Entrepreneurs, Idiosyncratic investment risk, and Monetary policy*

Zhesheng Qiu
City University of Hongkong

Yicheng Wang
Peking University HSBC Business School

February 24, 2022

Abstract

Entrepreneurs are an important part of the U.S. economy. We first document new empirical facts that entrepreneurs' income and consumption are fluctuating pro-cyclically over the business cycles and also responding strongly to monetary policy shocks. This is somewhat in contrast to the conventional wisdom that entrepreneurs are rich and their consumption can be smoothed and insured very well. We instead highlight that, when heterogenous entrepreneurs face idiosyncratic investment shocks and credit market frictions, a large fraction of them are actually being constrained and in turn consumption smoothing is a second-order issue relative to the investing motive. Both analytical results from a simple framework and quantitative analysis through a new Heterogeneous Agent New Keynesian model (HANK) model suggest that this is true. In addition, for the transmission of monetary policy shocks and the macroeconomic implications, we show entrepreneurs' heterogeneity is important to take into account.

Key words: Entrepreneur, Incomplete markets, Monetary Policy, Heterogeneous Agent New Keynesian model

JEL: D31, E12, E21, E24, E43, E52

*Emails: zheshqiu@cityu.edu.hk and wangyc@phbs.pku.edu.cn. Yicheng Wang acknowledges the financial support from the National Natural Science Foundation of China (Grant number 72150003).

1 Introduction

Entrepreneurs, owning and managing their private businesses, are an important part of the U.S. economy. They own more than one third of capital, hire more than one half of workers, and spend more than others on average.[1] However, entrepreneurs' decisions on consumption and investment in the face of idiosyncratic risks, and subsequently, the role of entrepreneurs over business cycles are not studied extensively yet. In this paper, we contribute on further understanding entrepreneurs' decisions and their macroeconomic implications both empirically and quantitatively.

Using micro-level data from Internal Revenue Service (IRS) and also the Consumer Expenditure Survey (CEX), we first document new empirical evidence: entrepreneurs' income and consumption are much more procyclical than workers, both unconditionally over the business cycle and also conditional on identified monetary policy shocks. These suggest that entrepreneurs face volatile business income over the business cycle, and their consumptions are not well insured — while in much of the business cycle studies, entrepreneurs are typically assumed to be risk neutral so that their individual consumption smoothing is not studied (e.g., as in the seminal work Bernanke et al. (1999) and the subsequent literature), or entrepreneurs are assumed to be wealthy enough to bear aggregate fluctuations (e.g., Broer et al. (2020) for a recent example)[2]. In this paper, we depart from these literature and study individual entrepreneurs' consumption and investment behavior in the face of idiosyncratic risks, with well disciplined cross-sectional heterogeneity in income and wealth.

We then first theoretically study risk-averse entrepreneurs' consumption and investment decisions in a simple but quite standard framework. It helps us build up some simple intuitions on the mechanisms. The model is deliberately kept simple for analytical results, but still sufficiently rich to capture the interactions between entrepreneurs' consumption and investment decisions with possible collateral constraint. We focus on two polar cases. We show that, when entrepreneurs are unconstrained in the credit market, for small changes in interest rates or wages, entrepreneurs behave as if they are in a complete-market environment with perfect consumption smoothing, and their consumption and investment decisions are completely independent. Entrepreneurs' marginal propensity to consume (MPC) is simply the interest rate in their Euler equations, just like in the case for a repre-

[1] See more information on entrepreneurs in the section of calibration.

[2] Mostly for modelling simplicity, the literature typically assumes a representative and wealthy entrepreneur.

sentative household, and the marginal propensity to invest (MPI) is 0. However, for the other extreme, when entrepreneurs are always being constrained, their consumption and investment choices interact. Lower real interest rate may depress consumption through substitution effect simply due to the fact that entrepreneurs would like to invest more. In this case, entrepreneurs' marginal propensity to consume (MPC) and marginal propensity to invest (MPI) are linked together and MPI could be very high.

We then build a Heterogeneous Agent New Keynesian model (HANK) with heterogenous entrepreneurs, in addition to heterogenous workers (as in Aiyagari (1994) and Kaplan, Moll, and Violante (2018)). Quantitatively, we would like to ask: when taking into account of entrepreneurs' cross-sectional heterogeneity in productivity and net worth, are entrepreneurs' consumption and investment decisions quantitatively important over business cycles? How much do they contribute to the transmission of monetary policy? Which elements are particularly important for the contributions? In particular, in the model, we assume entrepreneurs are risk averse, face idiosyncratic productivity shocks, choose consumption, saving, and investment optimally. Also, the markets are incomplete for entrepreneurs (like Aiyagari (1994)), and they face collateral constraints in external financing. Quantitatively, the model is able to capture the income and wealth distributions for workers and entrepreneurs from micro-level data very well.

Based on the quantitative model, we first find that, in the steady state, we have about 63% entrepreneurs being constrained. On average, the MPI is about 43% (weighted by individual wealth); while for constrained entrepreneurs, the average weighted MPI is almost doubled, as high as 86%; in sharp contrast, the average weighted MPI for unconstrained entrepreneurs is about 30%. For MPC, we see in general there are not large differences between constrained and unconstrained entrepreneurs. Across different percentiles of wealth, MPI on average is decreasing; this pattern also holds for MPC. Thus, entrepreneurs value quite differently with the injection of additional liquid assets: for example, for those at 10^{th} percentile of the wealth distribution, the valuation would be on average three times higher than the average for all entrepreneurs.

Next we study the transition dynamics of the economy when there is a one-time unexpected expansionary monetary shock (about 0.6 percentage points). We find entrepreneurs' business income increase substantially on impact, larger than that for workers' wages, and entrepreneurs increase consumption and investment. These facts are consistent with our empirical documents. Investigating further, we find that, since there are a significant fraction of entrepreneurs constrained in the steady state at the time of shocks and have very high levels of MPI, with the decreased real interest rates and decreased real wages, entrepreneurs' business income will increase to a great extent and the increase in investment on impact is substantial. We further look at the distributional impact of monetary policy shocks across different groups. Ex-ante constrained have much higher increase in investment and consumption; For firms with initial net worth in the first quintile, their firm capital on average increases about 3% in the peak, almost 4 times higher than firms with net worth in the highest quintile. We also confirm our results with several robustness check.

In short summary, in this paper, we are motivated empirically that entrepreneurs' income and consumption are fluctuating pro-cyclically over the business cycles and also responding strongly to monetary policy shocks. This is somewhat in contrast to the conventional wisdom that entrepreneurs are rich and their consumption can be smoothed and insured very well. We instead highlight that, when heterogenous entrepreneurs face idiosyncratic investment shocks and credit market frictions, a large fraction of them are actually being constrained and consumption smoothing is a second-order issue relative to the investing motive. Both analytical results through a simple but quite standard framework and the quantitative analysis through a realistic structural model suggest that this is true. In addition, for the transmission of monetary policy shocks and the macroeconomic implications, we show entrepreneurs' heterogeneity is important to take into account.

Relating to the literature. Our paper is related to several strands of literature. First, a large literature suggests that entrepreneurs face substantial liquidity constraints for investing, either at the time of entry into entrepreneurship or during the process of operations; thus, exogenous changes in housing prices or credit market conditions may affect their entry and investment incentives. (e.g., among others, see Quadrini (2000), Cagetti et al. (2006), Hurst and Lusardi (2004), Buera and Moll (2015), Moll (2014), Khan and Thomas (2013), Schmalz et al. (2017), Adelino et al. (2015)).[3] In our paper, we build on this literature, incorporating entrepreneurs' realistic heterogeneity, and in the exercise of monetary policy, the change in nominal interest rates can also impact entrepreneurs differently.

Our empirical facts are related to a few papers in the literature. Parker and Vissing-Jorgensen (2009) documents that for households with high consumption on average, their consumption is more exposed to fluctuations in aggregate shocks than that of low-consumption households in the Consumer Expenditure (CEX) Survey. However, they

[3]Also, Angeletos and Calvet (2006) and Angeletos (2007) find in a steady state, entrepreneurs' idiosyncratic investment risk can lower aggregate savings in the equilibrium.

do not focus on entrepreneurs. We also show this holds unconditionally and conditional on monetary policy shocks. Coibion et al. (2017) shows contractionary monetary policy shocks decrease business income, but they focus on the effects of monetary policy on U.S. income and consumption inequality empirically. Instead, we focus on entrepreneurs' heterogeneity and the macroeconomic implications over business cycles, empirically, in theory and quantitatively.

Our paper is also related to the recent literature of HANK. Seminar papers, like Kaplan et al. (2018) and Auclert (2019) focus on household side – households' balance sheet and households' optimization on liquid and illiquid assets may have important macroeconomic implications. Ottonello and Winberry (2018) show that, both empirically and quantitatively, firms with low default risk are more responsive to monetary shocks since their marginal costs of investment are relatively flat. Kekre and Lenel (2020) consider heterogeneity in households' marginal propensity to take risk (e.g., through specifications in preferences) and study the transmission of monetary policy through risk premia in a heterogeneous agent New Keynesian environment. In comparison, our paper features endogenous heterogeneity for entrepreneurs in taking risks and in being financially constrained or not. We also show that this is important for the transmission of monetary policy. In another related paper, Bassetto et al. (2015) studies the effects of credit shocks in a model with heterogeneous entrepreneurs, financing constraints, and a realistic firm-size distribution (but not in a New Keynesian environment). They show that negative shocks can have a very persistent effect and the speed of recovery crucially depends on the extent to which the shock erodes entrepreneurial wealth. We also show that in the steady state and during the transitions, the effects of monetary shocks impact entrepreneurs differently and initial net worth plays a very important role in determining the magnitudes of responses.

The rest of the paper organizes as follows. In section 2, we provide our new empirical facts on entrepreneurs' business income and consumption. In section 3, we then present a simple framework and analyze entrepreneurs' decisions in theory. We then present the quantitative structural model in section 4, calibrate the model in section 5, investigate the model's properties in the steady state in section 6, and study the transmission of monetary shocks in section 7. We conclude in section 8. All supplementary proofs and supporting results are delegated to the Appendix.

2 Empirical Facts for entrepreneurs' income and consumption

In this section, we use various publicly available data and show two stylized facts for entrepreneurs' income and consumption.[4] Nevertheless, we try to use the best publicly available data as much as possible, and our messages are consistent. For the definition of entrepreneurs, following influential works in the literature (Quadrini (2000), Cagetti et al. (2006), Hurst and Lusardi (2004), and others), we think of entrepreneurs including those who own a significant percentage of their private businesses and also actively manage the businesses. This is also the concept we will have in the model.

Fact 1: Entrepreneurs have more cyclical income than workers, unconditionally and conditional on identified monetary policy shocks.

We first investigate entrepreneurs' income over the business cycles. To start, U.S. Bureau of Economic Analysis (BEA) provides seasonally adjusted annual data on non-farm proprietors' income, which is a comprehensive and consistent economic measure of the income earned by all U.S. unincorporated non-farm businesses.[5] Wages are from BLS (Average Hourly Earnings of Production and Non-supervisory Employees) for private sector employees, seasonally adjusted at annual frequency. Both measures are deflated with CPI.

In Figure 1, intuitively we can see the growth rates for business income fluctuate much more than wages. Statistically, Table 1 shows that, the mean and median of the annual growth rates for business income are about 10 times bigger than those for wages; for standard deviation, it is about 4 times bigger. Similar pictures also emerge in Table 8 in the appendix for quarterly frequency.

[4]To study entrepreneurs' income and consumption dynamics over the business cycles, we do not have a perfect empirical data set that has all the variables and relevant information. For example, the Consumer Expenditure Survey (CEX) data is a short panel and it only has 4 quarters for each household; PSID does not have good information on entrepreneurs' consumption and it is biennial.

[5]The data we use is from 1965 to present. BEA's measure of non-farm proprietors' income provides a comprehensive and consistent economic measure of the income earned by all U.S. unincorporated non-farm businesses. The featured measure — non-farm proprietors' income with inventory valuation adjustment and capital consumption adjustment is not directly affected by changes in tax laws, is adjusted for non-reported and misreported income, and excludes dividend income, capital gains and losses, and other financing flows and adjustments, such as deduction for "bad debt." Thus, the measure is a particularly useful analytical indicator of the health of noncorporate businesses and provides both a complement and contrast to the NIPA measure of corporate profits. See more details in Chapter 11 in the NIPA handbook 2019; Also see NIPA tables 1.10, 1.11, 1.12, 1.13, 1.16 and 6.12.

Figure 1: US Business income and wages over the business cycle; BEA and BLS data

Table 1: Annual growth rates for business income and wage (%)

	Mean	S.D.	Percentiles				
			10^{th}	25^{th}	50^{th}	75^{th}	90^{th}
Business income	2.23	6.18	-5.25	-1.66	3.43	6.25	9.22
Wage	0.22	1.56	-1.34	-0.60	0.39	1.10	1.93

Unfortunately, the BEA measure only focuses unincorporated private businesses and misses other types of private businesses (such as partnerships, S corporations and some other limited liability businesses for entrepreneurs.[6]). In addition, it is aggregate data and

[6]See Dyrda and Pugsley (2019) for discussions on recent forms of organizations for U.S. private businesses, and Cooper et al. (2016) on U.S. business ownerships and taxation. Also see more statistics from U.S. census on this topic: "https://www.census.gov/programs-surveys/susb/technical-documentation/data-user-

we do now know whether and how different entrepreneurs may have different patterns of business income dynamics. To explore further, we use micro-level data from the Internal Revenue Service (IRS), as in Piketty et al. (2018). In particular, data is from the samples of tax returns that have been created by the Statistics of Income (SOI) division of the Internal Revenue Service (IRS). We use these SOI individual income tax files provided from Piketty et al. (2018). The observation unit in the data is a tax filer and filer identities are anonymous, so the data is essentially a repeated cross-sectional data. It includes all filed business income, from unincorporated non-farm businesses as well as from other "Pass-through" businesses like partnerships and S-corporations, or any other business income. Note we do not include capital gains or dividends from investing and holding shares of private or public firms. The disadvantage of this data is that, it is only for those who do file taxes[7], and the information is at individual filer level, not household level.

Figure 2: US Business income and wages over the business cycle; IRS data

resources/legal-form-of-organization.html".

[7]According to Piketty et al. (2018), about 10-15% of adults aged 20 and above do not file tax returns.

In Figure 2, we confirm that using tax data, we can also observe larger fluctuations in business income comparing to workers' wages. For example, the growth rates for average business income are lower than -10% in the recessions around 1980, and are close to about 8% in the Great recession around 2008. In addition, we group entrepreneurs by different wealth percentiles (e.g., top 1%, 5%, 10%, 25%, 50%), and inspect the patterns for average business income within each group over time. The results are in Figure 14 in the appendix. By and large, we find entrepreneurs in the top percentile groups all have large fluctuations in business income comparing to wages.[8] Admittedly, since here we are using these cross-sectional data on tax filers, different groups over time may consist of quite different individuals, so we cannot speak too much at this point for individual-level business income risks. In DeBacker et al. (2012), using a new, large, and confidential panel of US income tax returns from IRS and extensive econometric estimation methods, they also show that, individual-level business income is much riskier than labor income. Our facts on business income is also consistent with the finding in Heathcote and Perri (2018), where they show that U.S. households' wealth is low in periods of recessions (like 1980 and 2008); presumably, business income and business wealth are important for households' total wealth.

Next, we show that, using identified monetary policy shocks, business income also respond more than wages. To do so, we follow the influential work Romer and Romer (2004) to identify innovations to monetary policy purged of anticipatory effects related to economic conditions. We also follow Nakamura and Steinsson (2018) and obtain identified monetary policy shocks by using high-frequency financial market data on interest rates. For outcome variables of interest, we consider aggregate time series on GDP, consumption, business income and wages. Table 8 provides summary statistics for the data time series we use. As mentioned before, the standard deviation for business income is almost 5 times bigger of that for wages. Following the local projection method in Jordà (2005), we set the

[8]For the bottom 50%, the fluctuations in business income are even noisier, and we suspect that this is mainly due to the fact that there are many small entrepreneurs in and out of the data over time, and the average business income for the bottom group are typically negative in the recessions. That is why we often see almost -200% growth rates.

regression equation as follows:

$$\begin{aligned}\log(\text{Outcome}_{t+h}) - \log(\text{Outcome}_{t+h-1}) &= \\ \sum_{j=1}^{J} \alpha_j^h \times [\log(\text{Outcome}_{t-j}) - \log(\text{Outcome}_{t-j-1})] \\ + \sum_{i=1}^{I} \beta_i^h \times \text{MP}_{t-i} + \mu_h + \epsilon_{t+h}, h &= 0, 1, ..., H,\end{aligned}$$

where MP are quarterly monetary policy innovations. As a benchmark, set $J = 2$ and $I = 20$. We estimate the system of equations across horizons jointly and for standard errors we allow for arbitrary serial and cross-sectional correlations across horizons and time. We are mostly interested in the accumulated impulse responses, which could be constructed using $\beta_1^h, 0 \leq h \leq H$.

In Figure 17, we report the estimation results for the accumulated impulse responses. Evidently, we find that responding to an expansionary monetary policy shock with 100 basis point, aggregate GDP, consumption and wages respond gradually and reach a level of 0.8% roughly 5 quarters after the shock. However, for business income, the responses are much larger, reaching almost 5% in the peak in about 10 to 15 quarters. Thus, its magnitude of responses is almost about 8 to 10 times larger than wages. We also confirm our results are robust to different lags for the monetary policy shocks in the regression (see Figure 19 in the appendix for $I = 15$ and Figure 21 in the appendix for $I = 25$). In the benchmark we have used shocks based on Romer and Romer (2004); the results based on Nakamura and Steinsson (2018) are similar and are reported in Figure 23 in the appendix.[9]

Fact 2: Entrepreneurs have more cyclical consumption than workers, unconditionally and conditional on identified monetary policy shocks.

We further investigate entrepreneurs' consumption dynamics. Intuitively, even if entrepreneurs have more volatile income, ex ante we do not know whether and how their consumption fluctuations would be different from that for workers: on one hand, entrepreneurs on average are richer and they could insure against large income shocks; on the other hand, it is possible that when entrepreneurs have different skills and investment opportunities, their business income are partly endogenously determined by their optimal choices in investing. Thus, entrepreneurs' consumption could also be more volatile as the benefits of smoothing consumption could be potentially dominated by pursuing higher

[9]Since the time series data for this quarterly monetary policy shocks are more limited, we use fewer lags in the regressions ($I = 10$).

profits. Empirically, we use micro-level data from the Consumer Expenditure Survey (CEX) from the US, the largest and best available data set for studying consumption. Our data is from the years 1980 to 2006, using the raw data provided in Heathcote et al. (2010). In the data, a household is interviewed at most for 4 consecutive quarters and we have roughly 15000 households before 1999 and 20000 after that. Each quarter the household members is asked to report consumption expenditures information but income questions are only asked to the households during the first and fourth interview. We define entrepreneurs in the short panel of CEX as those with any positive business income in any period within the panel of observations, or without any wage income but with positive labor income (for more details on variable definitions, please see the data appendix).

We first intuitively inspect the cyclical property of consumption, for entrepreneurs and for workers. To do this, we simply regress consumption growth rate on Aggregate GDP growth rate, and we include a rich set of controls on exogenous variables which would potentially affect consumption growth as well: dummy variables for interview year and month, rural area, region, change of family sizes, reference person's sex, education and ages. The results are reported in Table 9 in the appendix. We find that, indeed, entrepreneurs have more cyclical consumption than workers. By and large, we find that, for 1% increases in Aggregate GDP, entrepreneurs' consumption growth rate on average is about 0.7 percentage points higher than that for workers. This result also holds across a variety of robustness checks and specifications: In column (1), we report the benchmark results for consumption on non-durable goods; In column (2) we do not include any controls; In column (3) we add consumption in housing services, namely housing rents for renters and imputed equivalent housing rents for home owners, to non-durable goods consumption; In column (4) we use all expenditures reported in the CEX, including non-durable goods consumption, consumption in housing services, and all other durable goods consumptions (such as appliance, vehicles, and so on); In column (5) we restrict the sample with no family changes; In column (6) we exclude households with total expenditures in the top 5% of the cross-sectional distribution each year; In column (7) we exclude households with total expenditure in the top 10%; In column (8) we regress only for households with income larger than the median values each year. In column (9) we only include households with income smaller than the median; column (10) is only for households with total expenditure larger than the median; column (11) is only for households with total expenditure smaller than the median.

Our result on entrepreneurs' cyclical consumption is also consistent with findings from Parker and Vissing-Jorgensen (2009). In particular, Parker and Vissing-Jorgensen find that the consumption of high-consumption households is more exposed to fluctuations in aggregate consumption and income than that of low-consumption households in the Consumer Expenditure (CEX) Survey.

Lastly, we study how consumptions respond to well identified monetary policy shocks for differen groups. As before,we again use the identified monetary policy shocks based on Romer and Romer (2004) and Nakamura and Steinsson (2018). The regression specification is simple and follows:

$$\Delta \log(c_{i,t}) = \sum_{k=-1,-2,-3,-4} \beta_k \times \epsilon_{t+k} \times I_{entrepreneurs} + I_{entrepreneurs} + \delta_0 X_{i,t} \times I_{entrepreneurs} + \epsilon_{i,t},$$

where $I_{entrepreneurs}$ is the dummy variable for being entrepreneurs in the CEX data. We report $\sum\limits_{k=-1,-2,-3,-4} \beta_k$ for the cumulated responses (or the semi-elasticity) of entrepreneurs' consumption to monetary shocks, relative to that for workers. The results are in Table 10 in the appendix (and also the details on controls). We also find that entrepreneurs' consumption responses are much larger than that for workers. For a given monetary shock with 100 basis points lower, entrepreneurs' consumption growth rates on average would be higher for about 2 percentage points higher. We also confirm its robustness: in the Table column (1) we do not include any controls; column (2) include a rich set of controls and their interaction with dummy variable of being entrepreneurs; column (3) considers non-durable goods consumption and consumption in housing services; column (4) considers all household expenditures; column (5) excludes those households with consumption in the top 5% of the distribution each year; column (6) uses household consumption and survey sampling weights as the weight for household; column (7) excludes any periods in the recessions.

3 Investigating Entrepreneurs' optimizations through a simple model

To further understand entrepreneurs' consumption and investment decisions over business cycles, we first use a simple, quite standard theoretical framework. The model is deliberately kept simple for analytical results, but still sufficiently rich to capture the interactions between entrepreneurs' consumption and investment decisions with possible collateral constraint. It helps us build up some simple intuitions on the model mechanisms.

3.1 Basic Setups

We assume there is an infinitely lived representative entrepreneur with perfect foresight on the paths of gross real interest rate $\{R_t\}$ and real wage rate $\{w_t\}$. It chooses the sequences of consumption $\{c_t\}$, savings $\{b_t\}$, investment $\{i_t\}$, capital $\{k_t\}$, and labor $\{n_t\}$ to solve

$$\begin{aligned}
&\max_{\{c_t,b_t,i_t,k_t,n_t\}} \sum_{t=0}^{+\infty} \beta^t u(c_t), && \\
\text{s.t.}\quad & c_t + b_t + i_t \leq R_t b_{t-1} + F(k_{t-1}, n_t) - w_t n_t, && \text{(budget constraint)} \\
& k_t \leq i_t - g\left(\frac{i_t}{k_{t-1}}\right) k_{t-1} + (1-\delta) k_{t-1}, && \text{(capital law of motion)} \\
& 0 \leq b_t + \Psi k_t, && \text{(collateral constraint)}
\end{aligned}$$

in which initial b_{-1} and k_{-1} are given, β is the discount factor, Ψ is the pledgeability rate of capital, δ is the depreciation rate of capital, $u(\cdot)$ is the utility function of consumption, $F(\cdot)$ is the production function defined on capital and labor, and $g(\cdot)$ is the cost function of capital adjustment. For well defined solutions, we impose Assumption 1 below.

Assumption 1. *Functions $\{u, F, g\}$ are twice continuously differentiable, among which $\{u, F\}$ are strictly increasing and strictly concave in $\mathbb{R}_{>0}$ and $\mathbb{R}^2_{>0}$, while g is strictly convex in $\mathbb{R}$. These functions satisfy* $\lim_{c\to+0} u'(c) = +\infty$, $\lim_{c\to+\infty} u'(c) = +0$, $F(\lambda k, \lambda n) = \lambda F(k, n)\ \forall \lambda > 0$, $\lim_{k\to+0} F_k(k, n) = +\infty$, $\lim_{k\to+\infty} F_k(k, n) < 1$, $F(1, x) \equiv f(x)$, *and* $g(\delta) = g'(\delta) = 0$.

The solution of the problem is a set of functions mapping $(b_{t-1}, k_{t-1}, \{R_{t+\tau}, w_{t+\tau}\})$ to $\{c_t, b_t, i_t, k_t, n_t\}$. We focus on the first order perturbation solution around the steady state, in which the total differential representation of $\{c_t, k_t\}$ that captures the impulse responses of consumption and investment is

$$\begin{bmatrix} dc_t \\ dk_t \end{bmatrix} = \begin{bmatrix} \frac{\partial c_t}{\partial b_{t-1}} & \frac{\partial c_t}{\partial k_{t-1}} \\ \frac{\partial k_t}{\partial b_{t-1}} & \frac{\partial k_t}{\partial k_{t-1}} \end{bmatrix} \begin{bmatrix} db_{t-1} \\ dk_{t-1} \end{bmatrix} + \sum_{\tau=0}^{+\infty} \begin{bmatrix} \frac{\partial c_t}{\partial R_{t+\tau}} & \frac{\partial c_t}{\partial w_{t+\tau}} \\ \frac{\partial k_t}{\partial R_{t+\tau}} & \frac{\partial k_t}{\partial w_{t+\tau}} \end{bmatrix} \begin{bmatrix} dR_{t+\tau} \\ dw_{t+\tau} \end{bmatrix}.$$

For deeper understanding, we decompose the partial derivatives with respect to $\{R_{t+\tau}, w_{t+\tau}\}$ into income effects ∂^I and substitution effects ∂^S with $\partial = \partial^I + \partial^S$.

This decomposition has real-world implications. For instance, a cut in real interest rate raises the return rate of investment through substitution effect and alleviates the interest payment burden through income effect; a decline of real wage rate increases the cash flows of entrepreneurs through income effect and also the profit margins of production through substitution effect. A better understanding of theses channels can provide deeper insights into how monetary policy transmits through entrepreneurs.

To justify the perturbation, we only consider situations in which a steady state exists, and the collateral constraint is either always not binding or always binding. We denote all steady state variables using subscripts "ss", and omit variables in functions evaluated at the steady state. With such notations, Lemma 1 below characterizes the incomes effects.

Lemma 1 (Income Effects). *Consider a hypothetical lump-sum transfer $Tr_{t+\tau}$ to the budget at period $t+\tau$, then the income effects satisfy*

$$\begin{bmatrix} \frac{\partial^I}{\partial R_{t+\tau}} & \frac{\partial^I}{\partial w_{t+\tau}} \end{bmatrix} = - \begin{bmatrix} (-b_{ss}) \frac{\partial}{\partial Tr_{t+\tau}} & n_{ss} \frac{\partial}{\partial Tr_{t+\tau}} \end{bmatrix}.$$

Intuitively, higher real interest rate increases interest payment and higher real wage rate increases labor cost, both of which reduce entrepreneur income. Theoretically, Lemma 1 is consistent with Slutsky Decomposition in consumer theory, in which substitution effect is defined as decision changes induced by price changes when utility is kept constant by a hypothetical lump-sum transfer, and income effect is defined as the residual of substitution effect in total effects. The hypothetical transfer in Lemma 1 is the negative of that in Slutsky Decomposition when price changes are infinitesimal. We prove it in the appendix.

