{aligned}$$

The mappings from sectoral markup wedges into allocations in the *equivalent economy* reduce to:

$$\widehat{C}_i(\widehat{\boldsymbol{\mu}})-\widehat{C}(\widehat{\boldsymbol{\mu}})=\widehat{P}_C(\widehat{\boldsymbol{\mu}})-\widehat{P}_i(\widehat{\boldsymbol{\mu}})=\sum_{k=1}^n(\lambda_k-\ell_{i,k})\widehat{\mu}_k+o(\|\widehat{\boldsymbol{\mu}}\|),$$

$$\widehat{L}_i(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}}) = \widehat{P}_i(\widehat{\boldsymbol{\mu}}) - \widehat{W}(\widehat{\boldsymbol{\mu}}) - \widehat{\mu}_i = \sum_{k=1}^{n} \ell_{i,k}\widehat{\mu}_k - \widehat{\mu}_i + o(\|\widehat{\boldsymbol{\mu}}\|),$$

$$\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}}) = \widehat{P}_i(\widehat{\boldsymbol{\mu}}) - \widehat{P}_j(\widehat{\boldsymbol{\mu}}) - \widehat{\mu}_i = \sum_{k=1}^{n}(\ell_{i,k} - \ell_{j,k})\widehat{\mu}_k - \widehat{\mu}_i + o(\|\widehat{\boldsymbol{\mu}}\|).$$

Accordingly, we derive the same efficiency wedge component of welfare loss for closed economies as in Rubbo (2023), *viz.*:

$$\widehat{C}(\widehat{\boldsymbol{\mu}}) - \widehat{L}(\widehat{\boldsymbol{\mu}}) = -\frac{1}{2}\sum_{i=1}^{n}\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2 - \frac{1}{2}\sum_{i=1}^{n}\lambda_i\widehat{\mu}_i^2 - \sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\ell_{i,j}\widehat{\mu}_i\widehat{\mu}_j + \frac{1}{2}\Big(\sum_{i=1}^{n}\lambda_i\widehat{\mu}_i\Big)^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2). \tag{M.19}$$

We further follow La'O and Tahbaz-Salehi (2022) to introduce the pricing error $\{\overline{e}_i\}_i$ and link them to sectoral markup wedges as below:

$$\overline{e}_i = \sum_{j=1}^{n}\ell_{i,j}\widehat{\mu}_j, \qquad \widehat{\mu}_i = \overline{e}_i - \sum_{j=1}^{n}\omega_{i,j}\overline{e}_j, \qquad \sum_j \beta_j\overline{e}_j = \sum_{i=1}^{n}\lambda_i\widehat{\mu}_i. \tag{M.20}$$

Combining equations (M.20) and (M.19), we derive the same efficiency wedge component of welfare loss for closed economies as in Rubbo (2023), *viz.*:

$$\widehat{C}(\widehat{\boldsymbol{\mu}}) - \widehat{L}(\widehat{\boldsymbol{\mu}}) = -\frac{1}{2}\sum_{i=1}^{n}\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2 - \frac{1}{2}xvar_0(\overline{\mathbf{e}}) + \frac{1}{2}\sum_{i=1}^{n}\lambda_i\, xvar_i(\overline{\mathbf{e}}) + o(\|\widehat{\boldsymbol{\mu}}\|^2),$$

where $xvar_0(\overline{\mathbf{e}})$ and $xvar_i(\overline{\mathbf{e}})$ are the same short-hand notations as in La'O and Tahbaz-Salehi (2022), *viz.*:

$$xvar_0(\overline{\mathbf{e}}) = \sum_{j=1}^{n}\beta_j\overline{e}_j^2 - \Big(\sum_{j=1}^{n}\beta_j\overline{e}_j\Big)^2, \quad \text{and}$$

$$xvar_i(\overline{\mathbf{e}}) = \sum_{j=1}^{n}\omega_{i,j}\overline{e}_j^2 - \Big(\sum_{j=1}^{n}\omega_{i,j}\overline{e}_j\Big)^2, \quad \text{for } i \in \{1, 2, \cdots, n\}.$$

M.2. Proof of Propositions 5 and H.1: The optimal monetary policy

The optimal monetary policy maximizes the welfare loss (up to the second-order approximation) in equation (39) subject to the sectoral Phillips curves (up to the first-order approximation) in equation (34):

$$\max_{\widehat{C}^{gap},\widehat{\mathbf{P}}} \Big\{ -\frac{1}{2}\Big(\sigma - 1 + \frac{\varphi+1}{\Lambda_L}\Big)\widehat{C}^{gap}(\boldsymbol{\xi})^2 - \frac{1}{2}\widehat{\mathbf{P}}^\top\boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}\Big\}$$

$$\text{s.t.} \qquad \widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}},$$

where $\boldsymbol{\mathcal{L}} \equiv (\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})$. Denote $\boldsymbol{\eta}$ the vector of multipliers for the constraint of sectoral Phillips curves. The first-order conditions with respect to $\widehat{C}^{gap}$ and $\widehat{\mathbf{P}}$, respectively, are:

$$-\big[\sigma - 1 + (\varphi + 1)/\Lambda_L\big]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\eta}^\top \boldsymbol{\mathcal{B}} = 0, \tag{M.21}$$

$$-\boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \boldsymbol{\eta} = 0. \tag{M.22}$$

Substituting equation (M.22) into equation (M.21) to eliminate $\boldsymbol{\eta}$ yields:

$$\big[\sigma - 1 + (\varphi + 1)/\Lambda_L\big]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = 0. \tag{M.23}$$

Substituting equations (21) and (20) from Section 3 into equation (M.23) yields:

$$\Big\{\big[\sigma - 1 + (\varphi + 1)/\Lambda_L\big]\kappa_C^{-1}\boldsymbol{\mathcal{M}}_{OG}^\top(\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\Big\}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = 0.$$

Substituting the sectoral Phillips curves in equation (34) in equation (M.23) yields:

$$\big[\sigma - 1 + (\varphi + 1)/\Lambda_L + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\big]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}} = 0.$$

□