Before solving the entrepreneurs' problem, we simplify it by eliminating labor input decisions under constant returns to scale production function F in Lemma 2 below.

Lemma 2 (Production Function). *Denote $n(k_{t-1}, w_t) \equiv \arg\max_n \{F(k_{t-1}, n) - w_t n\}$. When production function F has constant returns to scale as in Assumption 1, the marginal production of capital under optimal labor input depends only on real wage rate but not on capital as below*

$$F_k(k_{t-1}, n(k_{t-1}, w_t)) = f({f'}^{-1}(w_t)) - w_t {f'}^{-1}(w_t) \equiv \widetilde{R}(w_t) - 1 + \delta.$$

In addition, we have the marginal effect of real wage rate on capital return rate satisfying

$$\widetilde{R}'(w_t) = -{f'}^{-1}(w_t) = \frac{F_{kn}(k_{t-1}, n(k_{t-1}, w_t))}{F_{nn}(k_{t-1}, n(k_{t-1}, w_t))}.$$

With Lemma 2, entrepreneurs' problem reduces to a dynamic portfolio choice problem with exogenous return rate of assets. A shock to real wage rate can be viewed as a shock to capital return rate. To have a brief idea of how sensitive $\widetilde{R}(w_t)$ is to w_t, we use Example 1.

Example 1 (Cobb-Douglas)**.** *Let* $F(k,n) = zk^{1-\alpha}n^{\alpha}$ *with* $z \in (0,+\infty)$ *and* $\alpha \in (0,1)$*, then*

$$w_{ss}\widetilde{R}'(w_{ss}) = -\frac{\alpha}{1-\alpha}[\widetilde{R}(w_{ss}) - 1 + \delta].$$

In Example 1, the elasticity of marginal product of capital ($MPK = \widetilde{R} - 1 + \delta$) with respect to real wage rate is determined by labor share α only. Since the steady state MPK is much smaller than 1, a 1% change in real wage rate can only have a very small marginal effect on the real return rate of capital $\widetilde{R}(w_{ss})$ (denoted as $\widetilde{R}_{ss}$ for short) in the steady state.

3.2 Unconstrained Vs. Constrained Entrepreneurs

To further understand the entrepreneurs' decisions, we provide the analytical solutions of all income and substitution effects for two polar cases in which entrepreneurs are permanently unconstrained and permanently constrained in Proposition 1 and 2, respectively.

Proposition 1 (Unconstrained)**.** *When* $R_{ss} = \frac{\widetilde{R}_{ss} - \Psi R_{ss}}{1-\Psi} = \beta^{-1}$ *and* $b_{t-1} + \Psi k_{t-1} > 0$*,*

$$\begin{bmatrix} \frac{\partial^I c_t}{\partial R_{t+\tau}} & \frac{\partial^I c_t}{\partial w_{t+\tau}} \\ \frac{\partial^I k_t}{\partial R_{t+\tau}} & \frac{\partial^I k_t}{\partial w_{t+\tau}} \end{bmatrix} = -\beta^{\tau} \begin{bmatrix} 1-\beta \\ 0 \end{bmatrix} \begin{bmatrix} -b_{ss} & n_{ss} \end{bmatrix},$$

$$\begin{bmatrix} \frac{\partial^S c_t}{\partial R_{t+\tau}} & \frac{\partial^S c_t}{\partial w_{t+\tau}} \\ \frac{\partial^S k_t}{\partial R_{t+\tau}} & \frac{\partial^S k_t}{\partial w_{t+\tau}} \end{bmatrix} = -\beta^{\tau} \mathbf{1}_{\tau \geq 1} \left\{ \beta c_{ss} \left(-\frac{u'}{c_{ss}u''}\right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \frac{k_{ss}}{g''} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & f'^{-1} \end{bmatrix} \right\}.$$

To make sure that entrepreneurs are not constrained in the steady state, we must have $\beta R_{ss} = \beta \widetilde{R}_{ss} = 1$, so that the return rates of savings and capital exactly counterbalance the discount factor β. As a result, (b_{ss}, k_{ss}) is indeterminate. When such an entrepreneur starts with $b_{t-1} + \Psi k_{t-1} > 0$ at period t, it is in a steady state $(b_{ss}, k_{ss}) = (b_{t-1}, k_{t-1})$, and any shock will drift it to a new steady state. When the shock is sufficiently small, the entire transition path will be unconstrained as well.

The unconstrained results in Proposition 1 isolate an intratemporal substitution effect in investment decisions. To see why, consider a real interest rate $R_{t+\tau}$ with $\tau \geq 1$ and the corresponding capital response k_t. The marginal cost of investment at period t evaluated at period $t+\tau$ is $R_{ss}^{\tau} \cdot g'(\frac{i_t}{k_{t-1}})$, while the marginal benefit of arbitrage is $R_{ss} - R_{t+\tau}$. Equalizing these two objects yields $\frac{\partial k_t}{\partial R_{t+\tau}} = -\beta^{\tau}\frac{k_{ss}}{g''}$, which is exactly the total effect. There are no other effects on investment because when the return rates of savings and capital are equal in the long run, capital is dominated by savings in the short run as an asset due to the adjustment cost $g'' > 0$, and hence will not be used for intertemporal consumption smoothing.

Proposition 1 does not help explain why entrepreneur's consumption is much more procyclical than workers'. To understand it, we consider the income effects and substitution effects separately. For income effects, we have $\frac{\partial c_t}{\partial Tr_{t+r}} = (1-\beta)\beta^\tau$, which is exactly identical to the counterpart of an unconstrained worker in a standard consumption-saving problem with complete markets. For substitution effects, $\frac{\partial^S c_t}{\partial R_{t+\tau}} = -\beta^{\tau+1} c_{ss}(-\frac{u'}{c_{ss}u''})$ for $\tau \geq 1$ is also the same as the worker's solution. These equivalence results are immediate consequences of excluding capital in intertemporal consumption smoothing.

Proposition 2 (Constrained)**.** *When* $R_{ss} < \frac{\widetilde{R}_{ss} - \Psi R_{ss}}{1-\Psi} = \beta^{-1}$ *and* $b_{t-1} + \Psi k_{t-1} = 0$*,*

$$\begin{bmatrix} \frac{\partial^I c_t}{\partial R_{t+\tau}} & \frac{\partial^I c_t}{\partial w_{t+\tau}} \\ \frac{\partial^I k_t}{\partial R_{t+\tau}} & \frac{\partial^I k_t}{\partial w_{t+\tau}} \end{bmatrix} = -\beta^\tau \frac{1}{1 + \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g''} \begin{bmatrix} 1 + \boldsymbol{1}_{\tau=0} \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g'' - \beta \\ -\frac{1}{1-\Psi}(\boldsymbol{1}_{\tau \geq 1} - \beta) \end{bmatrix} \begin{bmatrix} -b_{ss} & n_{ss} \end{bmatrix},$$

$$\begin{bmatrix} \frac{\partial^S c_t}{\partial R_{t+\tau}} & \frac{\partial^S c_t}{\partial w_{t+\tau}} \\ \frac{\partial^S k_t}{\partial R_{t+\tau}} & \frac{\partial^S k_t}{\partial w_{t+\tau}} \end{bmatrix} = -\beta^\tau \boldsymbol{1}_{\tau \geq 1} \frac{\beta c_{ss}(-\frac{u'}{c_{ss}u''})}{1 + \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g''} \begin{bmatrix} -1 \\ \frac{1}{1-\Psi} \end{bmatrix} \begin{bmatrix} \frac{\Psi}{1-\Psi} & \frac{1}{1-\Psi} f'^{-1} \end{bmatrix}.$$

When $\beta R_{ss} < 1$, real interest rate is too low to keep entrepreneurs' savings above the collateral constraint in the long run. As the collateral constraint binds, savings and capital can be viewed as a bundle $b_t + k_t = (1-\Psi)k_t$ with gross return rate $\frac{\widetilde{R}(w_{t+1}) - \Psi R_{t+1}}{1-\Psi}$. Since this return rate is equal to β^{-1} in the steady state, k_{ss} is indeterminate. For any steady state to start with, when a shock is sufficiently small, the transition path is constrained as well.

Unlike the standard borrowing constraint that blocks the intertemporal consumption smoothing, the collateral constraint prevents intratemporal arbitrage. When exploiting the return rate disparity between savings and capital, the reallocation between savings and capital with net wealth unchanged is no longer feasible or optimal, and the entrepreneurs have to trade intertemporally.

The intertemporal trade of capital in this permanently constrained case crucially relies on the capital adjustment cost g''. As $g'' \to 0$, the constrained entrepreneurs' problem is isomorphic to a standard consumption-saving problem under complete markets, while as $g'' \to +\infty$, the collateral constraint resembles a standard borrowing constraint that blocks intertemporal consumption smoothing.

When $g'' \in (0, +\infty)$, investment decision makes collateral constraint less effective in generating procyclical consumption. To see why, we consider the income and substitution effects in Proposition 2 separately. In income effects, a constrained entrepreneur has

$$mpc = 1 - \frac{\beta}{1 + \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g''}.$$

$g'' > 0$ implies $mpc > 1-\beta$, and that the constrained case has stronger income effects than the unconstrained case, while $g'' < +\infty$ implies $mpc < 1$, and that the income effects are reduced by investment. In substitution effects, $g'' < +\infty$ implies $\frac{\partial^S c_t}{\partial R_{t+\tau}} > 0$ and $\frac{\partial^S c_t}{\partial w_{t+\tau}} > 0$, which works against procyclical consumption. As a result, highly procyclical business income is necessary to generating highly procyclical consumption for this extreme case.

Unlike consumption, entrepreneurs' investment cyclicality relies less on the binding collateral constraint, because the substitution effects of the unconstrained entrepreneurs can be very strong compared with the constrained case. To show the difference, consider the size of $\frac{\partial^S k_t}{\partial R_{t+\tau}}$ for $\tau \geq 1$ in the constrained case below

$$\frac{\partial^S k_t}{\partial R_{t+\tau}} = -\beta^\tau \frac{\beta c_{ss}(-\frac{u'}{c_{ss}u''})}{1 + \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g''} \frac{\Psi}{(1-\Psi)^2} > -\Psi\beta^\tau \frac{k_{ss}}{g''}.$$

The substitution effect in the constrained case is no more than a fraction Ψ of that in the unconstrained case. The difference is $(1-\Psi)\beta^\tau \frac{k_{ss}}{g''}$ if measured in levels, and $(1-\Psi)\beta^\tau \frac{1}{\delta g''}$ if measured as a percentage of steady state investment. In standard calibration, the size of $\frac{1-\Psi}{\delta g''}$ can be large even compared with the cyclicality of business income in data.

3.3 Remarks

As a short summary, we have three takeaways from the simple theoretical analysis. First, the unconstrained case does not help explain why entrepreneurs' consumption is much more procyclical than workers'. Second, the collateral constraint for entrepreneurs is not as effective as the borrowing constraint in a standard consumption-saving problem, so that we need more procyclical business income to generate highly procyclical consumption. Third, entrepreneurs' investment procyclicality relies less on the collateral constraint.

Nevertheless, we should keep in mind that the two cases we consider are quite extreme. In reality, due to large idiosyncratic shocks, unconstrained and constrained statuses are not likely to be permanent, so that each individual entrepreneur is making decisions taking into account of future transitions, instead of in some steady state. Hence, the theoretical results in the simple model are at best only suggestive. Therefore, in the next section, we consider a more realistic quantitative model, in which whether entrepreneurs are constrained or not are all determined endogenously. And also, in the quantitative model, we can have the changes in real interest rates, real wage rate, business income, and individual decisions all in a general equilibrium environment.

4 Quantitative Model

Households There are a continuum of households in the economy. Households are heterogenous, differing from each other in idiosyncratic labor efficiency e and financial assets a. In period t, denote $\Pi_t = \frac{P_t}{P_{t-1}} - 1$ as the inflation rate between $t-1$ and t, where P_t is time t price for final goods. Denote i_t^a as the nominal interest rates between $t-1$ and t, and r_t as the real interest rates between $t-1$ and t, with $1 + r_t = \frac{1+i_t^a}{1+\Pi_t}$. A typical household faces the following optimization problem:

$$V_t(a, e) = \max_{c, a'} u(c) - v(n) + \beta E V_{t+1}(a', e') \tag{1}$$

$$c + a' = a \frac{1 + i_t^a}{1 + \Pi_t} + wen \tag{2}$$

$$a' \geq 0. \tag{3}$$

where she chooses final goods to consume, saving for the next period. Note that the budget constraint is written in units of final goods this period. The household faces possible borrowing constraint $a' \geq 0$, and also, households face idiosyncratic income shocks in e' going to the next period. Notice that in this economy, we assume individual household takes the amount of labor demand n as given, which is optimized and required by the labor unions; for the labor market arrangement, see more details below.

Entrepreneurs There are also a continuum of entrepreneurs in the economy. They differ in idiosyncratic productivity, firm size and debt liability positions, and they produce homogenous final goods. In the current period, denote the entrepreneur's idiosyncratic productivity as z, firm capital as k, total debt as b. Both of k and b are determined in the prior period. In period t, a typical entrepreneur has the following optimization problem:

$$V_t(b,k,z) = \max_{c,b',k',n} u(c) + \beta^E E V_{t+1}(b',k',z') \tag{4}$$

$$c + i + g(i,k) + \frac{1+i_t^a}{1+\Pi_t} b = b' + \exp(z)\left[(1-\alpha)k^{\frac{\varepsilon-1}{\varepsilon}} + \alpha n^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon\nu}{\varepsilon-1}} - wn, \tag{5}$$

$$i = k' - (1-\delta)k, \tag{6}$$

$$b' \leq \Psi k', 0 < \Psi < 1, k' \geq 0, \tag{7}$$

where entrepreneurs' production function is given by $f(z,k,n) \equiv \exp(z)\left[(1-\alpha)k^{\frac{\varepsilon-1}{\varepsilon}} + \alpha n^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon\nu}{\varepsilon-1}}$, with constant elasticity of substitution across capital and labor, ε, and labor share in the production is denoted by α. The adjustment cost function is assumed to be $g(i,k) = \frac{\Phi_k}{2}(\frac{i}{k} - \delta)^2 k$. In the credit market, entrepreneurs face a form of collateral constraint: $b' \leq \Psi k'$. That is, entrepreneurs can borrow at most up to Ψ fraction of firm capital. Following much of the literature (e.g., Buera and Moll (2015), Moll (2014), Khan and Thomas (2013)), we use this simple form to capture different degrees of credit market frictions when Ψ varies.

For the optimization, in short, the entrepreneur chooses labor demand n for this period's production (a static optimization problem), investment i for the next period, consumes, repays any interest payments in real terms and borrows b' for the next period (save if $b' < 0$). In the process of adjusting capital stock, she faces adjustment costs as $g(i,k)$. Notice that the entrepreneur has to choose capital stock for next period's production in this period; thus, she faces possible idiosyncratic investment risk. This setting is different from most of macroeconomic models for entrepreneurs (such as Quadrini (2000), Cagetti et al. (2006) and Moll (2014)). With the environment of incomplete markets for idiosyncratic investment risk and entrepreneurs being risk averse, entrepreneurs with different state of (a,k,z) will value liquid assets quite differently (see more discussions in section 6.2). Finally, for the convenience of analysis later on, economic profits or business income for entrepreneurs in the current period (consistent with BEA and NIPA definitions), can be defined as

$$\exp(z)\left[(1-\alpha)k^{\frac{\varepsilon-1}{\varepsilon}} + \alpha n^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon\nu}{\varepsilon-1}} - wn - (\delta + r_t)k,$$

where we net of all the costs for current production.

Corporate firms To better match data quantitatively, assume there is a sector of corporate firms. A representative corporate firm operates with a decreasing-return-to scale technology, faces no credit market frictions (as in Cagetti et al. (2006) and Zetlin-Jones and Shourideh (2017)), and discount future dividends using real interest rate in the financial market. The representative corporate enters time t with capital stock K_t, chooses dividend payout D_t, capital stock for the next period K_{t+1}, and labor input N_t to maximize the present value of discounted dividends $V_t(K_t)$:

$$
\begin{aligned}
V_t(K_t) &= \max_{\{K_{t+1}, N_t\}} D_t + \frac{1}{1+r_{t+1}} V_{t+1}(K_{t+1}).,\\
D_t &= F(K_t, N_t) - W_t N_t - I_t - g\left(I_t, K_t\right)\\
I_t &= K_{t+1} - (1-\delta)K_t,
\end{aligned}
$$

where production function is assumed to be $F(K,N) \equiv \exp(z^c)\left[K^{1-\alpha}N^{\alpha}\right]^{\nu}$, with $\nu \leq 1$ and $\exp(z^c)$ denotes firm productivity. In a very simple setting as the benchmark, following much of the literature, we assume all firm dividends are taxed away and spent by the government.[10] Denote the representative firm's optimal labor demand as N_t^c, investment demand as I_t^c and capital stock as K_t^c.

Labor market and Wage setting

We assume nominal wages are sticky when the economy is out of steady state, and we have a form of Wage Philips curve for the dynamics of nominal wages. In particular, following the New Keynesian literature with Calvo wages (see, e.g., Christiano et al. (2016), Auclert et al. (2018) and Auclert et al. (2020), among others), we assume that there is a

[10] Alternatively, we can assume that corporate firms issue shares in the equity market, and there is a competitive financial intermediation sector in the economy. The representative financial intermediation firm (FF) takes all deposits from households, lends to all entrepreneurs if there is any demand, and purchase equity shares for the representative corporate firm. Denote the after-dividend share price at time t as p_t, we should have the flow budget constraint for FF:

$$
\int a_t(1+r_t)m^W \mu_t^W(a,e) - \int_t b_t(1+r_t)m^E \mu_t^E(a,k,z) \quad = \quad p_t + (D_t),
$$

where the equation states that, after receiving dividends at the end of this period, the total liability on the left hand side is equal to total assets on the right hand side. At the end of time t, the financial intermediation firm purchases equity shares and takes deposits for the next period; the flow budget constraint is:

$$
p_t \quad = \quad \int a_{t+1} m^W \mu_t^W(a,e) - \int_t b_{t+1} m^E \mu_t^E(a,k,z),
$$

where at the end of time t, the total liability for FF is the right hand side of the second equation, and p_t is the total value of assets since FF purchases shares at price p_t for the next period. Lastly, asset pricing equation holds as $p_t = \frac{1}{1+r_{t+1}} E_t(p_{t+1} + D_t - Tax_t)$, since no arbitrage condition should hold and the financial intermediation sector is assumed to competitive and frictionless. These setups follow much of the literature, such as Kaplan et al. (2018) and Auclert et al. (2020).

continuum types of labor varieties in the economy, denoted as $j \in [0,1]$. At each period t, the representative labor packer collect N_{jt} units of labor variety for each type $j \in [0,1]$ to produce N_t units of composite labor inputs through a Dixit-Stiglitz aggregator

$$N_t = \left(\int_0^1 N_{jt}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

where $\varepsilon_w \in (1, +\infty)$ denotes the elasticity of substitution between labor varieties. Denote W_{jt} as the nominal wage rate of N_{jt} and w_j as the nominal wage rate of N_t. The standard competitive labor packer's problem yields

$$N_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\varepsilon_w} N_t, \quad W_t = \left(\int_0^1 W_{jt}^{1-\varepsilon_w} dj \right)^{\frac{1}{1-\varepsilon_w}}, \quad W_t N_t = \int_0^1 W_{jt} N_{jt} dj.$$

For each type of labor variety j, there is a labor union. Each union j has all workers as members, and chooses a nominal wage rate W_{jt} on behalf of them. The induced demand for labor variety N_{jt} is imposed uniformly on all union members. In another word, each worker is forced to supply N_{jt} for all $j \in [0,1]$.

Denote $\theta_w \in [0,1]$ as the probability that nominal wages cannot be adjusted in a quarter. Assume workers' disutility in supplying labor is given by $v(n) = \chi \frac{n^{\frac{1}{\xi}+1}}{\frac{1}{\xi}+1}$, in which $\chi > 0$ is a normalization parameter and $\xi > 0$ is the Frisch elasticity of labor supply. With these assumptions, in appendix C.4, we show that we have a form of wage Phillips Curve when the economy is out of steady state:

$$\hat{\Pi}_t^w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w \xi^{-1})} \left(\xi^{-1} \hat{N}_t + \sigma^{-1} \hat{C}_t^* - \hat{w}_t \right) + \beta \mathbb{E}_t \hat{\Pi}_{t+1}^w,$$

where all variables with hat denote the corresponding log-deviations from their steady-state values, $\hat{\Pi}_t^w$ denotes the wage inflation rate from $t-1$ to t and equals $\hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1}$; $\hat{N}_t$ denotes the log-deviation of aggregate labor demand; C_t^* is the weighted workers' consumption in period t, defined as $u'(C_t^*) \equiv \int_0^1 e_{it} u'(c_{it}) di$. When the economy is in the steady state (or wages are flexible), we have the following condition for labor supply:

$$w_{ss} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{v'(N_{ss})}{u'(C_{ss}^*)},$$

which resembles the usual optimality condition in a representative agent economy but it is slightly modified in our environment. Following Christiano et al. (2016), we set the probability of not resetting wage each quarter θ_w as 0.75, the Frisch elasticity of labor supply ξ as 1.00, and the elasticity of labor variety demand ε_w as 6.00.

Monetary Policy

For monetary policy, we follow the rule in Christiano et al. (2016) for a medium scale economy, and assume that nominal interest rates at time t are set according to

$$\hat{i}^a_{t+1} = \rho_i \hat{i}^a_t + (1-\rho_i)(\varphi_\pi \hat{\Pi}_t + \varphi_Y \hat{Y}_t) - \sigma_i \epsilon_t, \tag{8}$$

with parameter values $(\rho_i, \varphi_\pi, \varphi_Y, 400\sigma_i) = (0.77, 2.02, 0.01, 0.64)$. We could have some other alternative monetary policy rules and we confirm our main results are not affected by these different settings.[11]

Stationary Economy. In most of our exercise for dynamics below, we assume the economy starting from a stationary status, or the steady state. In particular, the Stationary Economy is characterized a set of stationary measures, m^W for the measure of workers, m^E for the measure of entrepreneurs, $\mu^W(a,e)$ for the steady-state density function for the distribution of workers over individual states (a,e), and similarly $\mu^E(a,k,z)$ for the distribution of entrepreneurs (using net worth, $a \equiv k - b$ as a transformation for convenience; see Appendix C.1 for the derivation of entrepreneurs' optimization). In addition, the Stationary competitive equilibrium consists of a set of value functions and policy functions for workers, entrepreneurs and firms, and all of them take r and w as given and optimize as described previously. $\Pi = 0$ in the steady state. In the labor market, total labor supply from workers equal to total labor demand:

$$m^W \int e n^d \mu^W(a,e) = m^E \int n \mu^E(a,k,z) + N^c.$$

[11] For example, as in Kaplan et al. (2018), we assume that nominal interest rates at time t are set according to

$$\begin{aligned} \hat{i}^a_{t+1} &= \varphi_\pi \hat{\Pi}_t + \eta_t, \\ \eta_t &= \rho_i \eta_{t-1} - \sigma_i \epsilon_t. \end{aligned}$$

with parameters $(\rho_i, \varphi_\pi, 400\sigma_i) = (0.61, 1.25, 1.00)$. Alternatively, we could have

$$\hat{i}^a_{t+1} = \rho_i \hat{i}^a_t + (1-\rho_i)\varphi_\pi \mathbb{E}_t \hat{\Pi}_{t+1} - \sigma_i \epsilon_t.$$

with parameters set at $(\rho_i, \varphi_\pi, 400\sigma_i) = (0.86, 1.93, 1.00)$, mostly taken from existing literature. This version of Taylor rule follows Christiano et al. (2016) in modeling interest rate persistence to make sure that the sign of nominal interest rate response is more stable. It has nominal interest rate adjustment depending on inflation expectations instead of current inflation to avoid impact from the initial jump in inflation.

In the financial assets market, we should have market clearing as $m^W \int a\mu^W(a,e) = m^E \int b\mu^E(a,k,z)$. Lastly, for final goods, we need to have:

$$\begin{aligned} Y &= m^E \int f(z,k,n)\mu^E(a,k,z) + F(K^c, N^c) \\ &= m^W \int c\mu^W(a,e) + m^E \int (c + i + g(i,k))\mu^E(a,k,z) + I^c + G + g\left(I^c, K^c\right). \end{aligned}$$

5 Model Calibration

First, we set several parameters according to quite standard literature. Risk aversion parameter σ is set to 2.0 for all households with CRRA utility over consumption. For households' borrowing constraint, we assume it is 0. We assume the labor efficiency process follows an AR(1) process, with persistence of 0.929 and standard deviation for the innovation at 0.227 in a quarterly setting (see Chang and Kim (2007)). The Frisch elasticity of labor supply is assumed to be 1. For firms and production, capital depreciation rate δ is set to 0.025 in a quarter, labor share parameter $\alpha = 0.66$, and the fraction of entrepreneurs is set to 0.1, according to the SCF estimates, thus the measure for workers is 0.9. All firms, including corporate firms, have decreasing return to scale ν at 0.85: as pointed out by Quadrini (2000) and Cagetti et al. (2006), this number should be in the range of 0.8 and 0.9. On the other hand, since we do not model monopolistic competition for the firms, we think our parameter is consistent with the estimates of markups for intermediate goods firms, as in Basu and Fernald (1997), Burstein and Hellwig (2008) and Atkeson and Kehoe (2005). We use the employment share of corporate firms in the data to guide us on its productivity level z^c. For the entrepreneurs' production function, our benchmark model assume it is Cobb-Douglas production, i.e., the constant elasticity of substitution between capital and labor ε is set to 1.0. For the idiosyncratic productivity process z, we assume persistence of 0.86 and standard deviation for the innovation at 0.2 in a quarter (see Arellano et al. and Clementi and Palazzo (2016)). For collateral constraint, Ψ is set to 0.35 so that firms at most borrow up to 35% of the assets, similar to the values such as in Zetlin-Jones and Shourideh (2017).

For (Φ_k, β, β^E), where Φ_k is the investment adjustment cost parameter in the function $g(i,k) = \frac{\Phi_k}{2}(\frac{i}{k})^2 k$. β is the discount factor for workers, and we allow entrepreneurs' discount factor β^E to be slightly different; we do this mainly for a quantitative purpose so that entrepreneurs will not have strong incentives to save too much and are not collateral constrained (see related discussions in Cagetti et al. (2006) and Arellano et al.). We select these parameters to match three moments: the equilibrium quarterly real interest rate is around 1%, the standard deviation of investment/capital ratio is about 0.35 (as in Cooper and Haltiwanger (2006) and Ottonello and Winberry (2018)), the wealth share for entrepreneurs as a whole. The fitted values are $(1.02, 0.949, 0.926)$, respectively.

In the following table, we compare several moments from the data and the model, and we find our model can perform reasonably well. For example, regarding wealth distribution in the cross-section, all entrepreneurs as a whole accounts for about 45 percent of total wealth in the model, vs. 44 percent in the data. In addition, within entrepreneurs, we are also able to capture the distribution reasonably well and recall that we are not targeting at these moments: for those entrepreneurs with wealth above the 90^{th} percentile (of all households), the wealth share is about 41.4% in the model and 42.1% in the data; for the 50^{th} percentile these numbers are 45.3% vs. 43.8%, respectively. For all households, the wealth share for those above the 50^{th} percentile is about 94.3% in the model and is about 99% in the data; for the 90^{th} percentile, the model's number is 56.2%, smaller than 85% in the data –most likely, this is because the model is not desired to model those extremely rich workers but instead focusing more on entrepreneurs. We also compare income distributions in the model and in the data. They are also fairly close. For example, in the last panel we look at the income shares within entrepreneurs. For those entrepreneurs with income higher than the 90^{th} percentile of the whole population, the model implies that their income share (relative to all entrepreneurs) is about 92.4%, and the data counterpart is about 83.8%. For other percentiles, the model is also close to the data.

Table 2: **Data and Model moments**

Moments	Data	Model
Firm debt to assets	0.35	0.28
Employment share for corporations	0.48	0.48
Std(investment ratio)	0.35	0.33
Median(Income/Wealth) for Entrepreneurs	0.06	0.09
Wealth shares		
All Entrepreneurs	43.9%	45.6%
Entrepreneurs; Wealth above 90^{th} percentile	42.1%	41.4%
Entrepreneurs; Wealth above 80^{th} percentile	43.0%	43.7%
Entrepreneurs; Wealth above 50^{th} percentile	43.8%	45.3%
All HHs; Wealth above 90^{th} percentile	85.8%	56.2%
All HHs; Wealth above 50^{th} percentile	99.0%	94.3%
Income shares For All HHs		
Income above 90^{th} percentile	58.1%	54.5%
Income above 80^{th} percentile	67.1%	65.2%
Income above 50^{th} percentile	87.5%	85.8%
Income shares within Entrepreneurs		
Income above 90^{th} percentile	83.8%	92.4%
Income above 80^{th} percentile	89.2%	97.0%
Income above 50^{th} percentile	98.2%	99.1%

6 Understanding the model

6.1 Entrepreneurs' optimizations

Since entrepreneurs face collateral constraints,$\Psi k' - b' \geq 0$ (or $a' - (1-\Psi)k' \geq 0$), these constraints may be binding in the optimization problem. Denote $\mu u'(c)$ as the associated multiplier for the collateral constraint, scaled by current marginal utility, and the optimality conditions for consumption and investment are thus given as follows:

$$u'(c) = \mu u'(c) + \beta E u'(c\prime)(1+r'), \mu \geq 0. \tag{9}$$

$$\beta E \frac{u'(c\prime)}{u'(c)}[\frac{\partial \pi'}{\partial k'} - (r' + \delta) - g_2(i', k')] = g_1(i, k) + (1 - \Psi)\mu. \tag{10}$$

Note that with the adjustment cost function $g(i,k) = \frac{\Phi_k}{2}(\frac{i}{k} - \delta)^2 k$, the marginal adjustment cost in the current period is $g_1 \equiv \frac{\partial g(i,k)}{\partial i} = \frac{\partial g(i,k)}{\partial k'} = \Phi_k(\frac{i}{k} - \delta)$, and when changing k', the associated marginal adjustment cost in the next period is denoted as $g_2(i', k')$, where $g_2 \equiv \frac{\partial g(i,k)}{\partial k} = \frac{\Phi_k}{2}(\frac{i}{k} - \delta)^2 - \Phi_k(\frac{i}{k} - \delta)\frac{k'}{k}$.

Equation 9 states that, in the case of not being constrained ($\mu = 0$), entrepreneurs' consumption are smoothed; and if they are constrained ($\mu > 0$), current consumption is lower, and on average the consumption path is relatively steeper (i.e., $\frac{\beta(1+r')Eu'(c\prime)}{u'(c)} = 1 - \mu < 1$). Intuitively, entrepreneurs are willing to sacrifice current consumption and save for the next period so that she could produce more when productivity is relatively high. Turn to optimal investment. the right-hand side of equation 10, $g_1(i,k) + (1 - \Psi)\mu$, represents the marginal cost of investment: $g_1(i,k)$ is the marginal adjustment cost in the current period, and μ is the shadow cost of one more unit investment into k', which is involving changing relative consumption path between today and tomorrow. Note that since entrepreneurs can use firm assets as collateral for borrowing, the effective shadow cost of one more unit investment thus is reduced and becomes $(1 - \Psi)\mu$. The left-hand side of equation 10 is the expected, discounted marginal return for investment. In particular, $[\frac{\partial \pi'}{\partial k'} - (r' + \delta) - g_2(i', k')]$ is the contingent marginal return to capital in the next period, net of any opportunity costs (similar to rental costs) for capital and any marginal adjustment costs in the next period due to changes in investment in the current period ($i = k' - (1 - \delta)k$).

We can examine entrepreneurs' optimal decisions numerically in Figures 25 and 26 in Appendix C.7. In both figures, the left-hand panel is for optimal consumption decisions c, the middle panel for optimal saving a' and decisions on k', and the right-hand panel plots investment ratios $\frac{i}{k}$. Since entrepreneurs may have different individual state variables in (a, k, z), we plot those decisions as functions of a holding constant k and z. Figure 25 is for relatively small k, roughly at 50% of aggregate firm capital $m^E \int k\mu^E(a,k,z)$, and Figure 26 is for larger k, at about two times of aggregate firm capital. Solid black line is for entrepreneurs with productivity higher than mean level with $2\sigma_z$, dashed and dotted line is for entrepreneurs with mean productivity, and dashed red line is for low productivity entrepreneurs (lower than the mean with $2\sigma_z$). A few points are worth noting: (1) when entrepreneurs have higher productivity in the current period, they earn higher profits for given levels of k. This will push up their consumption and saving in general. For investing, since z is persistent, so entrepreneurs have incentives to invest more when z is higher and whenever they can. (2) For given productivity and capital levels, when entrepreneurs have too little net worth in a, they are more likely to be constrained. This can been seen in the middle panel where the functions for a' and $(1-\Psi)k'$ are overlapping with each other when a is small; In contrast, when a is large, entrepreneurs are not constrained and a' is higher than $(1-\Psi)k'$. As discussed before, when being constrained, consumption is more likely to be depressed and is more likely to be concave numerically when a is small. Similarly, we find that investment ratio is also more likely to be lower (even negative investing) and much more concave numerically. In addition, when entrepreneurs have larger k in the current period, the region of net worth for being constrained is larger than that for smaller k. For given k but different levels of z, entrepreneurs with higher z are more likely to be constrained. (3) Lastly, since there are adjustment costs for investing, the investment ratio in general is smoothed out even when net worth is very large. Quantitatively, this also will prevent entrepreneurs accumulating wealth through investment too fast and thus grow out of financial constraints.

6.2 Distributions for Entrepreneurs in the steady state

Next we examine the equilibrium steady state distribution for entrepreneurs. Note that since different entrepreneurs have different idiosyncratic productivity shocks in the steady state, individual characteristics for entrepreneurs are changing over time, even though aggregate variables and the distribution over entrepreneurs' state variables ($\mu^E(a,k,z)$) are constant over time. In Figure 3 we plot the steady state distribution over the space of (net) financial assets and firm capital a and k (in logs).[12] First note that, entrepreneurs in the equilibrium are quite concentrated, around the line for $a=(1-\Psi)k$, or

$$\log(a) = \log(1-\Psi) + \log(k).$$

That is, most entrepreneurs are being constrained or are close to be constrained in the equilibrium. We show later that this feature will have important aggregate implications when there are small shocks in factor prices for entrepreneurs. Also, we see that the dispersion of entrepreneurs over the space is large, suggesting firm heterogeneity is important in the equilibrium. Comparing to the simple model in the previous section and most of the representative-firm model in the literature, we see our quantitative model can help us capture the entrepreneur and firm heterogeneity.

[12]Note that in these figures the distribution includes all possible idiosyncratic productivity levels in z.

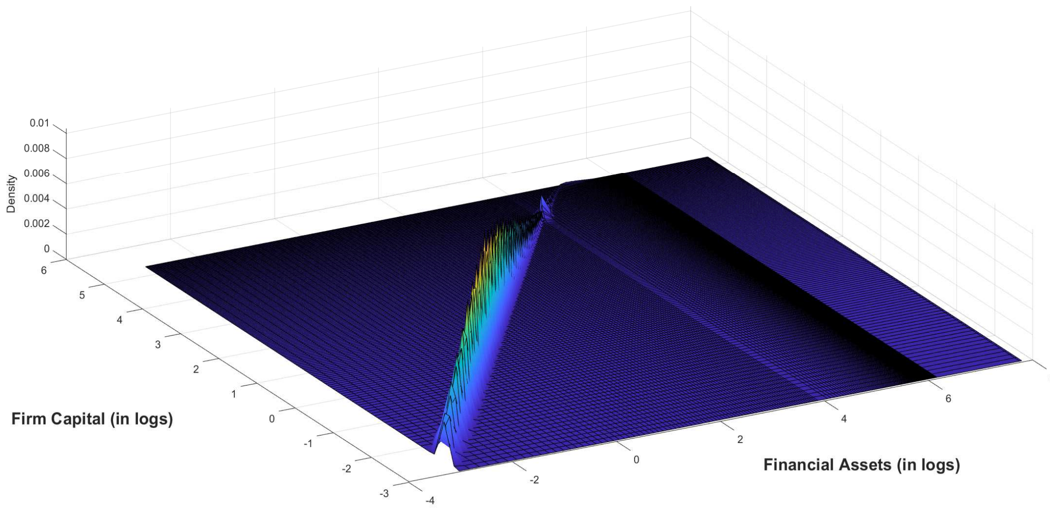


Figure 3: Steady state distribution for entrepreneurs

Since in the equilibrium most entrepreneurs are constrained or close to be constrained, naturally we would like to know how much entrepreneurs would value one more dollar in their current budget. Technically, we compute Marginal Propensity to invest (MPI) and Marginal Propensity to consume (MPC) for entrepreneurs, by using $\frac{\Delta i(a,k,z)}{\Delta}$ where Δ is small enough. Building on the insights from the recent macro literature on HANK models (e.g., Kaplan et al. (2018), Auclert et al. (2018), among others), we know MPI and MPC are important concepts in determining changes in aggregate demand in investment and consumption. In addition, in our environment, MPI is important for the persistence of propagating initial macroeconomic shocks, since entrepreneurs' investment will have persistent impacts on future periods' firm capital, labor hiring and entrepreneurs' consumption as well. Thus we provide more details for MPI in Figure 4 and for MPC in Figure 5.

A few points are worth noting: (1) For MPI, we see on average the magnitude is large; MPI slightly decreases if the combination of a and k is away from the line for those being constrained. Also note that for each point in the this figure, it represents the averages across entrepreneurs with the same level of (a, k) but different levels of individual z. (2) It is not necessarily the case that entrepreneurs with larger financial assets would have lower MPI. This is because these entrepreneurs may also have large firm capital, and in order to maintain smoothing capital stock in the next period, they have to invest more than others, and they will also be likely constrained by the collateral requirements. (3) Theoretically, in Appendix C.2, we show that there is a simple relationship between MPI and MPC for

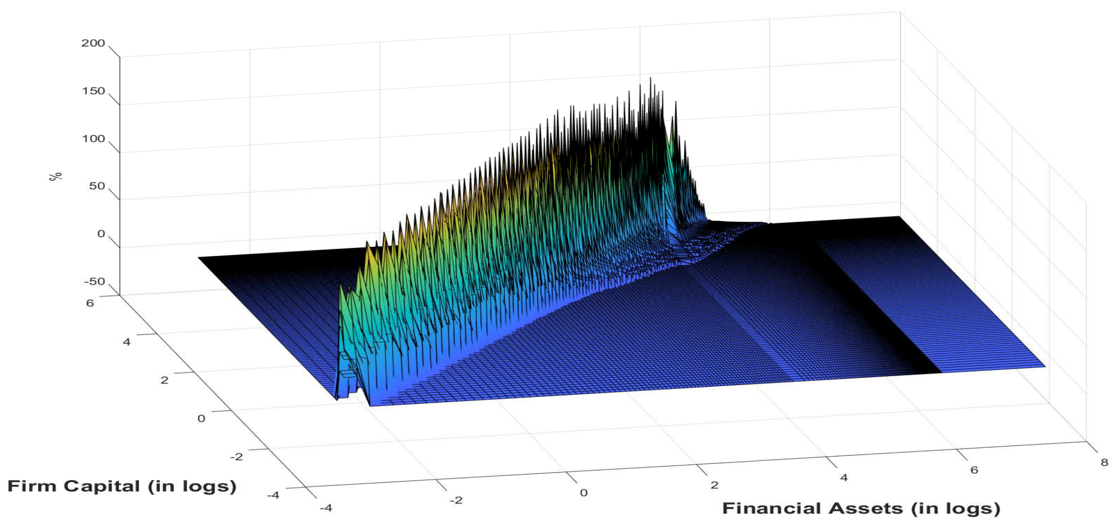


Figure 4: Marginal Propensity to invest (MPI) for entrepreneurs

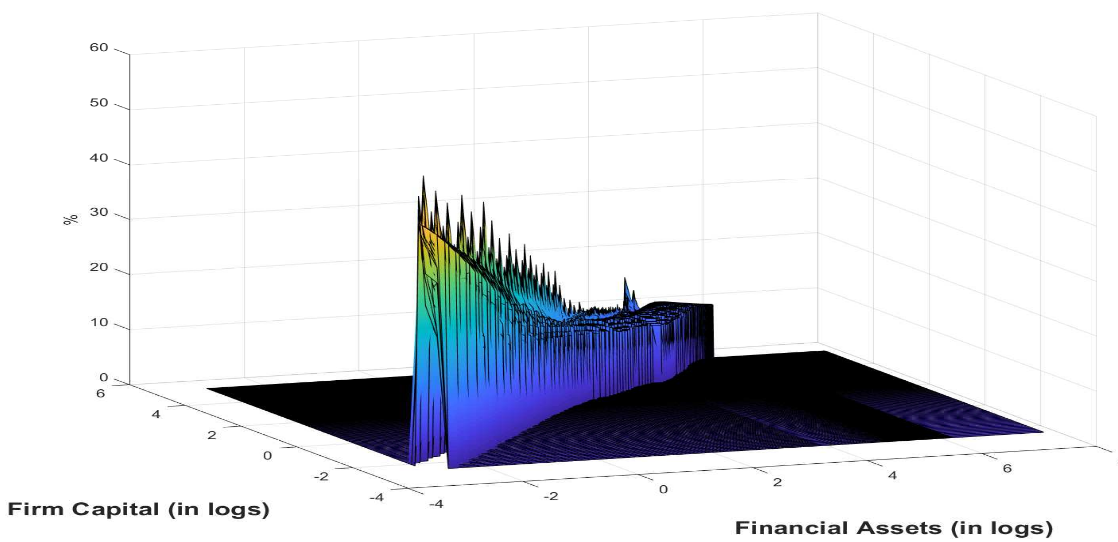


Figure 5: Marginal Propensity to consume (MPC) for entrepreneurs

constrained entrepreneurs:

$$MPI = \frac{1 - MPC}{[1 - \Psi + \Phi_k(\frac{i}{k} - \delta)]}. \tag{11}$$

Intuitively, when absent from adjustment costs ($\Phi_k = 0$), constrained entrepreneurs will spend some fraction of an additional dollar for consumption, reflected in MPC, and invest the remaining parts. Since entrepreneurs can borrow up to the collateral limit in the credit market, his or her additional investment would be $\frac{1-MPC}{1-\Psi}$. In terms of numbers, with Ψ calibrated to about 0.35, the effect could be large and additional investment could be about 150% of the remaining dollar. With adjustment costs ($\Phi_k > 0$), high-productivity entrepreneurs will likely have $\frac{i}{k} - \delta > 0$ and thus MPI is slightly lower than otherwise. For more details on simple analytical expressions of MPI for those constrained entrepreneurs, we derived in Appendix C.2. Lastly, we can see that for entrepreneurs, MPC are also quite different across the distribution, and MPC decreases on average when entrepreneurs have larger net worth a.

Table 3: MPI and MPC for entrepreneurs

	Weighted MPC	MPI	Unweighted MPC	MPI
A. Entrepreneur				
All	12.80%	43.76%	19.00%	74.74%
Constrained (63.2%)	14.66%	86.44%	21.79%	87.87%
Unconstrained	11.45%	30.98%	14.22%	52.15%
B. Worker				
All	13.08%		37.48%	
Constrained (27.2%)	75.42%		75.96%	
Unconstrained	12.71%		23.07%	

Numerically, in Table 3 we report MPI and MPC for entrepreneurs with and without using net worth a as weights. A few points stand out: (1) In the equilibrium we have about 63% entrepreneurs being constrained. On average, the weighted MPI is about 43%; while for constrained entrepreneurs, the average weighted MPI is almost doubled, as high as 86%; in sharp contrast, the average weighted MPI for unconstrained entrepreneurs is about 30%. We can also look at those statistics without using net worth a as weights. The average MPI is about 74%, much higher than the weighted counterpart; this reflects the facts that unconstrained entrepreneurs have relatively large financial net worth and relatively small firm capital. In addition, recall that previously we discussed about the theoretical relationship between MPI and MPC for constrained entrepreneurs; numerically, MPI and MPC sums to about 100% on average (see the unweighted cases). This suggests that even though constrained entrepreneurs will use collateral as leverage, the role of adjustment costs is also important and it lowers investment in general. (2) For entrepreneurs' MPC, we see in general there are not large differences between constrained and unconstrained entrepreneurs. Intuitively, when entrepreneurs are constrained, the marginal return of investment is relatively large, outweighing the benefits of raising consumption today. (3) Although it is not the focus of this paper, we also provide statistics for MPC for workers.

Roughly we have about 27% workers being credit constrained; also, we note that the difference in MPC between constrained and unconstrained workers on average is much larger than that for entrepreneurs. This also suggests that it is important to take into account of entrepreneurs' heterogeneity in a quantitative model.

Table 4: MPI and MPC: Distributional Statistics

	Mean	Std	Min	Max
	A. MPI for Entrepreneurs			
10^{th} percentile	79.2	25.2	26.4	164.2
25^{th} percentile	82.6	25.4	26.5	174.5
50^{th} percentile	83.9	25.9	24.5	189.0
75^{th} percentile	75.9	32.4	12.7	184.6
90^{th} percentile	59.3	30.9	4.6	142.4
	B. MPC for Entrepreneurs			
10^{th} percentile	28.1	3.8	19.2	45.9
25^{th} percentile	23.4	3.0	16.4	35.1
50^{th} percentile	17.6	1.8	14.3	23.6
75^{th} percentile	14.0	0.9	12.0	15.8
90^{th} percentile	11.8	0.4	10.6	13.7

Since we model entrepreneurs' heterogeneity along three different dimensions: financial assets a, firm capital stock k and individual productivity z, we also look into entrepreneurs' MPI and MPC in the cross-sectional distribution. These results are reported in Table 4. First, we sort equilibrium entrepreneurs by their financial assets a. Conditional on different percentiles of a in the steady state, there still will be dispersions in MPI and MPC across entrepreneurs since their k and z are different. For each different percentiles of a for entrepreneurs, in Table 4 we report the average, standard deviation, and the extreme values for MPI (in the upper panel) and MPC (in the lower panel). A few points are well worth noting: (1) across different percentiles of a, on average, the magnitude of MPI is large, roughly 4 times higher of average values in MPC. This is consistent with the fact that most entrepreneurs are constrained and have profitable opportunities to explore in the next period. (2) across different percentiles of a, MPI on average is decreasing; this pattern also holds for MPC. Perhaps this is somewhat intuitive, since entrepreneurs on average have more net financial assets at hand and, if they have high firm productivity, in general they could invest more and are less likely to be constrained. (2) For the dispersion of MPI, first, the extreme values in the whole cross-sectional distribution could differ dramatically. They could range from a very small value of 4.6% to a very large number of 189%. Across different percentiles of a, the dispersions slightly increase in a: for example, when a is relatively small at the 10^{th} percentile, the dispersion is about 25.2%, while it increases to about 30.9% for entrepreneurs at the 90^{th} percentile. In contrast, the pattern of dispersion for MPC is different and it decreases over different percentiles of a. This suggests the cross-sectional heterogeneity in individual characteristics of (a, k, z) increases as a increases.

We can also inspect how much different entrepreneurs value one more dollar given to them. In particular, we use the following formula to compute the change in (subjective) valuations of Δ increase in financial assets,

$$\frac{V(a+\Delta, k, z) - V(a, k, z)}{u'(\bar{c})},$$

where the change in valuation is scaled by $u'(\bar{c})$, and $\bar{c}$ is the average consumption level in the population.[13] We experiment with different magnitudes of Δ: in the upper panel Δ equals about 1% of output in the model economy[14], and in the lower panel Δ depends on individual characteristics and equals 1% of firm capital k for each entrepreneur. The results are now reported in Table 5.

In the table, a few points stand out: (1) Entrepreneurs value quite differently with the injection of additional liquid assets. For example, for those at 10^{th} percentile, the change in value function would be on average three times of average marginal utility (3.268 in the upper panel); when financial assets increase, on average entrepreneurs value less on the additional liquid assets. When financial assets are at the 90^{th} percentile, the increase in valuation is almost gone (only about 0.1% of average marginal utility). (2) In addition, we note that the dispersion in valuation increases across different entrepreneurs even conditional on a is also large; for example, when at the 25^{th} percentile, the standard deviation is almost as large as 30%, reflecting firm herogeneity in the cross section. As a increases, the dispersion also decreases. (3) When the injected liquid assets Δ is 1% of individual firm capital (the lower panel), we see the patterns for the changes in valuation are similar as before; however, since entrepreneurs with small financial assets also have small firm capital in the equilibrium ($a \geq (1-\Psi)k$), these entrepreneurs in turn will receive less injection than others. That is why we see the average numbers are smaller for those at

[13] We didn't use individual consumption to scale (like $\frac{V(a+\Delta,k,z)-V(a,k,z)}{u'(c(a,k,z))}$) because by doing that, when Δ is small enough, $\frac{V(a+\Delta,k,z)-V(a,k,z)}{u'(c(a,k,z))}$ will converge to $1+r$ according to the envelop theorem.

[14] If we used US data in 2010s, that would be about 500 dollars.

Table 5: Value of liquidity for Different Entrepreneurs

A. Additional liquid assets by 1% of GDP				
	Mean	Std	Min	Max
10^{th} percentile	3.268	2.417	0.075	12.588
25^{th} percentile	0.415	0.312	0.019	1.484
50^{th} percentile	0.048	0.034	0.004	0.157
75^{th} percentile	0.005	0.003	0.001	0.016
90^{th} percentile	0.001	0.001	0.000	0.002

B. Additional liquid assets by 1% of k				
	Mean	Std	Min	Max
10^{th} percentile	0.528	0.398	0.014	2.561
25^{th} percentile	0.224	0.170	0.011	0.938
50^{th} percentile	0.093	0.065	0.008	0.335
75^{th} percentile	0.036	0.024	0.005	0.123
90^{th} percentile	0.016	0.009	0.003	0.048

the 10^{th} and 25^{th} percentiles but larger for those at the 75^{th} and 90^{th} percentiles. In sum, this table intuitively illustrates the distributional importance of additional liquid assets for different entrepreneurs. In the following exercises, we show that how entrepreneurs respond to aggregate shocks are closely related to the distributional facts that a substantial percentage of them are constrained and have high values in liquid assets (income).

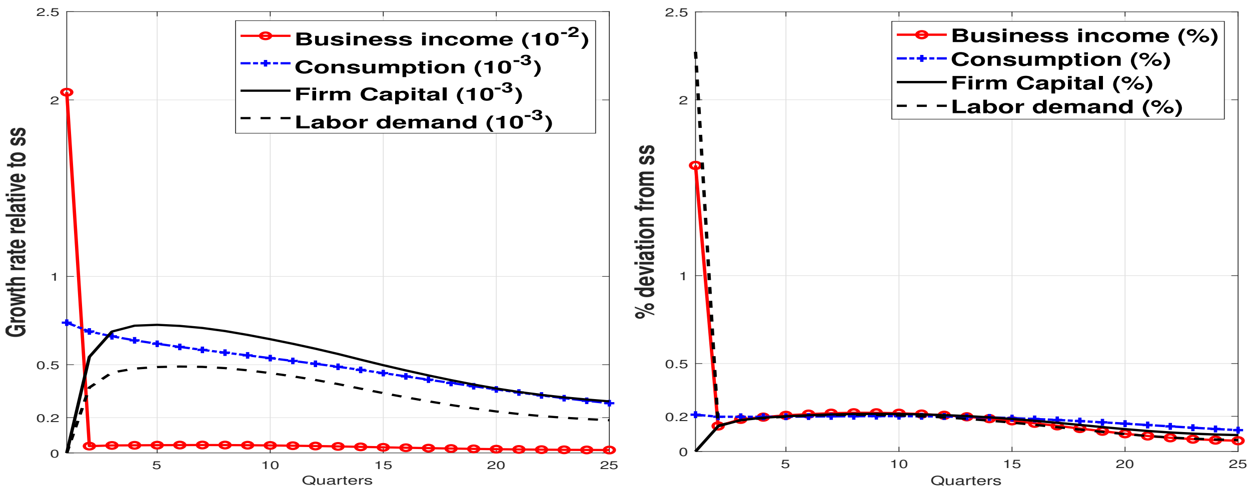
7 Quantitative results with Monetary policy shocks

7.1 Entrepreneurs' transitional dynamics: partial equilibrium

Next we study the transition dynamics of the economy when there is a one-time unexpected monetary shock. We consider an experiment in which at time $t = 0$, there is a quarterly innovation to the Taylor rule, $\epsilon_t = -1\%$ in the monetary policy rule. To understand the model further, we first study hypothetical cases with partial equilibrium analysis: suppose there is a one-time unexpected decrease in real wages or real interest rates, we investigate how entrepreneurs respond.[15]

[15] In terms of the methodology for solving transitional dynamics, we combine a more traditional way (backward iteration and forward iteration for a large panel of individuals from the steady state) and the method used in Auclert et al. (2019).

Figure 6: One-time, unexpected change in r (left) and in w (right)

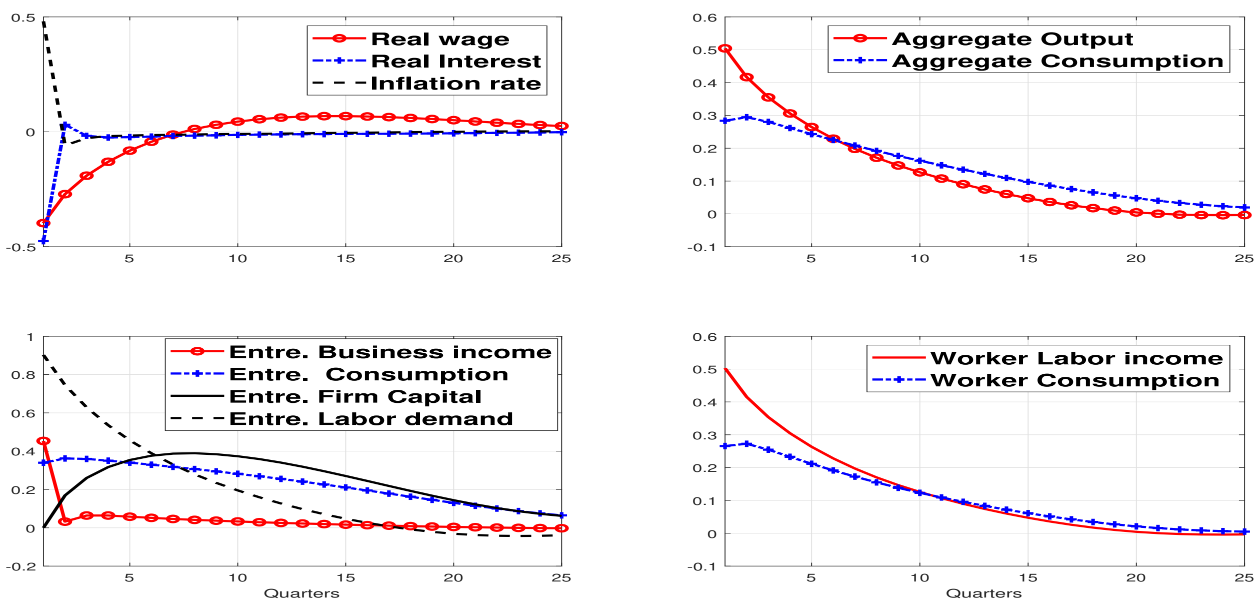


In the left panel of Figure 6, we assume r_t decreases about -0.64% in annualized terms ($\epsilon_t = -1\%$, and $(1 + r_{ss}\sigma_i)$) only for time $t = 0$. The figure plots the responses of the aggregate variables for entrepreneurs' business income, firm capital stocks, labor demand and consumption. All variables are expressed in terms of growth rates relative to their corresponding steady-state values. Since r_t decreases in the first period, entrepreneurs' production costs decreases and business income increases, even though there is no change in capital stock or labor hiring in the first period. After the first period, there is no further change in r_t relative to the steady state; however, the dynamics for the aggregate variables will be persistent for several years. The magnitude of business income changes is large and that is why we used a different scale in the left panel. On average, business income jumps for about 2% on impact but quickly diminishes after; Firm capital, labor demand and consumption all increase, around 0.05% on peak. Entrepreneurs gradually increase their firm capital until period 5 and hire more labor; after time 5, firm capital and labor demand gradually returns to the steady state. The main reason for this pattern is that, a significant percentage of entrepreneurs are constrained initially; when business income is increased in the first period, constrained entrepreneurs will not consume all of the increased liquid wealth immediately. Instead, they invest on firm capital since the marginal return is relatively high. Therefore, we observe that a one-time decrease in real interest rates has long-lasting impact on entrepreneurs. Lastly, for the aggregate consumption for all entrepreneurs, it increases upon impact since there is still a sizable fraction of entrepreneurs are relatively unconstrained.

In the right panel of Figure 6, we assume w_t decreases about 1% relative to its steady state for one period. Intuitively, when labor cost is lower, labor demand is increased, and entrepreneurs' business income increases in the first period. The responses of labor demand is mainly driven by how important labor in the production function and how elastic optimal labor demand responds to change in wages (e.g., labor share α and constant elasticity of substitution across capital and labor, ε in the production function). As before, we see that even though this is one-time shock in wages and there are no changes in interest rates or productivity, firm capital changes is still persistent since constrained entrepreneurs have relatively high MPI.

7.2 Aggregate responses

Figure 7: Responses to Expansionary monetary policy shocks



Turning to aggregate equilibrium responses for the one-time unexpected monetary shock, we plot several key aggregate variables' dynamics in Figure 7. Intuitively, we can understand these dynamics with several steps of thoughts. When there is an unexpected expansionary monetary shock in the beginning of period 1, consider first holding current inflation and current output constant (hence at their steady state values). This expansionary shock will immediately impact the risk-free nominal interest rates $\hat{i}^{a}_{t+1}$, as we can see in Equation 8 for monetary policy rule. The decreases in nominal interest rates in turn will induce the real interest rates r_{t+1} to be lower than its steady state value. Therefore, workers would like to raise their current consumption relative to their future consumption due to the inter-temporal substitution effects. For entrepreneurs, decreases in real interest rates induce them to invest more today since the cost of investing is lower; the same happens to corporate firms as well. All these increases current aggregate demand for final goods and tend to push inflation at time 1 $\hat{\Pi}_t$ upward and real wage lower than its steady state. In turn, entrepreneurs and firms will increase their labor demand with lower labor costs and produce more.

These would be the first-round of impacts on the economy. Going to more general equilibrium analysis, it is more complicated. However, a few points are still worth noting: (1)"business income and MPI": with decreased real interest rates in r_t and decreased real wages, entrepreneurs' current business income will increase (e.g., $\left(a_j - k_j\right) dr_t - n_j dw_t$, for entrepreneur j with a_j, k_j, n_j). As we have seen in the steady state analysis, we know there are a significant fraction of entrepreneurs constrained at the time of shocks and have very high levels of MPI. Thus, for these entrepreneurs, the increase in investment on impact is substantial. For the total investment changes by entrepreneurs, denoted as dI^E, we can see it more clearly in the following equation:

$$
\begin{aligned}
dI^E &\approx \int MPI_j \times d\Omega_j \\
&= \int MPI_j \times \left[\left(a_j - k_j\right) dr_t - n_j dw_t\right],
\end{aligned} \tag{12}
$$

where we sum up all the first-order changes in investment across all entrepreneurs. (2) "Factor income redistribution": for those entrepreneurs with debts (i.e., $k_j > a_j$), the unexpected shock will reduce their interest payments, and the workers (savers) in general would lose due to this real interest rates changes. This effect could be large for some entrepreneurs with high productivity and also high level of debts.

For the periods after the initial shock, first, since the monetary policy shock is relatively short-lived (e.g., the persistence parameter of ρ_i usually is just around 0.6 for a quarter) and the price of final goods is assumed to be flexible, we see that inflation rates are quite flexible to adjust: it increases on impact and quickly phases out. However, nominal wages are sticky in the economy; In turn, the equilibrium path for real wages are also quite smooth, with a little bit of overshooting around quarter 10.

Overall, for the magnitudes, inflation jumps on impact for about 0.5% and real wages decreases by a little less than 0.5% on impact. Entrepreneurs' business income jumps up with a large magnitude, close to about 4%[16]. Entrepreneurs' labor demand increases by

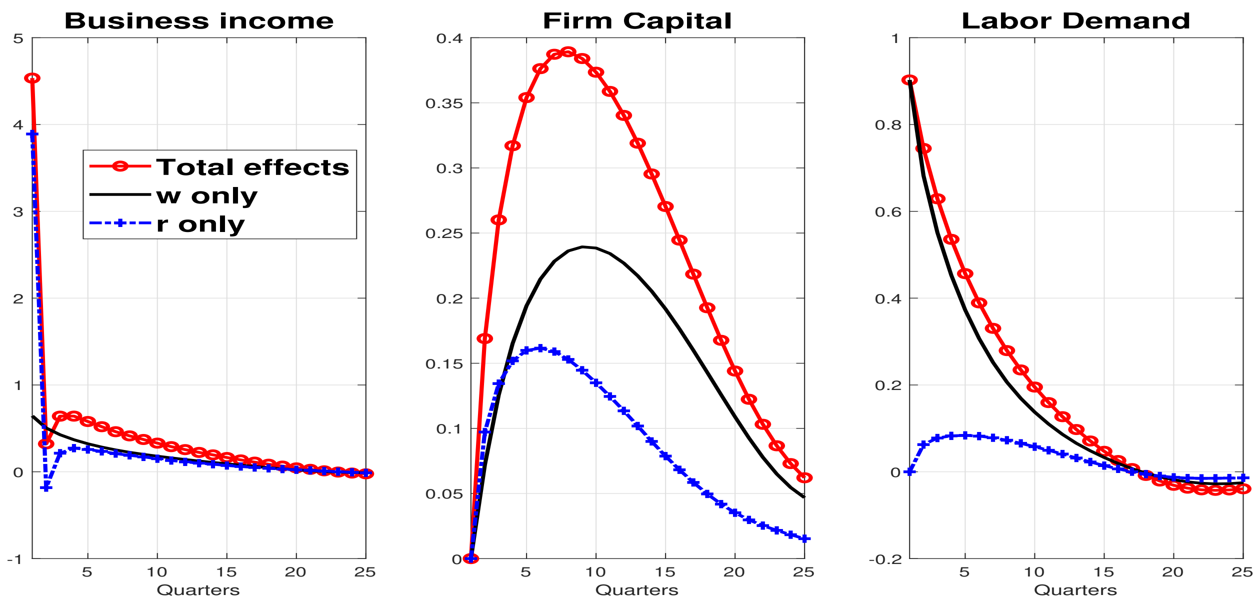
[16] as before, the scale used for business income is different from other variables

about 1% and capital stock moves slowly over time, reaching about 0.4% at its peak level. Also note that the responses of business income are much more volatile than consumption, supporting our empirical findings.

7.3 Decomposition of Aggregate responses

To understand more for entrepreneurs' responses in the general equilibrium, we further decompose their responses due to changes in real interest rates and changes in real wages. The results are reported in Figure 8.

Figure 8: Decomposition



For the partial equilibrium effects due to the changes in real interest rates, first note that in the first period the real interest rate decreases with a relatively large magnitude, and after that, the changes in real interest rates are relatively small and quite smooth. In turn, the responses of business income are following the opposite pattern of interest rates. In particular, the partial equilibrium effect on business income is large, with more than 4% deviation from its steady state in the first period. Even though it looks like a one-time windfall, those constrained entrepreneurs will save most of the increased business income and invest into the next periods. Therefore, aggregate firm capital for all entrepreneurs increases smoothly and reaches its peak about 5 quarters later, and the responses for labor demand follow closely firm capital's pattern but with much smaller magnitudes. Overall, the effects from real interest rates' changes on capital account for roughly a half of the total effects.

For change in wages alone, the responses of business income are quite smooth and decrease over time gradually. Intuitively, when real wages decrease on impact and recovers slowly over time –largely due to the wage stickiness, entrepreneurs will take this opportunity, increase labor demand and firm capital investment, and their business income also increase. Overall, the responses in labor demand accounts for almost all of the total general-equilibrium effects, and the effects on capital due to real wage changes accounts for slightly more than 50%.

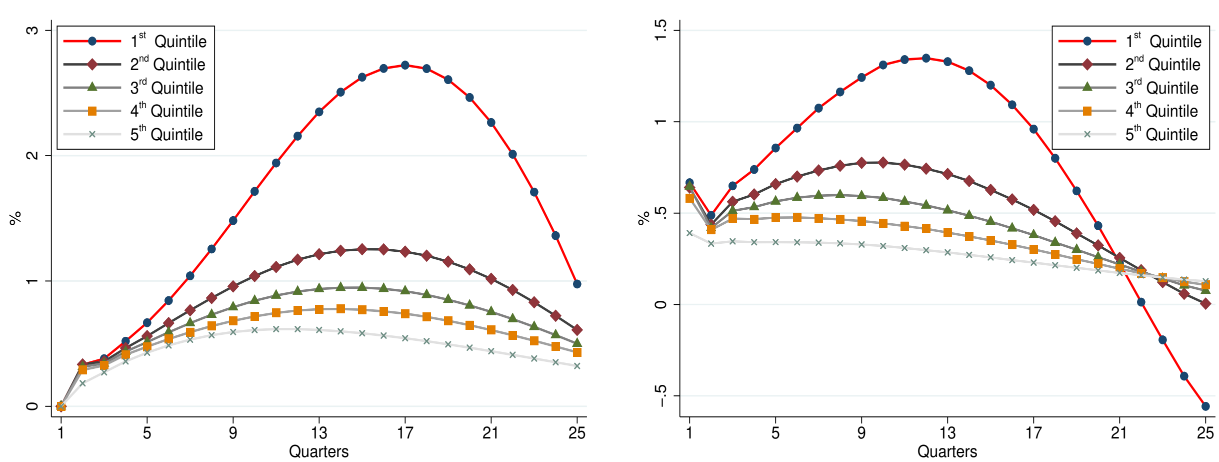
7.4 Distributional impact on Entrepreneurs

To further understand how different entrepreneurs react to the monetary shocks in the cross section, and how entrepreneurs' heterogeneity impact their responses, we further investigate the distributional impact of monetary shocks on different entrepreneurs. To do so, we divide entrepreneurs into different groups by their characteristics in the steady state. We first use entrepreneurs' initial net worth to group them into 5 quintile and then track their responses in firm capital and consumption over time by groups. The results are in Figure 9. For the sake of space, we only report the dynamics for firm capital and consumption here.

Figure 9: Cross-Sectional Responses: by Initial net worth

(a) Firm capital

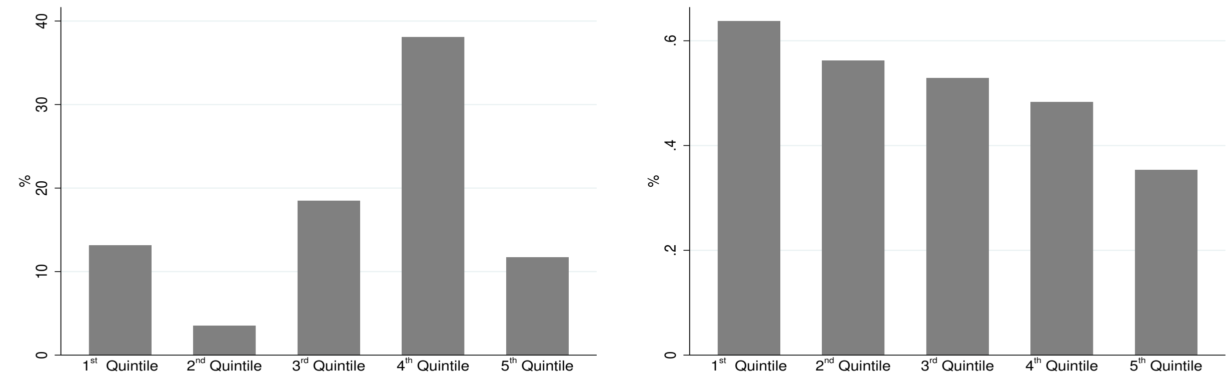
(b) Consumption



In Figure 9, we first compute the growth rates for firm capital (in the left panel) and

Figure 10: Initial impacts: by Initial net worth

(a) Business income (b) Consumption



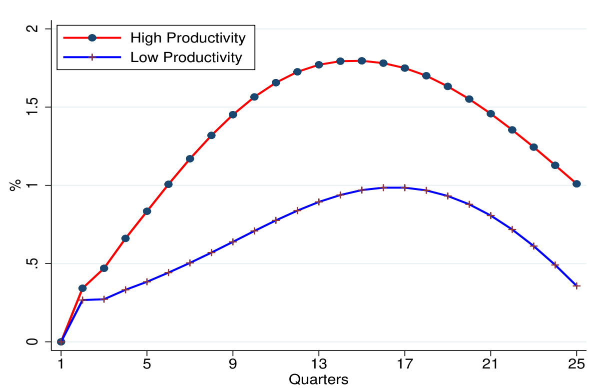
for consumption (in right panel) relative to their steady state values for each entrepreneur, and then report the average growth rates over time within each quintile. A few points stand out: (1) both the responses in firm capital and consumption are stronger with lower initial net financial assets. Intuitively, since entrepreneurs face collateral constraint in the credit market, lower net worth will limit the size of the firm and hence the marginal return to investment is relatively high. As net worth increases, on average the entrepreneurs have larger firm capital and also higher levels of consumption in the steady state; their responses to changes in factor prices are much more dampened. For consumption responses, the pattern is similar to that of firm capital. Note that even though the average growth rates for firm capital and consumption almost monotonically decrease as initial net worth increase, this is not necessarily the case for business income, since net worth is just one dimension of the heterogeneity across entrepreneurs and business income could be affected by productivity, financial assets, firm capital and labor hired. As shown in Figure 10 where we plot the initial impacts, the average growth rates in the first 4 quarters for different quintile. Business income (left panel) varies across different quintile and the 4^{th} quintile entrepreneurs have the largest growth. The pattern for consumption is not the case (see the right panel). These suggests the importance of having entrepreneurs' heterogeneity in the cross section.

We also divide entrepreneurs into two groups according to their firm productivity in the initial steady state, and track the average growth rates for firm capital and consumption over time in Figure 11. Intuitively, entrepreneurs with high productivity (higher than the median level) on average have higher marginal return to investment, else equal; however, entrepreneurs may also differ in net worth and in firm size initially, and those dimensions will also affect investment. Also note that even if some entrepreneurs have high productivity in the steady state, since productivity is stochastic and mean-reverting over time, those entrepreneurs will likely have same average productivity if time is long enough. Quantitatively, high-productivity group has larger responses in firm capital, almost doubled comparing to the other group; However, for consumption response, since it reflects more about life-time wealth changes, interestingly we see that the two groups have more or less similar magnitudes of responses.

Lastly, we divide entrepreneurs based on whether being financially constrained or not in the initial steady state, and track the average growth rates in Figure 12. One advantage of doing this is that, even though entrepreneurs may have many different dimensions of heterogeneity, financial constraint is a simple but an important summary characteristics for entrepreneurs. Also note that in the model simulated data, we can directly observe an entrepreneur is constrained if the collateral constraint binds and her multiplier is strictly positive (see Equation 9). Constrained entrepreneurs may have relatively high productivity and/or relatively low net worth in combination. As a result, we see those constrained expand firm capital quickly and the effects are more persistent comparing to other entrepreneurs. Constrained entrepreneurs also increases their consumption on average, but the initial changes in consumption are relatively small and there is a hump shape. This reflects those constrained entrepreneurs value highly for additional liquid wealth in the first few periods. This is in clear contrast to the other group of unconstrained entrepreneurs: the increase in investment is only mild, reaching a peak level roughly at one third of that for constrained entrepreneurs. Also, the consumption response for unconstrained entrepreneurs is smaller on impact, about half of the size for the constrained, and it gradually decays over time.

Figure 11: Cross-Sectional Responses: High Vs. Low productivity

(a) Firm capital



(b) Consumption

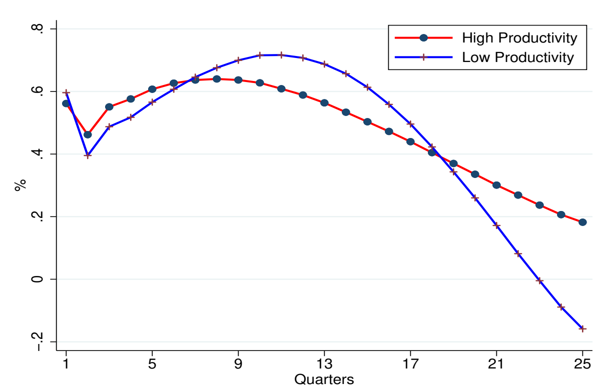
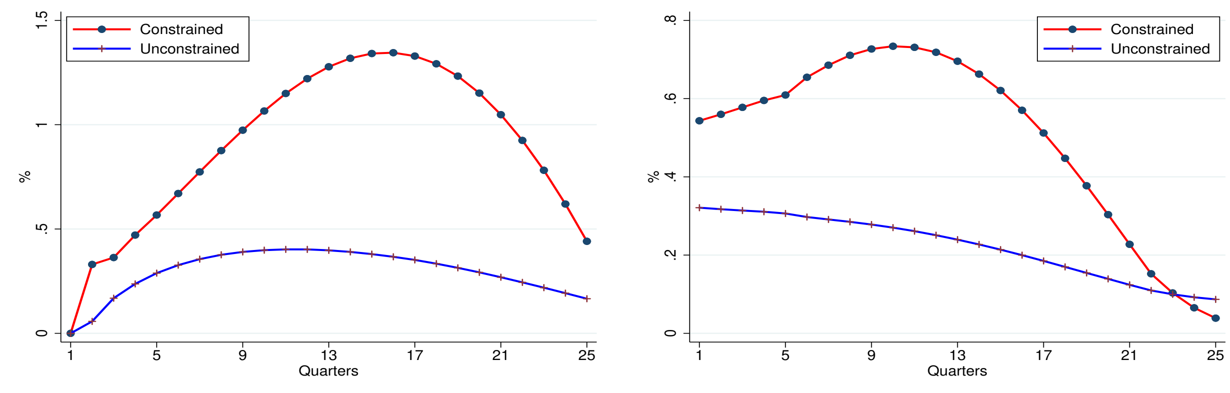


Figure 12: Cross-Sectional Responses: Constrained Vs. Unconstrained

(a) Firm capital

(b) Consumption



7.5 Initial Impact and Cumulative Impact

In our quantitative model, entrepreneurs' saving and investment activity today may have persistent effects on their household income and consumption. With expansionary monetary shocks, some constrained entrepreneurs may find their consumption and investment will be impacted persistently, their constraints are relaxed gradually over time since entrepreneurs can accumulate more net worth and overcome the collateral constraints, and in turn entrepreneurs will have more labor demand over time. This could be the case even for relatively short-lived shocks. Thus, in the face of expansionary monetary shocks, not only current aggregate demand will be impacted – which consists of workers' consumption demand and also entrepreneurs' investment and consumption demand, but also future aggregate demand will be affected. Comparing to much of the HANK literature (e.g., Kaplan et al. (2018) and Auclert et al. (2020)), this is a relatively new feature in our model. To best capture the persistent effects of monetary shocks, we compute the initial impact and cumulative impact on variable y for a given series of monetary shock as follows:

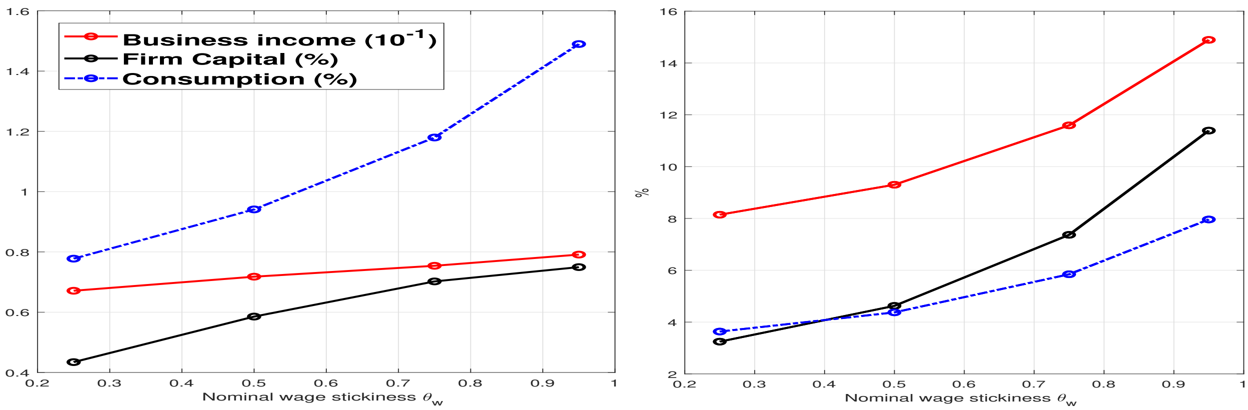
$$\sum_{t=1}^{t=T} \frac{\hat{y}_t}{(1+r_{ss})^{t-1}}$$

where $T = 4$ is for the initial impact and $T = +\infty$ is for the cumulative impact.

To illustrate the initial impact and cumulative impact, in Figure 13 we consider experiments with different degrees of nominal wage stickiness (θ_w), and for each value of θ_w we report the computed initial impact and cumulative impact. Our benchmark value for θ_w is 0.75. Later on we also consider other experiments: changes in the persistence of monetary

Figure 13: Initial Impact and Cumulative Impact: with different wage stickiness

(a) Initial Impact (b) Cumulative Impact



policy shocks, changes in workers' labor supply elasticity and so on.

In Figure 13, a few points stand out: (1) For a given value for θ_w, we find that the initial impacts on business income are large. For example, it increases more than 6.5% in the first 4 quarters, and increases for about 10.5% in the long run (for the benchmark value, also see Table 6). This is in contrast to the responses of firm capital and consumption, where they respond more slowly and more persistent. Business income's initial impact is larger in order of magnitude, but in the long run, consumption and firm capital have roughly similar magnitudes of responses. (2) Across different values of θ_w, more persistent wage stickiness gives larger initial impacts and also larger cumulative impacts. Note that the economy's steady state will not be affected by different values of wage stickiness. Intuitively, when there is nominal wage stickiness in the economy, the equilibrium path of real wages and real interest rates in the transitions will be affected. If real wages are lower than its steady state and are more persistent, entrepreneurs will take this opportunity, invest more in the firm capital, enjoy higher and more persistent business income and consumption (see more dynamics of these variables in Figure 29 in the Appendix).

7.6 Less Favorable Collateral condition

We have discussed that in the quantitative model a significant percentage of entrepreneurs are constrained in the credit market (recall subsection 6.2); in the face of an expansionary monetary shock, constrained entrepreneurs react much stronger than others (recall subsection 7.4). To have another view on the importance of collateral constraint on aggregate dynamics, we conduct a counterfactual exercise here. In particular, we change the degree of collateral constraint parameter Ψ, from benchmark value of 0.35 to 0.25 and keep all other model parameters the same as in the benchmark model. We re-compute the steady state and solve for the transitional dynamics with this less favorable collateral constraints.[17]

Intuitively, when entrepreneurs can borrow less for given values of firm capital in the credit market, they are more likely to be constrained for optimal investment. On the other hand, entrepreneurs can accumulate more net worth and try their best to overcome the credit market friction. Quantitatively we see the first effect dominates: in the steady state, entrepreneurs' total firm capital is reduced by about 27% from the corresponding benchmark value; the aggregate output is also lowered substantially, by about 34% from the benchmark case.

For the dynamics, we report the detailed results in Table 6, where we distinguish initial impacts and cumulated impacts as well. Comparing to the benchmark model, we find the new economy with less favorable borrowing condition (labeled as "Counterfactual") on average have both smaller initial and long-run responses in almost all of the variables in the table. For example, entrepreneurs' business Income is higher for about 6.5% relative to its steady state; for the benchmark model, the corresponding number is 7.5%. Firm capital's responses are somewhat similar across the two economies in the very short-run, and are significantly lower in the long-run since the collateral constraint in the new steady state is still tight. Intuitively, with tightened credit market condition, entrepreneurs are more likely to be constrained in expanding their firms in the face of monetary shocks. For aggregate variables, output increases about 1.43% relative to its steady state, and comparing to the benchmark economy, the increases is about 11% lower ((1.61%- 1.43%)/1.61%) in the short run; this difference is also present in the long run impacts. Similarly, aggregate consumption's increase is about 10% lower. Overall, we see that with less favorable borrowing condition, the economy's responses are weaker to the expansionary monetary shocks.

7.7 More complementary capital and labor inputs

We also consider an experiment with different ε, the elasticity of capital and labor in entrepreneurs' production. Our benchmark assumes a Cobb-Douglas production, $\varepsilon = 1.0$; here we assume capital and labor inputs are more complementary to each other and set $\varepsilon =$

[17] To ensure different models' similarity and comparability and also without loss of generality, our counterfactual exercise is around the neighborhood of the benchmark model, and we did not choose to completely shut down the credit markets.

Table 6: The impacts with less favorable Collateral borrowing

	Initial Impact		Cumulative Impact	
	Benchmark	Counterfactual	Benchmark	Counterfactual
Entrepreneurs' Business Income	7.54%	6.57%	11.59%	9.21%
Entrepreneurs' Firm Capital	0.70%	0.60%	7.37%	4.95%
Entrepreneurs' Labor demand	2.90%	2.58%	5.36%	2.66%
Entrepreneurs' Consumption	1.18%	1.15%	5.84%	3.49%
Aggregate Output	1.61%	1.43%	3.23%	3.01%
Aggregate Consumption	1.08%	0.99%	3.46%	2.69%

0.8. Intuitively, when capital and labor inputs are more complementary, and also capital is less flexible to adjust in the short run (due to the fact that entrepreneurs may be financially constrained by net worth and also the presence of adjustment costs), entrepreneurs have less flexibility to adjust their labor demand to take advantage of favorable changes in real interest rates and wages. In responding to monetary policy shocks, we report the results in Table 7 for initial impact and cumulative impact. Quantitatively, we see entrepreneurs have smaller responses in labor demand in the short run and in the long run as well. On impact, entrepreneurs' firm capital deviate from their corresponding steady state values only slightly and very similarly with different ε. Again, we see the responses in business income are substantially larger than those for aggregate output (and workers' labor income). Overall, with different complementarity, we find our results for entrepreneurs are still robust.

Table 7: More complementary capital and labor inputs

	Initial Impact		Cumulative Impact	
	Benchmark	Counterfactual	Benchmark	Counterfactual
Entrepreneurs' Business Income	7.54%	7.38%	11.59%	10.34%
Entrepreneurs' Firm Capital	0.70%	0.71%	7.37%	7.23%
Entrepreneurs' Labor demand	2.90%	2.67%	5.36%	4.59%
Entrepreneurs' Consumption	1.18%	1.07%	5.84%	4.45%
Aggregate Output	1.61%	1.51%	3.23%	3.03%
Aggregate Consumption	1.08%	1.02%	3.46%	3.05%

7.8 Alternative Monetary policy rules

We have assumed monetary policy following a rule as in Equation 8; alternatively, we also experiment with different monetary policy rules (also see the details in Section 4). In

Figure 27 in the appendix, we first consider a rule similar as in Kaplan et al. (2018),

$$\hat{i}^a_{t+1} = \varphi_\pi \hat{\Pi}_t + \eta_t,$$
$$\eta_t = \rho_i \eta_{t-1} - \sigma_i \epsilon_t,$$

where the monetary policy shocks are assumed to be an AR(1) process, and nominal interest rates $\hat{i}^a_{t+1}$ from the end of t to time $t+1$ responds to time t inflations and time t shocks. By and large, we find very similar responses for entrepreneurs' key variables, as well as for other aggregate variables.

In addition, we also consider another rule, similar to the one in the benchmark mode, as follows:

$$\hat{i}^a_{t+1} = \rho_i \hat{i}^a_t + (1-\rho_i)\varphi_\pi \mathbb{E}_t \hat{\Pi}_{t+1} - \sigma_i \epsilon_t.$$

This version of Taylor rule follows closely Christiano et al. (2016) in modeling interest rate persistence to make sure that the sign of nominal interest rate response is more stable. It has nominal interest rate adjustments depending on inflation expectations instead of current inflation to avoid impact from the initial jump in inflation. The results are reported in Figure 28 in the appendix. We confirm that the pattern for our quantitative results is robust to this alternative rule. Also note that due to the specific nature of this rule, inflation only jumps in the first period and then quickly converges to 0; real interest rates jumps downward in the first period and are quite smooth afterward. As a result, entrepreneurs' responses in business income and consumption are smoother, so do aggregate output and consumption.

7.9 Other robustness checks

We also consider other experiments to see how our quantitative results are affected by some important underlying parameters. In particular, we looked at the equilibrium dynamics with different degrees of nominal wage stickiness in Figure 29 in the appendix, with different workers' labor supply elasticity in Figure 30, and in Figure 31 with different persistence of nominal interest rates in the monetary policy rule.[18] We find our basic quantitative pattern is robust to these different experiments, although the magnitudes of responses depend on different underlying parameters. For example, with higher nominal

[18] For the sake of space, we only report a few key variables for entrepreneurs.

wage stickiness, we find the responses in entrepreneurs' key variables are larger in the short run; this is also the case for more persistent nominal interest rates. For different extents of workers' labor supply elasticity, however, we find the changes relative to the benchmark case are not so large; e.g., with Frisch labor supply elasticity at 0.5 or at 4.0, the differences are not very large.

8 Concluding remarks

In this paper, we first present new empirical facts from micro-level data, and we show that entrepreneurs' income and consumption fluctuate strongly over the business cycles and respond strongly to interest rate shocks. This suggest even though entrepreneurs are wealthy, it seems they cannot smooth out consumptions very well. This is in contrast to the conventional wisdom in much of the business cycle studies with entrepreneurs. We explore further and provide analytical results from a simple framework and quantitative analysis through a new Heterogeneous Agent New Keynesian model (HANK) model. We highlight that, when heterogenous entrepreneurs face idiosyncratic investment shocks and credit market frictions, a large fraction of them are actually being constrained and in turn consumption smoothing is a second-order issue relative to the investing motive. This is somewhat in a similar spirit as in Kaplan and Violante (2014) and Kaplan et al. (2018), where in their models households could invest in illiquid assets but with higher rate of return; as a result, a substantial fraction of households are "wealthy hand-to-mouth". Our quantitative exercises show that entrepreneurs' endogenous heterogeneity in productivity and wealth, is important for the transmission of monetary policy shocks and the macroeconomic implications. For future works, it would be interesting to explore further the macroeconomic implications of entrepreneurs' heterogeneity, e.g., how the unemployment dynamics in a search and matching framework would be affected, how should government authorities take this into account in designing optimal monetary policy and credit policy, and so on.

References

Adelino, M., Schoar, A., and Severino, F. (2015). House prices, collateral, and self-employment. *Journal of Financial Economics*, 117(2):288–306.

Angeletos, G.-M. (2007). Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic dynamics*, 10(1):1–30.

Angeletos, G.-M. and Calvet, L.-E. (2006). Idiosyncratic production risk, growth and the business cycle. *Journal of Monetary Economics*, 53(6):1095–1115.

Arellano, C., Bai, Y., and Kehoe, P. Financial frictions and fluctuations in volatility.

Atkeson, A. and Kehoe, P. J. (2005). Modeling and measuring organization capital. *Journal of Political Economy*, 113(5):1026–1053.

Auclert, A. (2019). Monetary policy and the redistribution channel. *American Economic Review*, 109:2333–2367.

Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2019). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. Technical report, National Bureau of Economic Research.

Auclert, A., Rognlie, M., and Straub, L. (2018). The intertemporal keynesian cross. Technical report, National Bureau of Economic Research.

Auclert, A., Rognlie, M., and Straub, L. (2020). Micro jumps, macro humps: monetary policy and business cycles in an estimated hank model. Technical report, National Bureau of Economic Research.

Bassetto, M., Cagetti, M., and De Nardi, M. (2015). Credit crunches and credit allocation in a model of entrepreneurship. *Review of Economic Dynamics*, 18(1):53–76.

Basu, S. and Fernald, J. G. (1997). Returns to scale in us production: Estimates and implications. *Journal of political economy*, 105(2):249–283.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393.

Broer, T., Harbo Hansen, N.-J., Krusell, P., and Öberg, E. (2020). The new keynesian transmission mechanism: A heterogeneous-agent perspective. *The Review of Economic Studies*, 87(1):77–101.

Buera, F. J. and Moll, B. (2015). Aggregate implications of a credit crunch: The importance of heterogeneity. *American Economic Journal: Macroeconomics*, 7(3):1–42.

Burstein, A. and Hellwig, C. (2008). Welfare costs of inflation in a menu cost model. *The American Economic Review*, 98(2):438–443.

Cagetti, M., De Nardi, M., et al. (2006). Entrepreneurship, frictions, and wealth. *Journal of political Economy*, 114(5):835–870.

Chang, Y. and Kim, S.-B. (2007). Heterogeneity and aggregation: Implications for labor-market fluctuations. *American Economic Review*, 97(5):1939–1956.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2016). Unemployment and business cycles. *Econometrica*, 84(4):1523–1569.

Clementi, G. L. and Palazzo, B. (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics*, 8(3):1–41.

Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.

Cooper, M., McClelland, J., Pearce, J., Prisinzano, R., Sullivan, J., Yagan, D., Zidar, O., and Zwick, E. (2016). Business in the united states: Who owns it, and how much tax do they pay? *Tax Policy and the Economy*, 30(1):91–128.

Cooper, R. W. and Haltiwanger, J. C. (2006). On the nature of capital adjustment costs. *The Review of Economic Studies*, 73(3):611–633.

DeBacker, J., Heim, B., Panousi, V., Ramnath, S., and Vidangos, I. (2012). The properties of income risk in privately held businesses.

Dyrda, S. and Pugsley, B. (2019). Macroeconomic perspective on the rise of pass-through businesses. Technical report, Society for Economic Dynamics.

Heathcote, J. and Perri, F. (2018). Wealth and volatility. *The Review of Economic Studies*, 85(4):2173–2213.

Heathcote, J., Perri, F., and Violante, G. L. (2010). Unequal we stand: An empirical analysis of economic inequality in the united states, 1967–2006. *Review of Economic dynamics*, 13(1):15–51.

Hurst, E. and Lusardi, A. (2004). Liquidity constraints, household wealth, and entrepreneurship. *Journal of political Economy*, 112(2):319–347.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American economic review*, 95(1):161–182.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, 108:697–743.

Kaplan, G. and Violante, G. L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239.

Kekre, R. and Lenel, M. (2020). Monetary policy, redistribution, and risk premia. *University of Chicago, Becker Friedman Institute for Economics Working Paper*, (2020-02).

Khan, A. and Thomas, J. K. (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, 104(10):3186–3221.

Nakamura, E. and Steinsson, J. (2018). High-frequency identification of monetary non-neutrality: the information effect. *The Quarterly Journal of Economics*, 133(3):1283–1330.

Ottonello, P. and Winberry, T. (2018). Financial heterogeneity and the investment channel of monetary policy. Technical report, National Bureau of Economic Research.

Parker, J. A. and Vissing-Jorgensen, A. (2009). Who bears aggregate fluctuations and how? *American Economic Review*, 99(2):399–405.

Piketty, T., Saez, E., and Zucman, G. (2018). Distributional national accounts: methods and estimates for the united states. *The Quarterly Journal of Economics*, 133(2):553–609.

Quadrini, V. (2000). Entrepreneurship, saving, and social mobility. *Review of Economic Dynamics*, 3(1):1–40.

Romer, C. D. and Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *American Economic Review*, 94(4):1055–1084.

Schmalz, M. C., Sraer, D. A., and Thesmar, D. (2017). Housing collateral and entrepreneurship. *The Journal of Finance*, 72(1):99–132.

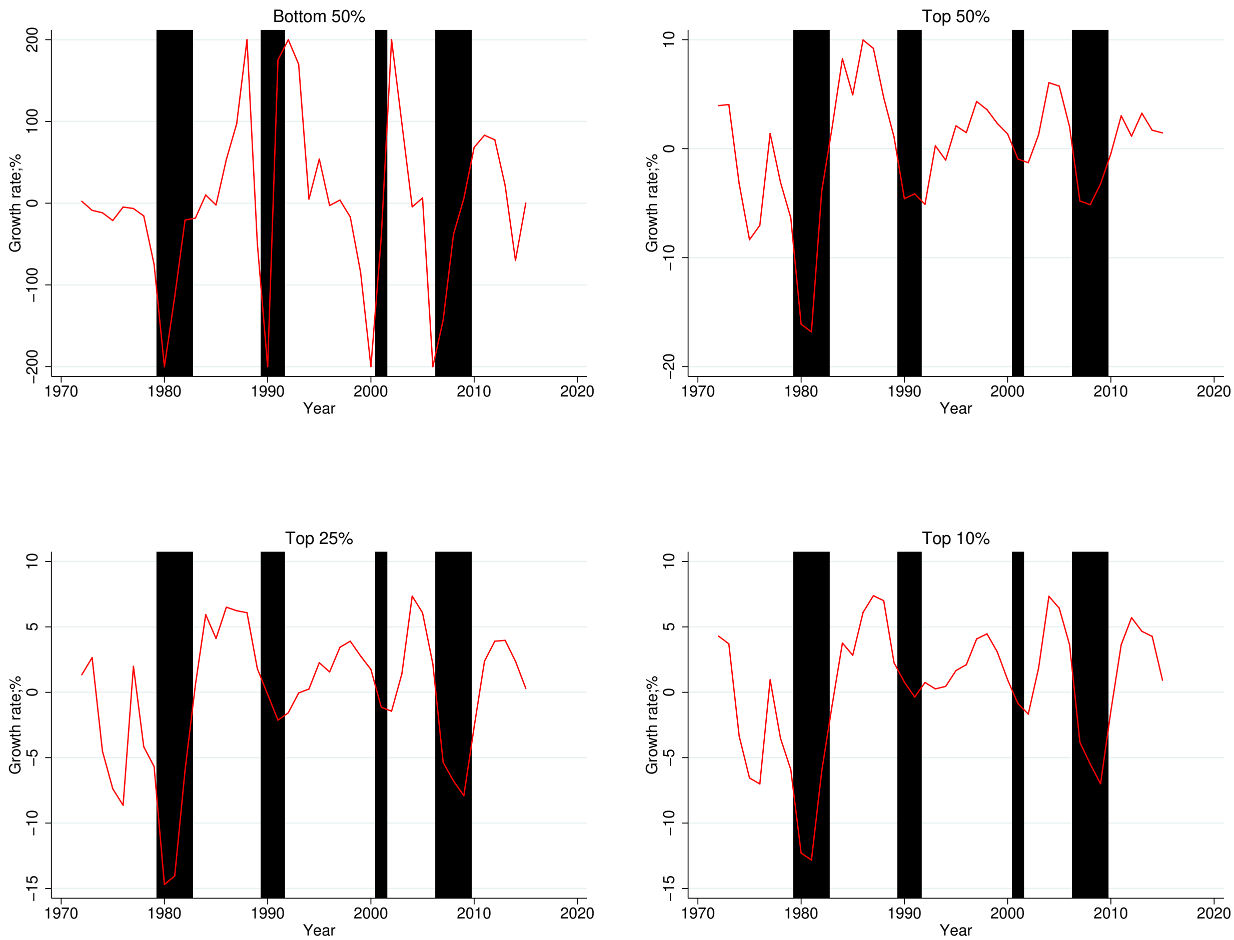
Zetlin-Jones, A. and Shourideh, A. (2017). External financing and the role of financial frictions over the business cycle: Measurement and theory. *Journal of Monetary Economics*, 92:1–15.

Appendices

A Appendix for Empirical Analysis

A.1 Business income vs. wages

Figure 14: Business income by entrepreneurs' wealth percentiles; IRS data



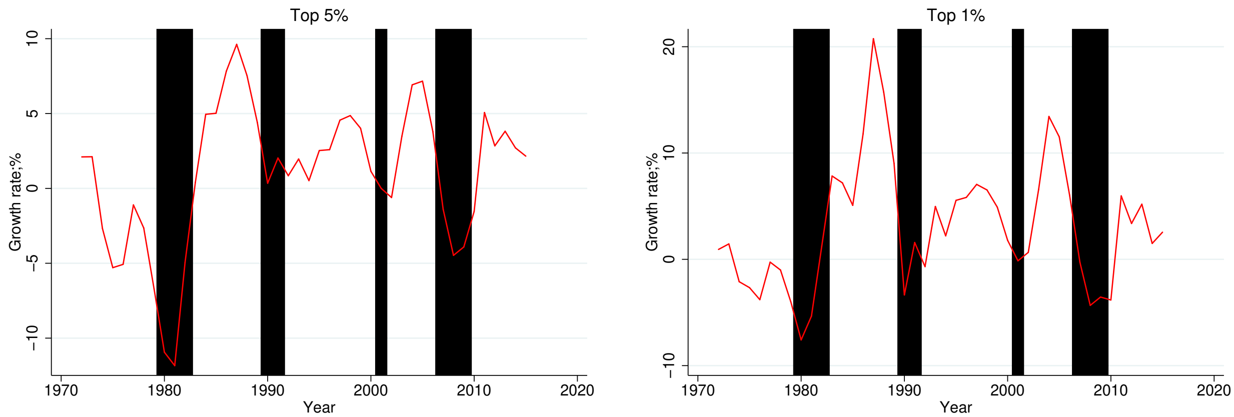
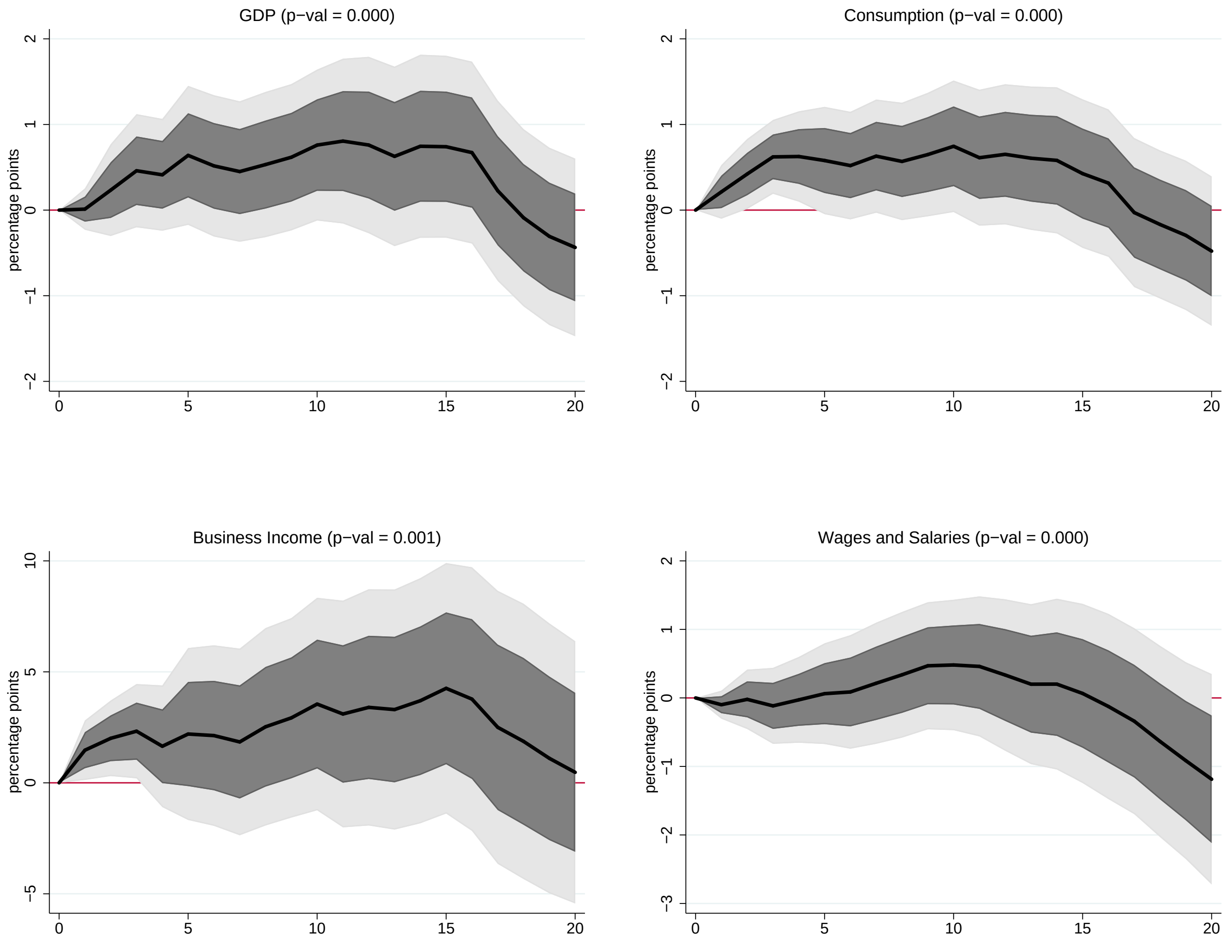


Table 8: Summary Statistics for Aggregate variables

	Obs.	Mean	S.D.	Percentiles						
				5^{th}	10^{th}	25^{th}	50^{th}	75^{th}	90^{th}	95^{th}
GDP growth	160	0.718	0.851	-0.845	-0.380	0.322	0.765	1.149	1.736	2.004
Consumption growth	160	0.793	0.682	-0.318	-0.001	0.442	0.844	1.137	1.671	1.835
Business income growth	160	0.596	4.079	-6.412	-3.353	-0.686	1.265	2.701	4.093	5.160
Wage income growth	160	0.643	0.822	-0.635	-0.454	0.052	0.783	1.217	1.580	1.901
Federal funds rates	160	6.493	3.393	1.438	2.302	4.612	5.698	8.398	10.558	13.582
Romer Romer Monetary shocks	159	-0.001	0.590	-0.805	-0.540	-0.226	0.000	0.241	0.537	0.775
Nakamura Monetary shocks	56	-0.195	0.590	-1.761	-1.080	-0.312	-0.027	0.071	0.282	0.473

NOTE: All aggregate variables are in terms of per capita and are deflated with CPI. For growth variables, the units are percent; for interest rates and shocks, the units are percentage points.

Figure 17: Aggregate responses to Romer and Romer (2004) Expansionary Monetary shocks



NOTE: Responses of Aggregate variables to Expansionary Monetary shocks of 100 basis points. The regression equation is as follows:

$$\begin{aligned}\log(\text{Outcome}_{t+h}) - \log(\text{Outcome}_{t+h-1}) &= \\ &\sum_{j=1}^{J} \alpha_j^h \times [\log(\text{Outcome}_{t-j}) - \log(\text{Outcome}_{t-j-1})] \\ + \sum_{i=1}^{I} \beta_i^h \times \text{MP}_{t-i} + \mu_h + \epsilon_{t+h}, h &= 0, 1, ..., H,\end{aligned}$$

The p-value is for the null hypothesis that the whole path of impulse responses is zero. The confidence interval in the graph are for 1 and 1.65 standard deviations, respectively. The figure reports the accumulated impulse responses to Monetary shocks based on the estimated values: $\widehat{\beta}_1^h(h = 0, 1, ..., H)$.

Figure 19: Robustness: Aggregate responses to Romer and Romer (2004) Expansionary Monetary shocks with shorter horizons

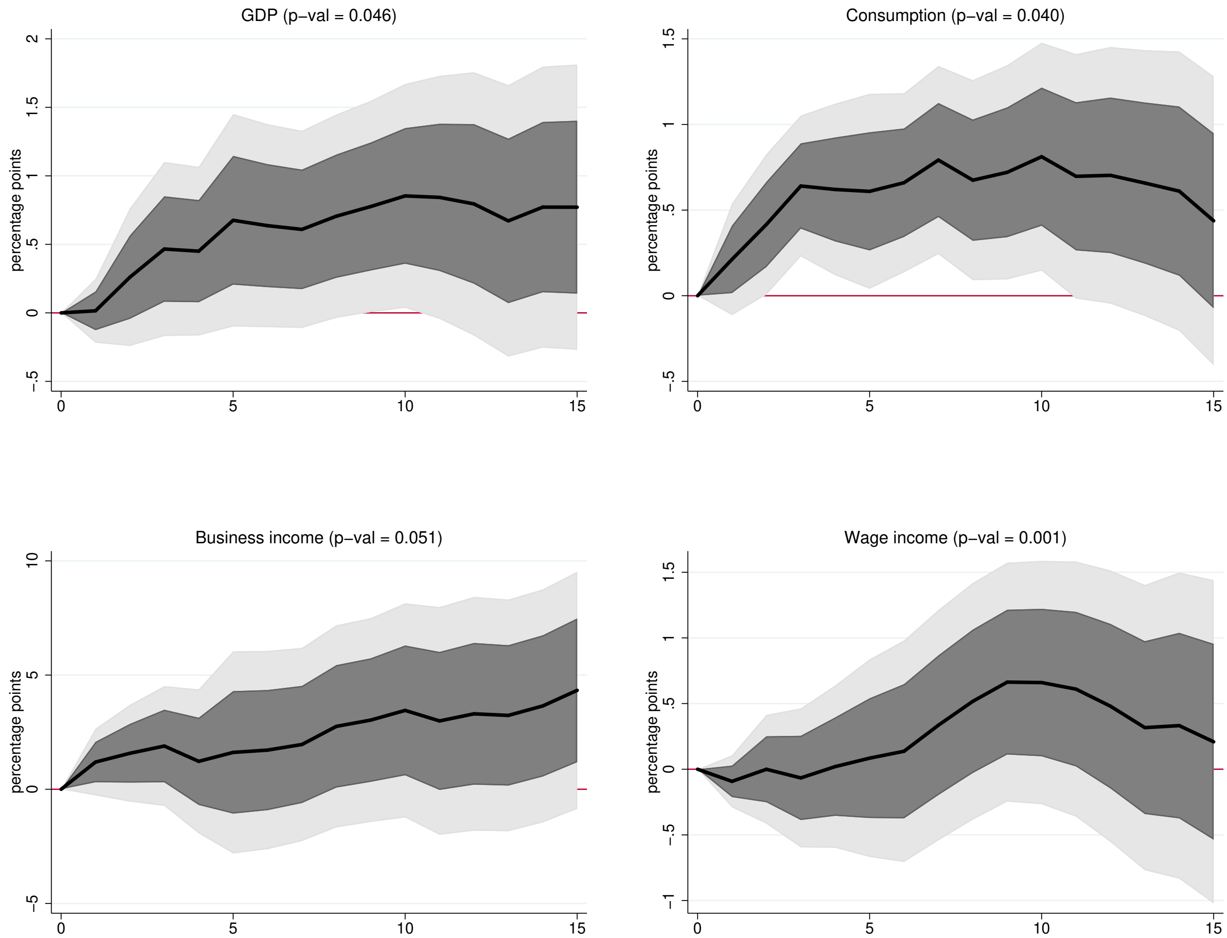


Figure 21: Robustness: Aggregate responses to Romer and Romer (2004) Expansionary Monetary shocks with longer horizons

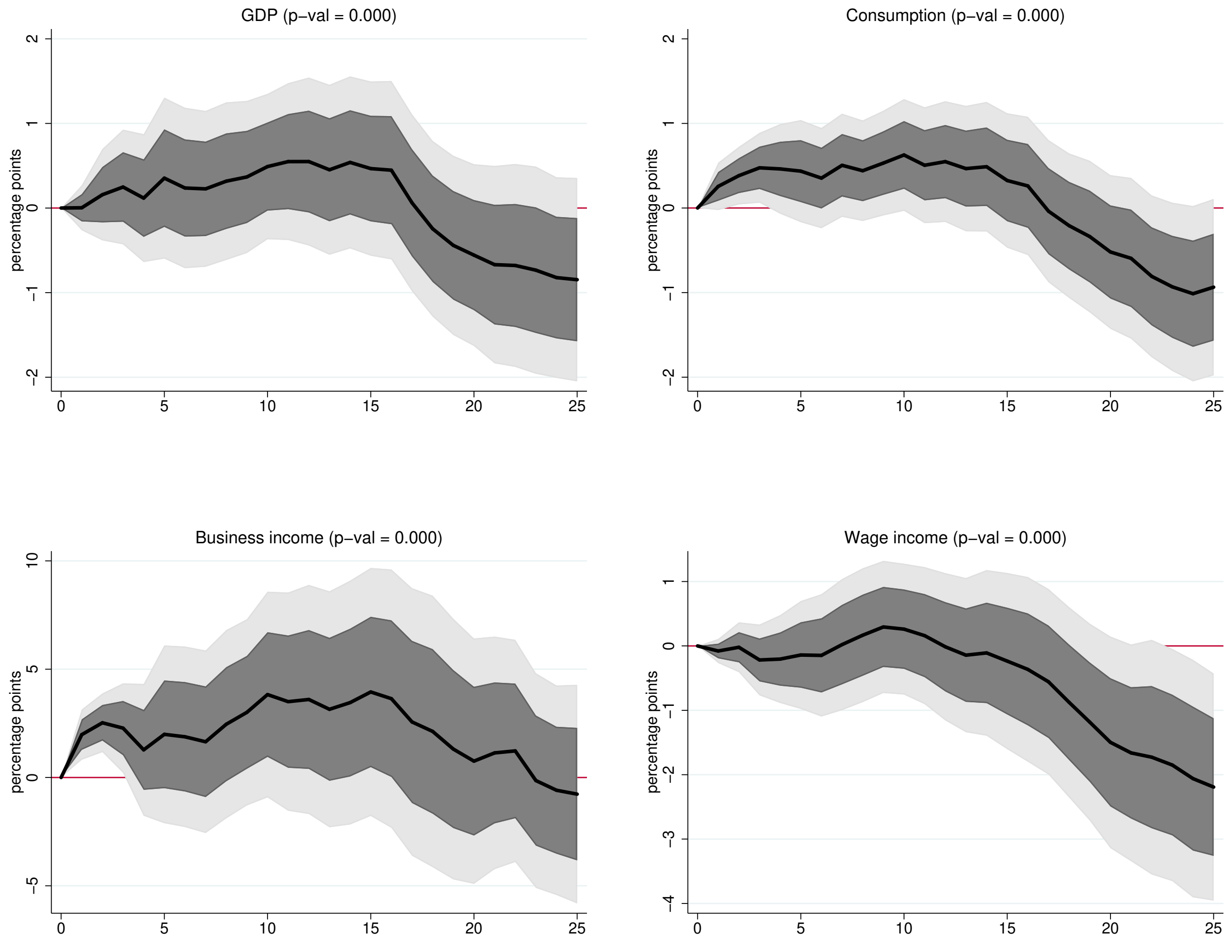
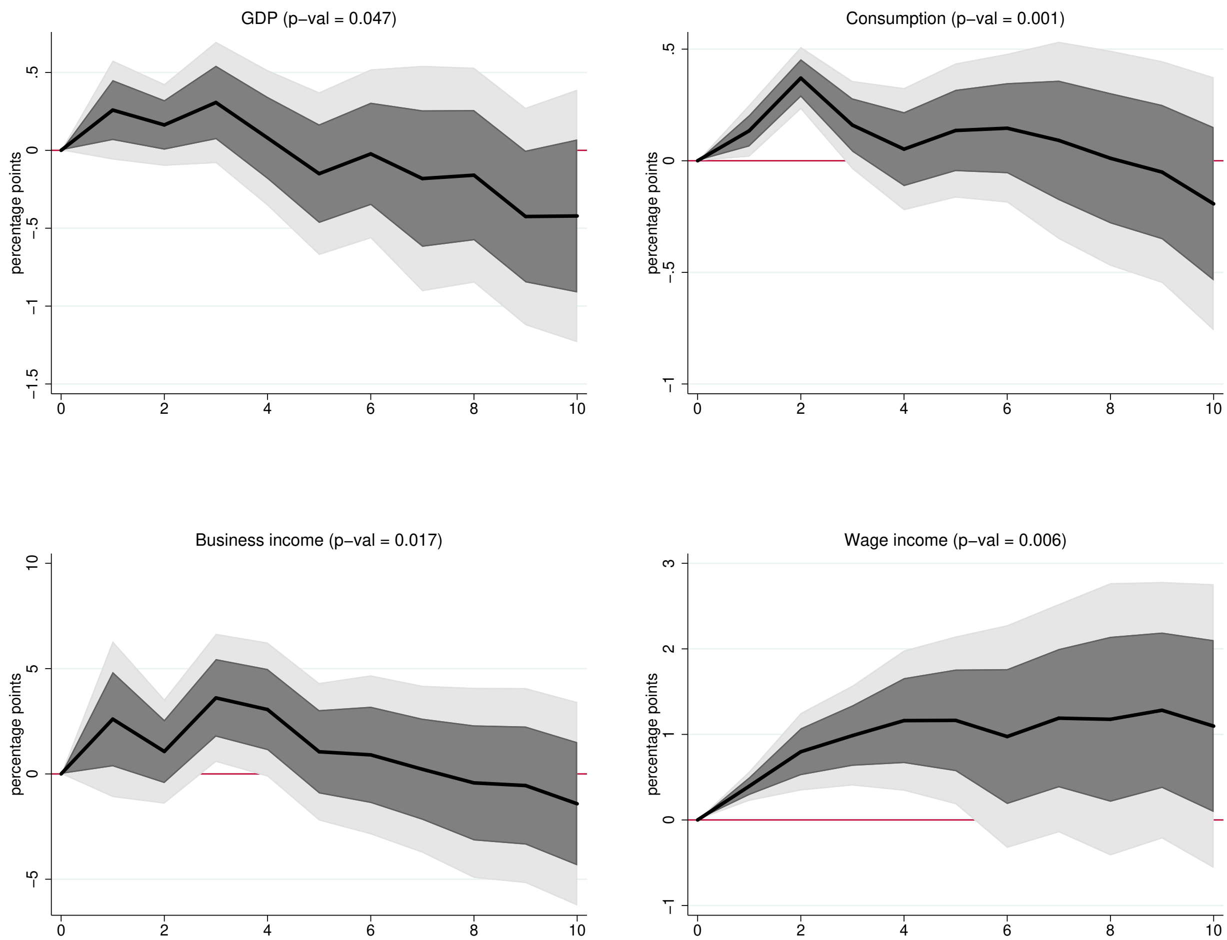


Figure 23: Aggregate responses to Nakamura and Steinsson (2018) Expansionary Monetary shocks



A.2 Consumption for entrepreneurs vs. workers

CEX data

Our CEX data is for the years 1980 to 2006, using the raw sample provided in Heathcote et al. (2010). In the data, a household is interviewed at most for 4 consecutive quarters and we have roughly 15000 households before 1999 and 20000 after that. We define entrepreneurs in the short panel of CEX as follows: those with any positive business income in any period within the panel of observations; no wage income but with positive labor income; or positive total working hours but 0 salaries (using INCOMEY in the individual MEMI file, and/or INCOMEY1 INCOMEY2 in the Fmli file). For other variables, we follow much of the analysis as in Heathcote et al. (2010). The definition of nondurable consumption expenditures includes the following categories: food and beverages (including food away from home and alcoholic beverages), tobacco, apparel and services, personal care, gasoline, public transportation, household operation and housing services, medical care, entertainment, reading material and education. Each observation is constructed by adding up household nominal expenditures in these categories during the three months period preceding the interview and then deflating the total using the CPI-Urban for that period. We also use other different measures for consumption for our robustness analysis: total expenditure (defined as FOODPQ + ALCBEVPQ + HOUSPQ + APPARPQ + TRANSPQ + HEALTHPQ + ENTERTPQ + PERSCAPQ + READPQ + EDUCAPQ + TOBACCPQ + MISCPQ + CASHCOPQ + PERINSPQ in the CEX data), and we use CPI-U to deflate the total expenditure; we also use nondurable consumption + imputed housing rent, deflated by CPI-U.

Table 9: Consumption growth for entrepreneurs is more cyclical

$g_{c_{i,t}}$	(1) Bench	(2) No control	(3) Housing rent	(4) Total Exp.	(5) No family change	(6) Exclude 5%	(7) Exclude 10%	(8) Top inc.	(9) Bottom inc.	(10) Top cons.	(11) Bottom cons.
Entrepreneurs											
GDP growth $g_{Y,t}$	1.017	1.235**	0.946	1.830**	1.018	1.036	0.882	-0.0245	1.323*	3.066**	0.355
	(0.664)	(0.533)	(0.743)	(0.809)	(0.679)	(0.665)	(0.669)	(1.429)	(0.784)	(1.397)	(0.900)
Obs.	21,802	28,766	21,802	21,825	21,592	21,223	20,073	4,354	17,448	4,148	17,654
R-squared	0.021	0.000	0.015	0.017	0.019	0.021	0.018	0.034	0.022	0.044	0.021
Worker											
GDP growth $g_{Y,t}$	0.328	0.649***	0.495*	1.396***	0.358	0.285	0.255	-0.637	0.508**	-0.442	0.528**
	(0.219)	(0.190)	(0.266)	(0.295)	(0.221)	(0.220)	(0.213)	(0.592)	(0.250)	(0.630)	(0.259)
Obs.	194,806	244,423	194,805	195,303	193,118	189,596	180,467	17,609	177,197	17,345	177,461
R-squared	0.014	0.000	0.007	0.009	0.013	0.014	0.013	0.029	0.013	0.030	0.014

NOTE: Robust standard errors are in parentheses, and notations for p-values are: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. "Control Variables" include: dummy variables for interview month and year, rural area, region, change of family sizes, reference person's sex, education and age groups.

Table 10: Consumption response to monetary shocks: Entrepreneurs vs. Workers

	(1) Nondurable	(2) Nondurable	(3) Nondurable+housing	(4) Total Exp.	(5) Exclude 5% Cons. growth	(6) Cons. weight	(7) Excluding Recession
Entr. vs. Worker	3.236***	2.019**	2.632***	3.762***	2.762**	3.179**	3.140**
	(0.832)	(0.887)	(0.978)	(1.106)	(1.099)	(1.376)	(1.412)
p-value	0.000330	0.0277	0.00996	0.00141	0.0156	0.0255	0.0314
With All controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Observations	30,636	24,994	24,994	25,039	20,933	20,933	19,920
R-squared	0.001	0.016	0.007	0.008	0.015	0.017	0.017

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

NOTE: The regression equation used is as follows:

$$\Delta \log(c_{i,t}) = \sum_{k=-1,-2,-3,-4} \beta_k \times \epsilon_{t+k} \times I_{Entr.} + I_{Entr.} + \delta_0 X_{i,t} \times I_{Entr.} + \epsilon_{i,t}$$

The table reports $\sum_{k=-1,-2,-3,-4} \beta_k \times I_{Entr.}$ for the cumulated responses (or the semi-elasticity) of entrepreneurs relative to workers. $I_{Entr.}$ is the dummy variable for being entrepreneurs in the CEX data. "With All controls" includes: demographic variables (household head education, sex, age groups, race, having kids in the family or not, change of total family numbers, being renters or not) and their interaction with dummy $I_{Entr.}$, other controls (recession dummies, quarter dummies, month dummies, region dummies, rural area dummies) and their interaction with dummy $I_{Entr.}$ The standard errors are clustered by time.

A.3 Empirical facts for entrepreneurs: SCF data

Table 11: Shares of wealth (and income) for entrepreneurs

	All	Top 50%	Top 20%	Top 10%
Net worth				
All HHs:	100.0 %	99.0 %	91.7 %	85.8 %
Entrepreneurs:	43.9 %	43.8 %	43.0 %	42.1 %
Total HH assets				
All HHs:	100.0 %	97.7 %	87.7 %	81.2 %
Entrepreneurs:	41.7 %	41.5 %	40.3 %	39.2 %
Total HH income				
All HHs:	100.0 %	87.5 %	67.1 %	58.1 %
Entrepreneurs:	27.9 %	27.4 %	24.9 %	23.4 %

NOTE: This table shows the shares for total HH net worth, Total HH assets and Total HH income from SCF data 1989-2016. For each survey year, we define the cross-sectional percentiles for real,total HH net worth (at the Top 50%, Top 20% and Top 10%) for all households. For a given variable, shares are computed relative to the total of the whole population. Sample weights are used. "Entrepreneurs" are defined as those households owning and actively managing their business.

Table 12: Business net worth and income within entrepreneurs

	Mean	Median	STD	P25	P50	P75	P90
All Entrepreneurs							
Business wealth/Total HH wealth:	0.37	0.27	1.03	0.00	0.05	0.59	0.83
Business income/Business wealth:	0.61	0.06	1.93	0.00	0.00	0.40	1.37
Net worth Top 10%							
Business wealth/Total HH wealth:	0.36	0.28	0.33	0.00	0.06	0.60	0.83
Business income/Business wealth:	0.49	0.05	1.67	0.00	0.00	0.29	1.03
Net worth Bottom 90%							
Business wealth/Total HH wealth:	0.40	0.24	1.54	0.00	0.04	0.56	0.84
Business income/Business wealth:	0.79	0.09	2.26	0.00	0.00	0.61	1.90

NOTE: This table shows the ratios of business income to different measures of wealth from SCF data 1989-2016. "**Business/Total wealth**" refers to the total value of business net worth/total HH net worth. "**Business income/net worth**" refers to the total value of business income/total business net worth; in the computation, we exclude those beyond the top 1% and below bottom 1% of this ratio. Sample weights are used.

B Appendix for more results on entrepreneurs' optimizations

B.1 Perturbation of the Problem

First Order Conditions. Consider the entrepreneurs' problem

$$\max_{\{c_t, b_t, i_t, k_t, n_t\}} \sum_{t=0}^{+\infty} \beta^t u(c_t),$$

$$\text{s.t.} \quad c_t + b_t + i_t \leq R_t b_{t-1} + F(k_{t-1}, n_t) - w_t n_t, \qquad (\mu_t^b)$$

$$k_t \leq i_t - g\left(\frac{i_t}{k_{t-1}}\right) k_{t-1} + (1-\delta) k_{t-1}, \qquad (\mu_t^k)$$

$$0 \leq b_t + \Psi k_t. \qquad (\mu_t^c)$$

The first order conditions are

$$0 = u'(c_t) - \mu_t^b,$$

$$0 = -\mu_t^b + \beta R_{t+1} \mu_{t+1}^b + \mu_t^c,$$

$$0 = -\mu_t^b + \mu_t^k \left[1 - g'\left(\frac{i_t}{k_{t-1}}\right)\right],$$

$$0 = -\mu_t^k + \beta \mu_{t+1}^b F_k(k_t, n_{t+1}) + \beta \mu_{t+1}^k \left[g'\left(\frac{i_{t+1}}{k_t}\right) \frac{i_{t+1}}{k_t} - g\left(\frac{i_{t+1}}{k_t}\right) + 1 - \delta\right] + \Psi \mu_t^c,$$

$$0 = F_n(k_{t-1}, n_t) - w_t,$$

$$c_t = R_t b_{t-1} + F(k_{t-1}, n_t) - w_t n_t - b_t - i_t,$$

$$k_t = i_t - g\left(\frac{i_t}{k_{t-1}}\right) k_{t-1} + (1-\delta) k_{t-1}.$$

Steady State Solution. In the steady state, we have

$$
\begin{aligned}
0 &= u'(c_{ss}) - \mu^b_{ss}, \\
0 &= -\mu^b_{ss} + \beta R_{ss}\mu^b_{ss} + \mu^c_{ss}, \\
0 &= -\mu^b_{ss} + \mu^k_{ss}, \\
0 &= -\mu^k_{ss} + \beta\mu^b_{ss}F_k(k_{ss}, n_{ss}) + \beta\mu^b_{ss}(1-\delta) + \Psi\mu^c_{ss}, \\
0 &= F_n(k_{ss}, n_{ss}) - w_{ss}, \\
c_{ss} &= R_{ss}b_{ss} + F(k_{ss}, n_{ss}) - w_{ss}n_{ss} - b_{ss} - i_{ss}, \\
k_{ss} &= i_{ss} + (1-\delta)k_{ss}.
\end{aligned}
$$

Eliminating $\{\mu^b_{ss}, \mu^k_{ss}, i_{ss}\}$ yields

$$
\begin{aligned}
1 &= \beta R_{ss} + \frac{\mu^c_{ss}}{u'}, \\
1 &= \beta(F_k + 1 - \delta) + \Psi\frac{\mu^c_{ss}}{u'}, \\
0 &= F_n - w_{ss}, \\
c_{ss} &= (R_{ss} - 1)b_{ss} + F - \delta k_{ss} - w_{ss}n_{ss}, \\
0 &= \mu^c_t(b_t + \Psi k_t).
\end{aligned}
$$

Eliminating μ^c_{ss} from the first two equations yields

$$
1 = \beta\frac{F_k + 1 - \delta - \Psi R_{ss}}{1 - \Psi}.
$$

Linear Perturbation. Perturbing the first order conditions around the steady state yields two Euler equations and a budget constraint

$$
\begin{aligned}
dc_t &= \beta R_{ss}dc_{t+1} + \frac{d\mu^c_t}{u''} + \beta\frac{u'}{u''}dR_{t+1}, \\
dc_t &= \beta(F_k + 1 - \delta)dc_{t+1} + \Psi\frac{d\mu^c_t}{u''} + \beta\frac{u'}{u''}\frac{F_{kn}}{F_{nn}}dw_{t+1} + \frac{u'}{u''}\frac{g''}{k_{ss}}[\beta(dk_{t+1} - dk_t) - (dk_t - dk_{t-1})], \\
dc_t &= -db_t - dk_t + R_{ss}db_{t-1} + (F_k + 1 - \delta)dk_{t-1} + b_{ss}dR_t - n_{ss}dw_t, \\
0 &= \mu^c_{ss}(b_{ss} + \Psi k_{ss}).
\end{aligned}
$$

In the unconstrained case, the first order conditions become

$$
\begin{aligned}
dc_t &= dc_{t+1} + \beta \frac{u'}{u''} dR_{t+1}, \\
dc_t &= dc_{t+1} + \beta \frac{u'}{u''} \frac{F_{kn}}{F_{nn}} dw_{t+1} + \frac{u'}{u''} \frac{g''}{k_{ss}} [\beta(dk_{t+1} - dk_t) - (dk_t - dk_{t-1})], \\
dc_t &= -(db_t + dk_t) + \beta^{-1}(db_{t-1} + dk_{t-1}) + b_{ss} dR_t - n_{ss} dw_t.
\end{aligned}
$$

In the constrained case, the first order conditions become

$$
\begin{aligned}
dc_t &= dc_{t+1} + \frac{\beta}{1-\Psi} \frac{u'}{u''} \left(\frac{F_{kn}}{F_{nn}} dw_{t+1} - \Psi dR_{t+1} \right) + \frac{u'}{u''} \frac{g''}{(1-\Psi) k_{ss}} [\beta(dk_{t+1} - dk_t) - (dk_t - dk_{t-1})], \\
dc_t &= -(1-\Psi)(dk_t - \beta^{-1} dk_{t-1}) + b_{ss} dR_t - n_{ss} dw_t, \\
0 &= db_{ss} + \Psi dk_{ss}.
\end{aligned}
$$

B.2 Proof to Lemma 1

Proof. Lemma 1 extends Theorem 2 in Auclert (2019) by allowing for persistent shocks. To prove it, we first reformulate the entrepreneurs' problem into a recursively, and include the hypothetical lump-sum transfer Tr_t in period t budget

$$
\begin{aligned}
V(b_{t-1}, k_{t-1}, \{R_{t+\tau}, w_{t+\tau}, Tr_{t+\tau}\}) = \max_{\{c_t, b_t, i_t, k_t, n_t\}} & \{u(c_t) + \beta V(b_t, k_t, \{R_{t+1+\tau}, w_{t+1+\tau}, Tr_{t+1+\tau}\})\}, \\
\text{s.t.} \quad c_t + b_t + i_t &\leq R_t b_{t-1} + F(k_{t-1}, n_t) - w_t n_t + Tr_t, \\
k_t &\leq i_t - g\left(\frac{i_t}{k_{t-1}}\right) k_{t-1} + (1-\delta) k_{t-1}, \\
0 &\leq b_t + \Psi k_t.
\end{aligned}
$$

Consider a perturbation $dR_{t+\tau}$. Denote $V_t \equiv V(b_{t-1}, k_{t-1}, \{R_{t+\tau}, w_{t+\tau}, Tr_{t+\tau}\})$. Applying the Envelop Theorem recursively yields

$$
\frac{\partial V_t}{\partial R_{t+\tau}} = \beta^\tau u'(c_{t+\tau}) b_{t-1+\tau}, \quad \frac{\partial V_t}{\partial Tr_{t+\tau}} = \beta^\tau u'(c_{t+\tau}).
$$

Let Tr_t be associated with R_t such that

$$
\frac{\partial V_t}{\partial R_{t+\tau}} + \frac{\partial V_t}{\partial Tr_{t+\tau}} \frac{\partial Tr_{t+\tau}}{\partial R_{t+\tau}} = 0 \quad \Longrightarrow \quad \frac{\partial Tr_{t+\tau}}{\partial R_{t+\tau}} = -b_{t-1+\tau}.
$$

According to the definition of income and substitution effects,

$$\frac{\partial}{\partial R_{t+\tau}} = \underbrace{-\frac{\partial Tr_{t+\tau}}{\partial R_{t+\tau}} \frac{\partial}{\partial Tr_{t+\tau}}}_{\text{income effects}} + \underbrace{\frac{\partial}{\partial R_{t+\tau}} + \frac{\partial Tr_{t+\tau}}{\partial R_{t+\tau}} \frac{\partial}{\partial Tr_{t+\tau}}}_{\text{substitution effects}}.$$

Consider a perturbation in $dw_{t+\tau}$. The Envelop Theorem under optimal labor implies that

$$\frac{\partial [F(k_{t-1+\tau}, n_{t+\tau}) - w_{t+\tau} n_{t+\tau}]}{\partial w_{t+\tau}} = -n_{t+\tau}. \quad \Longrightarrow \quad \frac{\partial V_t}{\partial w_{t+\tau}} = -\beta^\tau u'(c_{t+\tau}) n_{t+\tau}.$$

This rest of the proof for $dw_{t+\tau}$ is the same as that for $dR_{t+\tau}$.

As a summary, all shocks induce income effects through the budget and substitution effects through the Euler equations. We denote the shocks through income effects using d^I and those through substitution effects using d^S. By notation, $d = d^I + d^S$. Note that this decomposition method is also applicable to the fully-fledged quantitative model. □

B.3 Proof to Lemma 2

Proof. Due to constant returns to scale in Assumption 1,

$$\begin{aligned}
F_k(k_{t-1}, n(k_{t-1}, w_t)) &= \frac{F(k_{t-1}, n(k_{t-1}, w_t)) - F_n(k_{t-1}, n(k_{t-1}, w_t)) n_t}{k_{t-1}} \\
&= F\left(1, \frac{n(k_{t-1}, w_t)}{k_{t-1}}\right) - F_n\left(1, \frac{n(k_{t-1}, w_t)}{k_{t-1}}\right) \frac{n(k_{t-1}, w_t)}{k_{t-1}} \\
&= f\left(\frac{n(k_{t-1}, w_t)}{k_{t-1}}\right) - w_t \frac{n(k_{t-1}, w_t)}{k_{t-1}} \\
&= f(f'^{-1}(w_t)) - w_t f'^{-1}(w_t) \\
&\equiv \widetilde{R}(w_t) - 1 + \delta.
\end{aligned}$$

Directly taking derivatives in $\widetilde{R}(w_t)$ yields

$$\widetilde{R}'(w_t) = f'(f'^{-1}(w_t)) \frac{d f'^{-1}(w_t)}{d w_t} - f'^{-1}(w_t) - w_t \frac{f'^{-1}(w_t)}{d w_t} = -f'^{-1}(w_t).$$

We also have

$$\frac{F_{kn}(k_{t-1}, n(k_{t-1}, w_t))}{F_{nn}(k_{t-1}, n(k_{t-1}, w_t))} = \frac{\frac{\partial f(k_{t-1}/n(k_{t-1}, w_t))}{\partial k_{t-1}}}{\frac{\partial f(k_{t-1}/n(k_{t-1}, w_t))}{\partial n(k_{t-1}, w_t)}} = -\frac{n(k_{t-1}, w_t)}{k_{t-1}} = -f'^{-1}(w_t).$$

□

B.4 Proof to Example 1

Proof. Given production function $F(k,n) = zk^{1-\alpha}n^{\alpha}$, we have

$$\frac{[\widetilde{R}(w_t) + 1 - \delta]k_{t-1}}{w_t n(k_{t-1}, w_t)} = \frac{1-\alpha}{\alpha}.$$

Note that $\widetilde{R}'(w_t) = -\frac{n(k_{t-1},w_t)}{k_{t-1}}$. Hence,

$$\frac{\widetilde{R}(w_t) + 1 - \delta}{w_t \widetilde{R}'(w_t)} = -\frac{1-\alpha}{\alpha}.$$

□

B.5 Proof to Proposition 1

Proof. In the unconstrained case, combining the two Euler equations yields

$$(dk_t - dk_{t-1}) - \beta(dk_{t+1} - dk_t) = -\frac{k_{ss}}{g''} \cdot \beta \left(d^S R_{t+1} - \frac{F_{kn}}{F_{nn}} d^S w_{t+1} \right).$$

Solving $dk_t - dk_{t-1}$ forward yields

$$dk_t = dk_{t-1} - \frac{k_{ss}}{g''} \cdot \sum_{\tau=1}^{+\infty} \beta^{\tau} \left(d^S R_{t+\tau} - \frac{F_{kn}}{F_{nn}} d^S w_{t+\tau} \right).$$

This result is solved without using budget constraints, and hence is completely substitution effect. Given $\{dk_{t-1+\tau}\}$, the consumption-saving problem satisfies

$$\begin{aligned} dc_t &= dc_{t+1} + \beta \frac{u'}{u''} d^S R_{t+1}, \\ dc_t &= -(db_t + dk_t) + \beta^{-1}(db_{t-1} + dk_{t-1}) + b_{ss} d^I R_t - n_{ss} d^I w_t. \end{aligned}$$

This is just the perturbation of a standard consumption-saving problem in complete market if we view $db_t + dk_t$ as one object. Substituting the budget into the Euler equation yields

$$\begin{aligned} &[(db_t + dk_t) - (db_{t-1} + dk_{t-1})] - \beta[(db_{t+1} + dk_{t+1}) - (db_t + dk_t)] \\ &\quad = \beta \left[(b_{ss} d^I R_t - n_{ss} d^I w_t) - (b_{ss} d^I R_{t+1} - n_{ss} d^I w_{t+1}) + \beta(-\frac{u'}{u''}) d^S R_{t+1} \right]. \end{aligned}$$

Solving $(db_t + dk_t) - (db_{t-1} + dk_{t-1})$ forward yields

$$db_t + dk_t = db_{t-1} + dk_{t-1} + \sum_{\tau=0}^{+\infty} \beta^\tau \left[(\mathbf{1}_{\tau\geq 1} - \beta)(-b_{ss}d^I R_{t+\tau} + n_{ss}d^I w_{t+\tau}) + \mathbf{1}_{\tau\geq 1}\beta c_{ss}(-\frac{u'}{c_{ss}u''})d^S R_{t+\tau} \right].$$

Combine the solution of $db_t + dk_t$ with the budget yields the solution of dc_t

$$dc_t = (1-\beta)\beta^{-1}(db_{t-1} + dk_{t-1}) - \sum_{\tau=0}^{+\infty} \beta^\tau \left[(1 - \beta)(-b_{ss}d^I R_{t+\tau} + n_{ss}d^I w_{t+\tau}) + \mathbf{1}_{\tau\geq 1}\beta c_{ss}(-\frac{u'}{c_{ss}u''})d^S R_{t+\tau} \right].$$

The solution for db_t is given by $db_t = db_t + dk_t - dk_t$. □

B.6 Proof to Proposition 2

Proof. In the constrained case, combining the Euler equation and the budget yields

$$\left[1 + \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g''\right][(dk_t - dk_{t-1}) - \beta(dk_{t+1} - dk_t)] = \frac{\beta}{1-\Psi}[(b_{ss}d^I R_t - n_{ss}d^I w_t) - (b_{ss}d^I R_{t+1} - n_{ss}d^I w_{t+1})] - \frac{\beta^2}{(1-\Psi)^2}(-\frac{u'}{u''})\left(\Psi d^S R_{t+1} - \frac{F_{kn}}{F_{nn}}d^S w_{t+1}\right).$$

Solving $dk_t - dk_{t-1}$ forward yields

$$dk_t = dk_{t-1} + \frac{\sum_{\tau=0}^{+\infty}\beta^\tau \left[(\mathbf{1}_{\tau\geq 1} - \beta)(-b_{ss}d^I R_{t+\tau} + n_{ss}d^I w_{t+\tau}) + \mathbf{1}_{\tau\geq 1}\beta c_{ss}(-\frac{u'}{c_{ss}u''})\frac{\Psi d^S R_{t+1} - \frac{F_{kn}}{F_{nn}}d^S w_{t+1}}{1-\Psi} \right]}{(1-\Psi)\left[1 + \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g''\right]}.$$

Using the budget $dc_t = -(1-\Psi)(dk_t - \beta^{-1}dk_{t-1}) + b_{ss}d^I R_t - n_{ss}d^I w_t$, we have

$$dc_t = \frac{(1-\Psi)(1-\beta)}{\beta}dk_{t-1} - (-b_{ss}d^I R_t + n_{ss}d^I w_t) - \frac{\sum_{\tau=0}^{+\infty}\beta^\tau \left[(\mathbf{1}_{\tau\geq 1} - \beta)(-b_{ss}d^I R_{t+\tau} + n_{ss}d^I w_{t+\tau}) + \mathbf{1}_{\tau\geq 1}\beta c_{ss}(-\frac{u'}{c_{ss}u''})\frac{\Psi d^S R_{t+1} - \frac{F_{kn}}{F_{nn}}d^S w_{t+1}}{1-\Psi} \right]}{1 + \frac{\beta c_{ss}}{(1-\Psi)^2 k_{ss}}(-\frac{u'}{c_{ss}u''})g''}.$$

Given the solution of dk_t, we also have $db_t = -\Psi dk_t$. □

B.7 Hybrid Model

We consider a hybrid model as a linear combination of the two cases in the basic model. It has the potential to capture the interaction between consumption and investment decisions, to generate flexible marginal propensities to consume (mpc) and to invest (mpi), and to remove the undesirable feature that lower real interest rate depresses consumption. The model has difficulty in generating large consumption fluctuations with the substitution effect of real interest rate alone, and relies on the income effects of real interest rate and real wage rate.

In the hybrid model, we fix $\{R_{ss}, w_{ss}\}$ and assume two types of entrepreneurs denoted with subscripts "u" and "c", respectively. Type "u" entrepreneur is more patient and less productive with $R_{ss} = \widetilde{R}_{u,ss} = \beta_u^{-1}$, while Type "c" entrepreneur is less patient but more productive with $R_{ss} < \frac{\widetilde{R}_{c,ss} - \Psi R_{ss}}{1-\Psi} = \beta_c^{-1}$. We assume that these two types of entrepreneurs start with an unconstrained and a constrained steady state, respectively, so that Proposition 1 and 2 can be directly applied.

For algebra simplicity, we assume that the savings and capital of the two entrepreneur types are potentially different so as to support the same level of steady state consumption c_{ss}. Denote the share of constrained entrepreneurs as $\lambda \in [0,1]$ and the weighted average of $\{mpc, mpi\}$ as $\{MPC, MPI\}$. Corollary 1 illustrates the interaction between consumption and investment decisions based on $\{MPC, MPI\}$.

Corollary 1 (MPC and MPI)**.** *In the hybrid model, MPC is increasing in $-\frac{u'}{c_{ss}u''}g''$, while MPI is decreasing in it.*

Corollary 1 implies that IES and capital adjustment cost jointly affect MPC and MPI. The reason is that IES measures the tolerance for consumption non-smoothing, and capital adjustment cost captures the barrier to smoothing. These two effects reinforce each other. Moreover, we can calibrate $\{\lambda, g''\}$ to target on $\{MPC, MPI\}$ in Corollary 2.

Corollary 2 (Calibration)**.** *In the hybrid model, to target on $MPC, MPI \in [0,1]$, we have*

$$\lambda = \frac{MPC + (1-\Psi)MPI - (1-\beta_u)}{\beta_u} \in [1-\beta_u^{-1}, 1] \supset [0,1],$$

$$\frac{\beta c_{ss}}{(1-\Psi)^2 k_{c,ss}}\left(-\frac{u'}{c_{ss}u''}\right)g'' = \frac{\beta_c}{\beta_u}\left[1 + \frac{MPC - (1-\beta_u)}{(1-\Psi)MPI}\right] - 1 \in [\beta_c - 1, +\infty) \supset [0, +\infty).$$

Corollary 2 indicates that higher MPC requires more constrained entrepreneurs and higher capital adjustment cost, while higher MPI requires more constrained entrepreneurs but lower capital adjustment cost. Some calibrated $\{\lambda, g''\}$ is not feasible, which implies that some combination of $\{MPC, MPI\}$ may not be achievable in the hybrid model. To see how this model removes the undesirable feature that lower real interest rate depressing consumption through substitution effects, we refer to Corollary 3.

Corollary 3 (Removing Undesirable Feature)**.** *In the hybrid model, to prevent the substitution effect of lower real interest rate from depressing consumption for* $\tau \geq 1$*, we need*

$$\frac{\beta c_{ss}}{k_{c,ss}}\left(-\frac{u'}{c_{ss}u''}\right)g'' > (1-\Psi)^2\left[\left(\frac{\beta_c}{\beta_u}\right)^{\tau+1}\frac{\lambda}{1-\lambda}\frac{\Psi}{1-\Psi} - 1\right].$$

Corollary 3 implies that the share of constrained entrepreneurs induces the undesirable feature, while the capital adjustment cost helps remove it. Despite Corollary 3, when the pledgeability ratio Ψ is not close to 0, and the share of constrained entrepreneurs λ is high, the substitution effect of real interest rate on consumption is greatly dampened. To see how much of the effect is dampened, we have Corollary 4.

Corollary 4 (Dampened Substitution Effects)**.** *In the hybrid model,*

$$\lambda\frac{\frac{\partial^S c_{u,t}}{\partial R_{t+\tau}} - \frac{\partial^S c_{c,t}}{\partial R_{t+\tau}}}{\frac{\partial^S c_{u,t}}{\partial R_{t+\tau}}} = \lambda + \left(\frac{\beta_c}{\beta_u}\right)^{\tau+1}\frac{\Psi MPI}{\beta_c}.$$

Corollary 4 shows that the dampening ratio is determined by the share of constrained entrepreneurs λ and MPI. Since λ is likely to be large in the real world, and so is MPI, we conjecture that the stimulus effect of lower real interest rate on entrepreneur consumption through intertemporal substitution is at best very weak, and we need to find additional sources of consumption fluctuations such as business income fluctuations.

B.8 Proof to Corollary 1

Proof. The weight average MPC and MPI are

$$MPC = (1-\lambda)(1-\beta_u) + \lambda\left[1 - \frac{\beta_c}{1 + \frac{\beta_c c_{ss}}{(1-\Psi)^2 k_{c,ss}}\left(-\frac{u'}{c_{ss}u''}\right)g''}\right],$$

$$MPI = \lambda\frac{\frac{\beta_c}{1-\Psi}}{1 + \frac{\beta_c c_{ss}}{(1-\Psi)^2 k_{c,ss}}\left(-\frac{u'}{c_{ss}u''}\right)g''}.$$

□

B.9 Proof to Corollary 2

Proof. Base on Corollary 1,

$$MPC + (1-\Psi)MPI = (1-\lambda)(1-\beta_u) + \lambda.$$

The solution for λ is

$$\lambda = \frac{MPC + (1-\Psi)MPI - (1-\beta_u)}{\beta_u}.$$

The solution for g'' satisfies

$$\frac{\beta_c c_{ss}}{(1-\Psi)^2 k_{c,ss}}(-\frac{u'}{c_{ss}u''})g'' = \frac{\lambda\beta_c}{(1-\Psi)MPI} - 1 = \frac{\beta_c}{\beta_u}\left[1 + \frac{MPC-(1-\beta_u)}{(1-\Psi)MPI}\right] - 1.$$

□

B.10 Proof to Corollary 3

Proof. Negative substitution effects of real interest rate on consumption requires that

$$-(1-\lambda)\beta_u^{\tau+1}c_{ss}(-\frac{u'}{c_{ss}u''}) + \lambda\frac{\beta_c^{\tau+1}c_{ss}(-\frac{u'}{c_{ss}u''})}{1+\frac{\beta_c c_{ss}}{(1-\Psi)^2 k_{c,ss}}(-\frac{u'}{c_{ss}u''})g''}\frac{\Psi}{1-\Psi} < 0.$$

The solution for g'' proves it. □

B.11 Proof to Corollary 4

Proof. The dampening ratio is

$$\lambda\frac{\frac{\partial^S c_{u,t}}{\partial R_{t+\tau}} - \frac{\partial^S c_{c,t}}{\partial R_{t+\tau}}}{\frac{\partial^S c_{u,t}}{\partial R_{t+\tau}}} = \lambda + \lambda\frac{\beta_c^{\tau+1}\frac{\frac{\Psi c_{ss}}{1-\Psi}(-\frac{u'}{c_{ss}u''})}{1+\frac{\beta_c c_{ss}}{(1-\Psi)^2 k_{c,ss}}(-\frac{u'}{c_{ss}u''})g''}}{\beta_u^{\tau+1}c_{ss}(-\frac{u'}{c_{ss}u''})} = \lambda + \lambda\left(\frac{\beta_c}{\beta_u}\right)^{\tau+1}\frac{\frac{\Psi}{1-\Psi}}{1+\frac{\beta_c c_{ss}}{(1-\Psi)^2 k_{c,ss}}(-\frac{u'}{c_{ss}u''})g''}$$

$$= \lambda + \lambda\left(\frac{\beta_c}{\beta_u}\right)^{\tau+1}\frac{\frac{\Psi}{1-\Psi}}{\frac{\beta_c}{\beta_u}\left[1+\frac{MPC-(1-\beta_u)}{(1-\Psi)MPI}\right]} = \lambda + \left(\frac{\beta_c}{\beta_u}\right)^{\tau+1}\frac{\Psi MPI}{\beta_c}.$$

□

C Appendix for the Quantitative Model

C.1 Entrepreneurs' optimization

For entrepreneurs, first, the static profit optimization is:

$$\pi(k,z) \quad = \quad \max_n \exp(z)\left[(1-\alpha)k^{\frac{\varepsilon-1}{\varepsilon}} + \alpha n^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon\nu}{\varepsilon-1}} - wn, \tag{C.1}$$

with given capital stock k, the first order condition with respect to labor demand is:

$$FOC: 0 = \frac{\nu}{\frac{1-\alpha}{\alpha}\left(\frac{k}{n}\right)^{\frac{\varepsilon-1}{\varepsilon}} + 1}\frac{y}{n} - w \Rightarrow n = n^*(k,z). \tag{C.2}$$

also note that, the marginal return to capital is:

$$\frac{\partial \pi}{\partial k}|_{n=n^*(k,z)} \quad = \quad \frac{\nu\frac{1-\alpha}{\alpha}\left(\frac{k}{n}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{\frac{1-\alpha}{\alpha}\left(\frac{k}{n}\right)^{\frac{\varepsilon-1}{\varepsilon}} + 1}\frac{y}{k}. \tag{C.3}$$

For entrepreneurs' dynamic optimization, first, define $a = k - b \geq 0$, a is the total household net worth that we could take to the data. We could then rewrite the problem as:

$$V_t(a,k,z) = \max_{c,a',k',n} u(c) + \beta E V_{t+1}(a',k',z') \tag{C.4}$$

$$c + a' + g(i,k) = \pi_t(k,z;Y) + a\frac{1+i_t^a}{\Pi_t} - (\delta + \frac{1+i_t^a}{\Pi_t} - 1)k \tag{C.5}$$

$$a' \geq (1-\Psi)k', k' \geq 0, a' \geq 0. \tag{C.6}$$

Denote the multiplier μ for the inequality constraint $a' - (1-\Psi)k' \geq 0$. It is easy that we have the following set of first-order conditions for a' and k', taking into account of the inequality constraints:

$$\mu \geq 0 : a' - (1-\Psi)k' \geq 0, [a' - (1-\Psi)k']\mu = 0 \tag{C.7}$$

$$a' : \mu - u'(c) + \beta E u'(c\prime)(1+r') = 0, \tag{C.8}$$

$$k' : -(1-\Psi)\mu - u'(c)g_1(i,k) + \beta E u'(c\prime)[\frac{\partial \pi'}{\partial k'} - (r'+\delta) - g_2(i',k')] = 0, \tag{C.9}$$

and for the adjustment cost function, note that we have assumed the following functional forms:

$$g(i,k) = \frac{\Phi}{2}(\frac{i}{k} - \delta)^2 k, i = k' - (1-\delta)k \tag{C.10}$$

$$g_1 = \frac{\partial g(i,k)}{\partial i} = \frac{\partial g(i,k)}{\partial k'} = \Phi(\frac{i}{k} - \delta) \tag{C.11}$$

$$g_2 = \frac{\partial g(i,k)}{\partial k} = \frac{\Phi}{2}(\frac{i}{k} - \delta)^2 - \Phi(\frac{i}{k} - \delta)\frac{k'}{k}. \tag{C.12}$$

We can further simplify these conditions if the constraint is binding as follows; these formulas will be useful for numerical analysis. Redefining μ as $\widetilde{\mu}\beta E u'(c\prime)(1+r')$,

$$\text{if: } \mu > 0 : a' = (1-\Psi)k' \tag{C.13}$$

$$u'(c) = (1+\widetilde{\mu})[\beta E u'(c\prime)(1+r')] \tag{C.14}$$

$$\frac{\beta E u'(c\prime)[\frac{\partial \pi'}{\partial k'} - (r'+\delta) - g_2(i',k')]}{u'(c)} = g_1(i,k) + (1-\Psi)\frac{\widetilde{\mu}}{(1+\widetilde{\mu})}. \tag{C.15}$$

C.2 Marginal propensity to invest and consume for entrepreneurs

Following the transformation of the optimization problem previously and slightly abusing notation, denote economic profits as

$$\pi(k_t, n_t; w_t, r_t, z_t) \equiv \exp(z_t)\left[(1-\alpha)k_t^{\frac{\varepsilon-1}{\varepsilon}} + \alpha n_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon\nu}{\varepsilon-1}} - w_t n_t - (r_t+\delta)k_t. \tag{C.16}$$

Suppose there are one-period, transitory, small enough deviations for $\{dw_t, dr_t\}_{t=0}$, relative to the paths of $\{w_t, r_t\}_{t\geq 0}$. Denote the original solutions as $\{c_t, c_{t+1}, i_t, k_{t+1}, i_{t+1}, k_{t+2}, ...\}_{t\geq 0}$ and the deviations for the new solution as $\{dc_t, dc_{t+1}, di_t, dk_{t+1}, di_{t+1}, ...\}_t$

First, we have the following decomposition for entrepreneurs' economic profits:

$$d\pi = \frac{\partial \pi}{\partial w}dw + \frac{\partial \pi}{\partial r}dr + \frac{\partial \pi}{\partial k}dk + \frac{\partial \pi}{\partial n}dn. \tag{C.17}$$

In the first period of the experiment, k cannot be adjusted. When entrepreneurs can optimally choose n_t according to its FOC for a given k_t, $\frac{\partial \pi}{\partial n} = 0$, and $d\pi$ can be simplified to:

$$d\pi = -ndw - kdr. \tag{C.18}$$

Denote the entrepreneur's current wealth as Ω, the deviation is

$$d\Omega_t = a_t dr_t + d\pi. \tag{C.19}$$

Consider the case that the entrepreneur is constrained with $a' = (1-\Psi)k'$. For a small change of current wealth in the first period $d\Omega$, from the budget constraint we have

$$dc + da' + dg(i,k) = d\Omega,$$

using $da' = (1-\Psi)dk' = (1-\Psi)di$,and $dg(i,k) = \frac{\partial g(i,k)}{\partial i}di$, we can express marginal propensities as

$$MPC = 1 - MPI \times [1 - \Psi + \Phi(\frac{i}{k} - \delta)]. \tag{C.20}$$

For constrained (and unconstrained) entrepreneurs, the FOCs can be re-written as

$$\mu_t u'(c_t) = u'(c_t) - \beta(1+r)u'(c_{t+1}), 1 > \mu_t > 0, \tag{C.21}$$

where we scale the multiplier by current marginal utility $u'(c_t)$. And for investment,

$$\beta \frac{u'(c_{t+1})(1+r)}{u'(c_t)} \frac{\frac{\partial \pi'}{\partial k'} - (r+\delta) - g_2(i',k')}{(1+r)} = \\ g_1(i_t,k) + (1-\Psi)[1 - \beta \frac{u'(c_{t+1})}{u'(c_t)}(1+r)], \tag{C.22}$$

or alternatively,

$$\beta \frac{u'(c_{t+1})(1+r)}{u'(c_t)} NMRK_{t+1} = \\ g_1(i_t,k) + (1-\Psi)\mu_t, \tag{C.23}$$

where $NMRK_{t+1} \equiv \frac{\frac{\partial \pi'}{\partial k'} - (r+\delta) - g_2(i',k')}{(1+r)}$ denotes the net marginal return to capital (net of opportunity costs and next period's marginal adjustment cost) after discounting.

Log-linearizing the FOCs around the original solutions, we arrive at:

$$1 - \mu_t = \beta(1+r)\frac{u'(c_{t+1})}{u'(c_t)} \tag{C.24}$$

$$-d\mu_t = \frac{\beta(1+r)u'(c_{t+1})}{u'(c_t)} \left[\frac{-1}{\sigma_C} \left(\frac{dc_{t+1}}{c_{t+1}} - \frac{dc_t}{c_t} \right) \right], \tag{C.25}$$

and

$$(-d\mu_t)NMRK_{t+1} + \beta \frac{u'(c_{t+1})}{u'(c_t)}[\pi_{kk} di_t - g_{21} di_{t+1} - g_{22} di_t] \\ = \Phi \frac{1}{k} di_t + (1-\Psi) d\mu_t, \tag{C.26}$$

where $\sigma_C \equiv \frac{-u'}{cu''}$ denotes the elasticity of intertemporal substitution in consumption, $\pi_{kk} \equiv \frac{\partial^2 \pi(k_{t+1})}{\partial k \partial k}$ denotes the second derivative of profit respect to capital, and other derivatives are denoted as

$$g_1 \equiv \frac{\partial g(i,k)}{\partial i} = \frac{\partial g(i,k)}{\partial k'} = \Phi(\frac{i}{k} - \delta), g_{11} \equiv \frac{\partial^2 g(i,k)}{\partial i \partial i} = \Phi \frac{1}{k}$$
$$g_2 \equiv \frac{\partial g(i,k)}{\partial k} = \frac{\Phi}{2}(\frac{i}{k} - \delta)^2 - \Phi(\frac{i}{k} - \delta)\frac{k'}{k},$$
$$g_{21} \equiv \frac{\partial^2 g(i,k)}{\partial k \partial i} = -\Phi \frac{k'}{k}\frac{1}{k}, g_{22} \equiv \frac{\partial^2 g(i,k)}{\partial k \partial k} = \Phi \frac{k'}{k}\frac{i}{k^2}.$$

Assuming $di_{t+1} = 0$ for this transitory shock (numerically we can see this approximation is reasonably well), we can simplify the relation between di_t and $d\mu_t$.

$$\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} (\pi_{kk} - g_{kk}) - \Phi \frac{1}{k} \right] di_t \\ = [1 - \Psi + NMRK_{t+1}] \, d\mu_t. \tag{C.27}$$

To solve for an expression for MPI, we can first notice that, from envelope conditions we have:

$$1 - \mu_t = \beta(1+r)\frac{V_a(a_{t+1}, k_{t+1}, z)}{V_a(a,k,z)} \tag{C.28}$$

and the deviation when there is a small change in current wealth $d\Omega_t$ (change of da_t by $\frac{d\Omega_t}{1+r}$) is given by:

$$1-\mu_t = \beta(1+r)\frac{V_a(a_{t+1},k_{t+1},z)}{V_a(a,k,z)} \tag{C.29}$$

$$\frac{-d\mu_t}{1-\mu_t} = \frac{V_{aa}(a_{t+1},k_{t+1},z)(1-\Psi)+V_{ak}(a_{t+1},k_{t+1},z)}{V_a(a_{t+1},k_{t+1},z)}di_t \tag{C.30}$$

$$-\frac{V_{aa}(a,k,z)}{V_a(a,k,z)}\frac{d\Omega_t}{1+r} \tag{C.31}$$

Combining the relations between di_t and $d\mu_t$ we have the solution for MPI implied as:

$$\frac{V_{aa}(a,k,z)}{V_a(a,k,z)}\frac{1}{1+r} =$$

$$MPI \times \left\{ \frac{V_{aa}(a_{t+1},k_{t+1},z)(1-\Psi)+V_{ak}(a_{t+1},k_{t+1},z)}{V_a(a_{t+1},k_{t+1},z)} + \frac{\left[\beta\frac{V_a(a_{t+1},k_{t+1},z)}{V_a(a,k,z)}(\pi_{kk}-g_{kk})-\Phi\frac{1}{k}\right]}{(1-\mu_t)\left[1-\Psi+NMRK_{t+1}\right]} \right\}. \tag{C.32}$$

For unconstrained entrepreneurs, if there is some small enough change in current wealth $d\Omega_t$ (change of da_t by $\frac{d\Omega_t}{1+r}$), she will still have $a' > (1-\Psi)k'$ and is unconstrained. In our environment with z constant over time, we need to find dc_t, da_{t+1} and di_t:

$$dc_t + da_{t+1} + \frac{\partial g(i,k)}{\partial i}di_t = d\Omega_t. \tag{C.33}$$

For investment, the following optimality should continue to hold:

$$0 = \frac{\partial^2 \pi_{t+1}}{\partial k_{t+1}\partial k_{t+1}} - g_{22}(i_{t+1},k_{t+1}) - \Phi\frac{1+r}{k}. \tag{C.34}$$

thus, MPI should be 0. Then it's easy to find dc_t and da_{t+1} by

$$0 = \frac{V_{aa}(a_{t+1},k_{t+1},z)da_{t+1}+V_{ak}(a_{t+1},k_{t+1},z)di_t}{V_a(a_{t+1},k_{t+1},z)} - \frac{V_{aa}(a,k,z)}{V_a(a,k,z)}\frac{d\Omega_t}{1+r}. \tag{C.35}$$

C.3 Optimization for corporate firms

For corporate firms, denote the value function at time t with predetermined capital stock of K_t as $V_t(K_t)$, the dynamic problem is:

$$V_t(K_t) = \max_{\{K_{t+1},N_t\}} D_t + \tfrac{1}{1+r_{t+1}}V_{t+1}(K_{t+1}).,\tag{C.36}$$

$$D_t = F(K_t,N_t) - W_tN_t - I_t - g\left(\tfrac{I_t}{K_t},K_t\right) \tag{C.37}$$

$$I_t = K_{t+1} - (1-\delta)K_t, \tag{C.38}$$

Where r_{t+1} is the real risk-free interest rate across time t and $t+1$. Note that for our analysis of the dynamics, mostly the firm is with perfect foresight for all future variables in W_t and r_{t+1} after the initial shock, thus here we abstract away from expectations and stochastics.

The first order conditions and budget constraints are

$$0 = -1 - \frac{\partial g(\frac{I_t}{K_t}, K_t)}{\partial I} + \frac{1}{1+r_{t+1}} \left[F_K(K_{t+1}, N_{t+1}) - \frac{\partial g\left(\frac{I_{t+1}}{K_{t+1}}, K_{t+1}\right)}{\partial K} + (1-\delta) \right], \tag{C.39}$$

$$0 = F_N(K_t, N_t) - W_t, \tag{C.40}$$

$$0 = F(K_t, N_t) - W_t N_t - D_t - I_t - g\left(\frac{I_t}{K_t}, K_t\right), \tag{C.41}$$

$$I_t = K_{t+1} - (1-\delta)K_t. \tag{C.42}$$

Assuming standard adjustment cost function, let $g\left(\frac{I_t}{K_t}, K_t\right) = \frac{\Phi}{2}(\frac{I_t}{K_t} - \delta)^2 K_t$, we have $\frac{\partial g}{\partial I} = \Phi(\frac{I_t}{K_t} - \delta)$, $\frac{\partial g}{\partial K} = \frac{\Phi}{2}(\frac{I_t}{K_t} - \delta)^2 - \Phi(\frac{I_t}{K_t} - \delta)\frac{K_{t+1}}{K_t}$. In the steady state, $I_{ss} = \delta K_{ss}$, and $g\left(\frac{I_t}{K_t}, K_t\right) = \frac{\partial g}{\partial I} = \frac{\partial g}{\partial K} = 0$. The first-order condition for capital investment can be reduced to $r + \delta = F_K(K_{ss}, N_{ss})$.

With $F(K,N) = \exp(z^c)\left[K^{1-\alpha}N^{\alpha}\right]^{\nu}$, with $\nu \leq 1$. Use $WN = NF_N = \alpha\nu F$, we can solve for N_t for given K_t,

$$N_t = \left[\frac{\alpha\nu\exp(z^c)}{W_t}\right]^{\frac{1}{1-\alpha\nu}} K_t^{\frac{(1-\alpha)\nu}{1-\alpha\nu}},$$

and also note that $F_K(K,N)K = (1-\alpha)\nu F = (1-\alpha)\nu\exp(z^c)\left[K^{1-\alpha}N^{\alpha}\right]^{\nu}$, so we can simplify the marginal return to capital as

$$\begin{aligned} F_K(K_{t+1}, N_{t+1}) &= (1-\alpha)\nu F/K \\ &= \frac{1}{K}\frac{1-\alpha}{\alpha} W_{t+1} N_{t+1}, \\ &= \frac{1-\alpha}{\alpha} W_{t+1} \left[\frac{\alpha\nu\exp(z^c)}{W_{t+1}}\right]^{\frac{1}{1-\alpha\nu}} K_{t+1}^{\frac{\nu-1}{1-\alpha\nu}}. \end{aligned}$$

In the steady state, we can find the solutions for K_{ss} and N_{ss} given $\exp(z^c)$:

$$\begin{aligned} F_{ss} &= \exp(z^c)\left[K^{1-\alpha}N^{\alpha}\right]^{\nu} \\ &= \exp(z^c)[\frac{(1-\alpha)\nu F}{r+\delta}]^{(1-\alpha)\nu} \times [\frac{\alpha\nu F}{W}]^{\alpha\nu} \\ &\Rightarrow F_{ss}^{1-\nu} = \exp(z^c)[\frac{(1-\alpha)\nu}{r+\delta}]^{(1-\alpha)\nu} \times [\frac{\alpha\nu}{W}]^{\alpha\nu} \end{aligned}$$

It's useful to log-linearize the equilibrium conditions around the steady state. Use dx_t to denote the deviations from the corresponding steady state values, and use $\hat{x}_t$ to denote percentage deviations, $x_{ss}\hat{x}_t = dx_t$. The first order condition for labor demand gives:

$$0 = (1-\alpha\nu)dN_t/N_{ss} + dw_t/w_{ss} - (1-\alpha)\nu dK_t/K_{ss},$$

Using the first-order condition for investment demand, we can have

$$\begin{aligned} &\left[1 + \Phi(\frac{I_t}{K_t} - \delta)\right][1 + r_{t+1}] \\ = \quad & F_K(K_{t+1}, N_{t+1}) + (1-\delta) - \frac{\Phi}{2}(\frac{I_{t+1}}{K_{t+1}} - \delta)^2 + \Phi(\frac{I_{t+1}}{K_{t+1}} - \delta)\frac{K_{t+2}}{K_{t+1}} \\ = \quad & \frac{1-\alpha}{\alpha} W_{t+1}\left[\frac{\alpha\nu\exp(z^c)}{W_{t+1}}\right]^{\frac{1}{1-\alpha\nu}} K_{t+1}^{\frac{\nu-1}{1-\alpha\nu}} + (1-\delta) - \frac{\Phi}{2}(\frac{I_{t+1}}{K_{t+1}} - \delta)^2 + \Phi(\frac{I_{t+1}}{K_{t+1}} - \delta)\frac{K_{t+2}}{K_{t+1}}, \end{aligned}$$

log-linearizing it, we then further simplify it to

$$\begin{aligned}
& \Phi\delta(\hat{I}_t - \hat{K}_t)(1+r_{ss}) + dr_{t+1} \\
= \quad & (1-\alpha)\nu F_{ss}/K_{ss}[\frac{-\alpha\nu}{1-\alpha\nu}\hat{w}_{t+1} + \frac{\nu-1}{1-\alpha\nu}\hat{K}_{t+1}] \\
& +0+0+\Phi(\hat{I}_{t+1} - \hat{K}_{t+1})\delta.
\end{aligned}$$

Note that capital investment I_t satisfies

$$\begin{aligned}
dI_t &= dK_{t+1} - (1-\delta)dK_t, \\
\hat{I}_t &= [dK_{t+1} - (1-\delta)dK_t]/I_{ss} = \frac{1}{\delta}(\hat{K}_{t+1} - \hat{K}_t) + \hat{K}_t.
\end{aligned}$$

We lastly arrive at:

$$0 = -dr_{t+1} - \Phi(1+r_{ss})(\hat{K}_{t+1} - \hat{K}_t) \tag{C.43}$$

$$+(r_{ss}+\delta)[\frac{-\alpha\nu}{1-\alpha\nu}\hat{w}_{t+1} + \frac{\nu-1}{1-\alpha\nu}\hat{K}_{t+1}] + \Phi(\hat{K}_{t+2} - \hat{K}_{t+1}). \tag{C.44}$$

C.4 Calvo Wage in HANK

There is a continuum types of labor varieties denoted as $j \in [0,1]$. At each period $t \in \mathbb{N}$, the representative labor packer collect N_{jt} units of labor variety for each type $j \in [0,1]$ to produce N_t units of composite labor inputs through a Dixit-Stiglitz aggregator

$$N_t = \left(\int_0^1 N_{jt}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

where $\varepsilon_w \in (1, +\infty)$ denotes the elasticity of substitution between labor varieties. Denote W_{jt} as the nominal wage rate of N_{jt} and w_j as the nominal wage rate of N_t. The standard competitive labor packer's problem yields

$$N_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\varepsilon_w} N_t, \quad W_t = \left(\int_0^1 W_{jt}^{1-\varepsilon_w} dj\right)^{\frac{1}{1-\varepsilon_w}}, \quad W_t N_t = \int_0^1 W_{jt} N_{jt} dj.$$

For each type of labor variety j, there is a labor union. Each union j has all workers as members, and chooses a nominal wage rate W_{jt} on behalf of them. The induced demand for labor variety N_{jt} is imposed uniformly on all union members. In another word, each worker is forced to supply N_{jt} for all $j \in [0,1]$. Denote $\beta_w \in [0,1]$ as the discount factor of workers, $\theta_w \in [0,1]$ as the probability that nominal wage cannot be adjusted in a quarter, P_t as the aggregate price level, e_{it} as the idiosyncratic labor productivity level normalized by $\int_0^1 e_{it} di = 1$, $u(c_{it+s}^w)$ as the utility function for individual consumption, and $\int_0^1 v(N_{jt+s})dj$ as the disutility function for individual labor supply. The union for labor variety j trades off the marginal value of wage income and the marginal disutility of labor supply. It solves

$$\max_{W_{jt}} \mathbb{E}_t \sum_{s=0}^{+\infty} (\beta_w \theta_w)^s \int_0^1 \left[u'(c_{it+s}^w)\left(\frac{W_{jt}}{P_{t+s}} e_{it+s} N_{jt+s}\right) - v(N_{jt+s})\right] di, \quad \text{s.t. } N_{jt+s} = \left(\frac{W_{jt+s}}{W_{t+s}}\right)^{-\varepsilon_w} N_{t+s}.$$

Using notation $u'(C^*_{t+s}) \equiv \int_0^1 e_{it+s} u'(c^w_{it+s}) di$, the first order condition is

$$0 = \mathbb{E}_t \sum_{s=0}^{+\infty} (\beta_w \theta_w)^s \left[(1-\varepsilon_w) \frac{N_{jt+s}}{P_{t+s}} u'(C^*_{t+s}) + \varepsilon_w v'(N_{jt+s}) \frac{N_{jt+s}}{W_{jt}} \right].$$

To obtain closed form solution, impose functional form: $v(n) = \chi \frac{n^{\frac{1}{\xi}+1}}{\frac{1}{\xi}+1}$, $v'(n) = \chi n^{\frac{1}{\xi}}$, in which $\chi > 0$ is a normalization parameter and $\xi > 0$ is Frisch elasticity of labor supply. Denote $\Pi_{t,t+s}$ as the gross inflation rate from t to $t+s$, $w_t \equiv W_t/P_t$, and $w^\#_t$ as the solution for W_{jt}/P_t, then

$$(w^\#_t)^{1+\varepsilon_w \xi^{-1}} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\mathbb{E}_t \sum_{s=0}^{+\infty} (\beta_w \theta_w)^s \chi w_{t+s}^{\varepsilon_w(1+\xi^{-1})} \Pi_{t,t+s}^{\varepsilon_w(1+\xi^{-1})} N_{t+s}^{1+\xi^{-1}}}{\mathbb{E}_t \sum_{s=0}^{+\infty} (\beta_w \theta_w)^s u'^{*}_{t+s}) w_{t+s}^{\varepsilon_w} \Pi_{t,t+s}^{\varepsilon_w - 1} N_{t+s}}.$$

Note that in the special case or flexible wage in which $\theta_w = 0$, we have

$$w^\#_t = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\chi N_t^{\xi^{-1}}}{u'(C^*_t)} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{v'(N_t)}{u'(C^*_t)}.$$

This real wage is not identical to the equilibrium wage in standard flexible wage model. The main difference is that workers are not on their supply curve, and there are idiosyncratic labor wedges due to the assumption on unions. The recursive representation for global dynamics using auxiliary variables $\{H_{1t+1}, H_{2t+1}\}$ is

$$\begin{aligned}
(w^\#_t)^{1+\varepsilon_w \xi^{-1}} &= \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{H_{1t}}{H_{2t}}, \\
H_{1t} &= \chi w_t^{\varepsilon_w(1+\xi^{-1})} N_t^{1+\xi^{-1}} + (\beta_w \theta_w) \mathbb{E}_t \left[\Pi_{t+1}^{\varepsilon_w(1+\xi^{-1})} H_{1t+1} \right], \\
H_{2t} &= u'(C^*_t) w_{t+s}^{\varepsilon_w} N_{t+s} + (\beta_w \theta_w) \mathbb{E}_t \left[\Pi_{t+1}^{\varepsilon_w - 1} H_{2t+1} \right], \\
w_t^{1-\varepsilon_w} &= \theta_w \left(\frac{w_{t-1}}{\Pi_t} \right)^{1-\varepsilon_w} + (1-\theta_w)(w^\#_t)^{1-\varepsilon_w}.
\end{aligned}$$

The four equations above are similar to those in RANK. To obtain wage Phillips Curve useful for HANK, we log-linearize them assuming zero steady state inflation and $IES = \sigma$.

$$\begin{aligned}
(1+\varepsilon_w \xi^{-1}) w^\#_t &= (1-\beta_w \theta_w) \mathbb{E}_t \sum_{s=0}^{+\infty} (\beta_w \theta_w)^s \left[\xi^{-1} \hat{N}_{t+s} + \sigma^{-1} \hat{C}^*_{t+s} + \varepsilon_w \xi^{-1} \hat{w}_{t+s} + (1+\varepsilon_w \xi^{-1}) \hat{\Pi}_{t,t+s} \right], \\
\hat{w}_t &= \theta_w (\hat{w}_{t-1} - \hat{\Pi}_t) + (1-\theta_w) \hat{w}^\#_t.
\end{aligned}$$

Combining these two conditions yields

$$\hat{\Pi}^w_t = \frac{(1-\theta_w)(1-\beta_w \theta_w)}{\theta_w (1+\varepsilon_w \xi^{-1})} \mathbb{E}_t \sum_{s=0}^{+\infty} (\beta_w \theta_w)^s \left[\xi^{-1} \hat{N}_{t+s} + \sigma^{-1} \hat{C}^*_{t+s} - \hat{w}_{t+s} + (1+\varepsilon_w \xi^{-1}) \hat{\Pi}^w_{t,t+s} \right],$$

in which $\hat{\Pi}^w_t$ denotes the wage inflation rate from $t-1$ to t and $\hat{\Pi}^w_{t,t+s}$ demotes the wage inflation rate from t to $t+s$. Rewrite this equation one period forward

$$\hat{\Pi}^w_{t+1} = \frac{(1-\theta_w)(1-\beta_w \theta_w)}{\theta_w (1+\varepsilon_w \xi^{-1})} \mathbb{E}_{t+1} \sum_{s=0}^{+\infty} (\beta_w \theta_w)^s \left[\xi^{-1} \hat{N}_{t+1+s} + \sigma^{-1} \hat{C}^*_{t+1+s} - \hat{w}_{t+1+s} + (1+\varepsilon_w \xi^{-1}) \hat{\Pi}^w_{t+1,t+1+s} \right].$$

Due to the law of iterated expectations (LIE), we can take a difference $\hat{\Pi}^w_t - (\beta_w \theta_w) \hat{\Pi}^w_{t+1}$, and obtain the following wage Phillips Curve

$$\hat{\Pi}^w_t = \frac{(1-\theta_w)(1-\beta_w \theta_w)}{\theta_w (1+\varepsilon_w \xi^{-1})} \left(\xi^{-1} \hat{N}_t + \sigma^{-1} \hat{C}^*_t - \hat{w}_t \right) + \beta_w \mathbb{E}_t \hat{\Pi}^w_{t+1}.$$

C.5 Taylor rule and Wage rule

The log-linearized version for different Taylor rules are given by: (1) for using the rule in Kaplan et al. (2018), we have

$$\begin{aligned}\hat{\imath}^a_{t+1} &= \varphi_\pi \hat{\Pi}_t + \eta_t, \eta_t = \rho_i \eta_{t-1} - \sigma_i \epsilon_t \\ &\Rightarrow 0 = -\hat{r}_{t+1} - \hat{\Pi}_{t+1} + \varphi_\pi \hat{\Pi}_t + \rho_i(\hat{r}_t + \hat{\Pi}_t - \varphi_\pi \hat{\Pi}_{t-1}) - \sigma_i \epsilon_t.\end{aligned}$$

Note that $\hat{\imath}^a_{t+1}$ is the nominal interest rate from t to $t+1$, and the shock process is given by $\epsilon_{t=1} = 1\%, \epsilon_{t>1} = 0$. The realized real interest rate in the first period is: $0 = -\hat{r}_1 - \hat{\Pi}_1$.

(2) similarly, for using the rule in Christiano et al. (2016), we have:

$$\begin{aligned}\hat{\imath}^a_{t+1} &= \rho_i \hat{\imath}^a_t + (1-\rho_i)(\varphi_\pi \hat{\Pi}_t + \varphi_Y \hat{Y}_t) - \sigma_i \epsilon_t \\ &\Rightarrow 0 = -\hat{r}_{t+1} - \hat{\Pi}_{t+1} + \rho_i(\hat{r}_t + \hat{\Pi}_t) + (1-\rho_i)(\varphi_\pi \hat{\Pi}_t + \varphi_Y \hat{Y}_t) - \sigma_i \epsilon_t \\ 0 &= -\hat{r}_1 - \hat{\Pi}_1 \\ \epsilon_{t=1} &= 1\%, \epsilon_{t>1} = 0.\end{aligned}$$

(3) and lastly, to ensure a more stable path for the responses of nominal interest rates, similar to Christiano et al. (2016), we could have:

$$\begin{aligned}\hat{\imath}^a_{t+1} &= \rho_i \hat{\imath}^a_t + (1-\rho_i)\varphi_\pi \mathbb{E}_t \hat{\Pi}_{t+1} - \sigma_i \epsilon_t. \\ &\Rightarrow 0 = -\hat{r}_{t+1} - \hat{\Pi}_{t+1} + \rho_i(\hat{r}_t + \hat{\Pi}_t) + (1-\rho_i)\varphi_\pi \hat{\Pi}_{t+1} - \sigma_i \epsilon_t. \\ 0 &= -\hat{r}_1 - \hat{\Pi}_1, \\ \epsilon_{t=1} &= 1\%, \epsilon_{t>1} = 0.\end{aligned}$$

For the wage rules, denote the coefficient before households' consumption $\frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w \xi^{-1})}$ as c_w, and note that nominal wage growth rate $\hat{\Pi}^w_t$ is equal to $\hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1}$, we therefore have the following linear equation:

$$\begin{aligned}\hat{\Pi}^w_t &= \hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1} = c_w(\xi^{-1}\hat{N}_t + \sigma^{-1}\hat{C}^*_t - \hat{w}_t) + \beta\left(\hat{\Pi}_{t+1} + \hat{w}_{t+1} - \hat{w}_t\right) \\ &\Rightarrow 0 = c_w(\xi^{-1}\hat{N}_t + \sigma^{-1}\hat{C}^*_t - \hat{w}_t) + \beta\left(\hat{\Pi}_{t+1} + \hat{w}_{t+1} - \hat{w}_t\right) - \hat{\Pi}_t - \hat{w}_t + \hat{w}_{t-1} \\ &\Rightarrow 0 = c_w(\xi^{-1}\hat{N}_t + \sigma^{-1}\hat{C}^*_t) - \hat{\Pi}_t + \beta\hat{\Pi}_{t+1} + \hat{w}_{t-1} + \hat{w}_t(-c_w - \beta - 1) + \beta\hat{w}_{t+1}.\end{aligned}$$

C.6 Computational Appendix

We first discretize the space for a, k, z, e. We assume there are 100 points for a and 100 points for k, 5 points for z and 5 points for e. Space for a and k are first nonlinearly discretized, focusing on small values; a is between 0 and 50 and k is between 0 and 100. In addition, the two biggest points for a are modified to very large numbers (2400 and 2500), and similarly, the two biggest points for k are modified to very large numbers (800 and 1000). By doing this, we want to ensure that rich and/or highly productive entrepreneurs never choose the boundary points in each grid space. z and e are following AR(1) process and are discretized by Tauchen's method.

For workers' problem, we simply use endogenous grid method to solve for $V(a,e)$ in the steady state; out of steady state, this method is still used to solve for $V_t(a,e)$ when $V_{t+1}(a,e)$ and $\{w_t, R_t, N_t^d\}$ are given.

For entrepreneurs' problem, it is more involved, since we have two endogenous state variables, a and k. We still use a version of endogenous grid method to solve for $V_t(a,k,z)$. Specifically, first guess policy functions of $c(a,k,z)$, $a'(a,k,z)$, $k'(a,k,z)$ and $V(a,k,z)$. We then use the following steps to have value and policy functions convergence.

- For each (k,z) today, pick up any admissible points of (a',k') in the state space, for the unconstrained case, we can find the solution for implied a and the first order conditions should be satisfied:

$$0 = u'(c) - \beta E u'(c\prime)(1+r')$$
$$\beta E u'(c\prime)[\frac{\partial \pi'}{\partial k'} - (r'+\delta) - g_2(i',k')] = u'(c) g_1(i,k).$$

  Using the implied sequence of a, the implied c, and the associated a', k', for any asset on the original grid space, we can now use interpolation method. Call these updated polices as unconstrained candidates, and note that they are not necessarily the optimal policy.

- For the constrained case, similarly, from first order conditions, we have:

$$\text{if: } \mu > 0 : a' = (1-\Psi)k' \tag{C.45}$$
$$u'(c) = (1+\widetilde{\mu})[\beta E u'(c\prime)(1+r')] \tag{C.46}$$
$$\frac{\beta E u'(c\prime)[\frac{\partial \pi'}{\partial k'} - (r'+\delta) - g_2(i',k')]}{u'(c)} = g_1(i,k) + \frac{\widetilde{\mu}}{(1+\widetilde{\mu})}. \tag{C.47}$$

  Now the trick is to introduce possible μ (say, between 0 and 50 times of $\beta E u'(c\prime)(1+r')$), along with any admissible points of (a',k') in the state space with the constraint $a' = (1-\Psi)k'$. We can find the solution for implied a and the first order condition is satisfied. As before, using the implied sequence of a and the associated c, a', k', for any asset on the original grid space, we can use interpolation method. Call these updated polices as constrained candidates, and again note they are not necessarily the optimal policy either.

- Lastly, we compare these unconstrained and constrained candidates (along with corner solutions when a' or k' hit the corners of their discrete grid space). We compare the implied values for the value function and pick up the best choice. For all (k,z) today, we can update the policy functions and value functions until convergence.

C.7 Numerical results for the Quantitative model

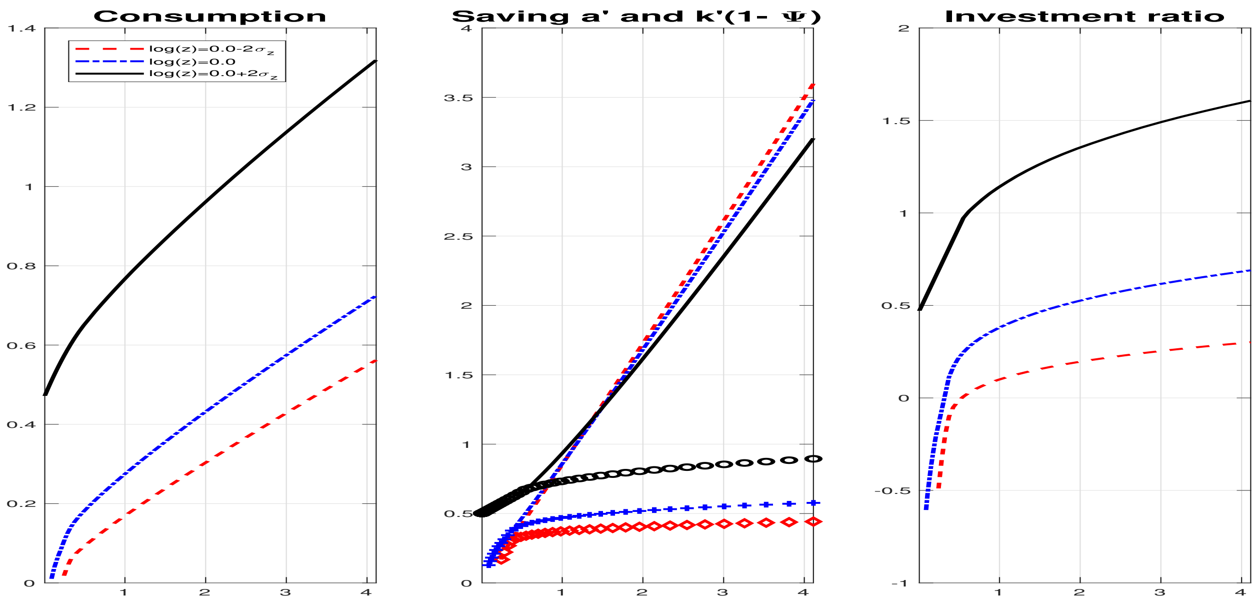


Figure 25: Entrepreneurs' optimal policies with small k

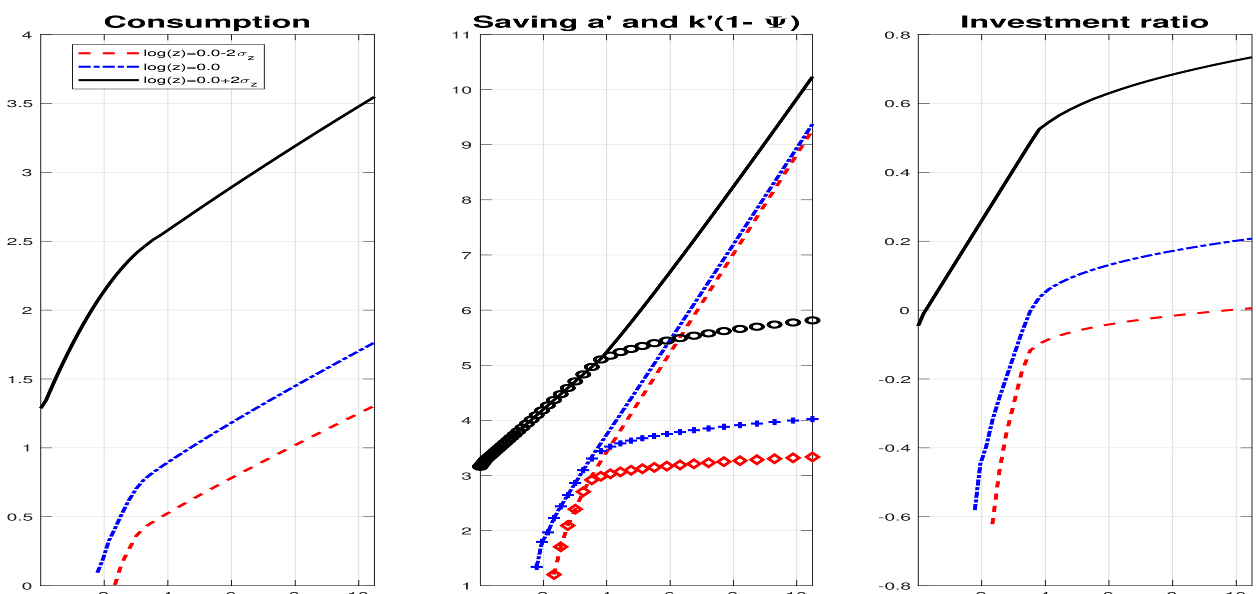


Figure 26: Entrepreneurs' optimal policies with large k

C.7.1 Alternative Monetary policy rules

Figure 27: Monetary policy rule as in Kaplan et al. (2018)

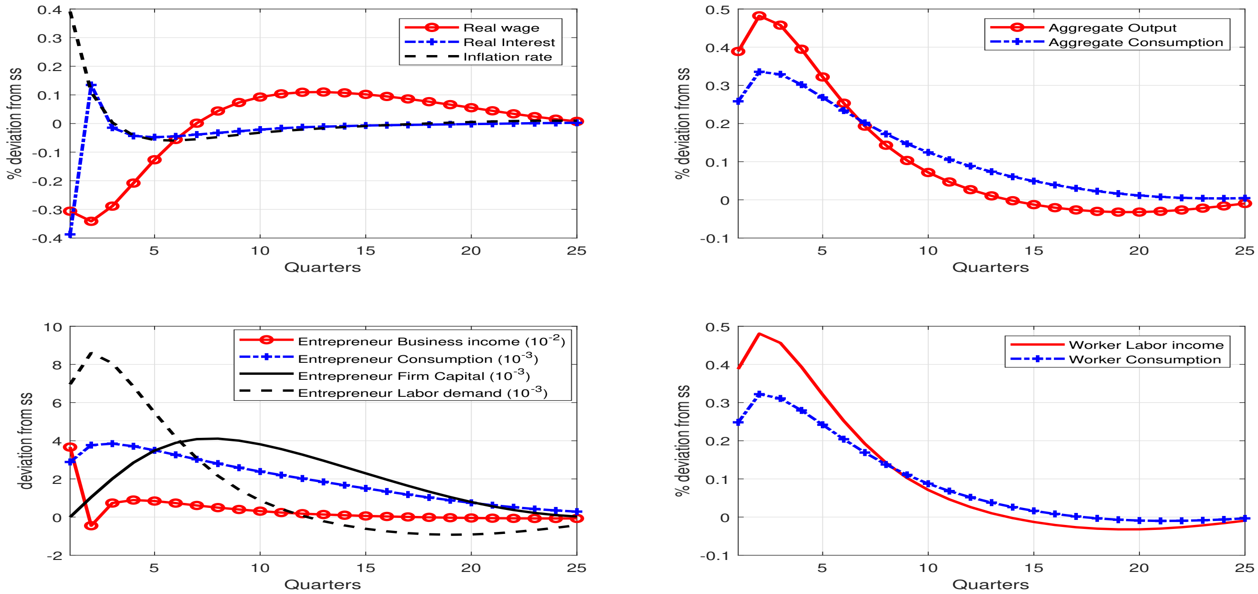
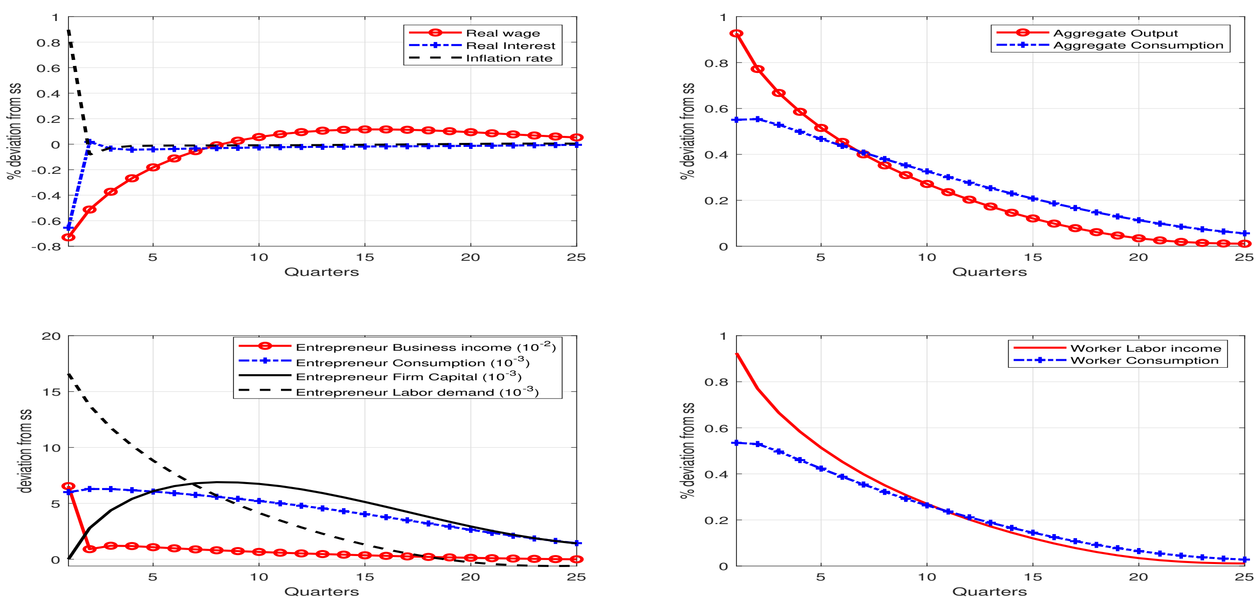


Figure 28: With inflation expectation in the monetary policy rule



C.7.2 Robustness

Figure 29: Entrepreneurs' responses with different wage stickiness

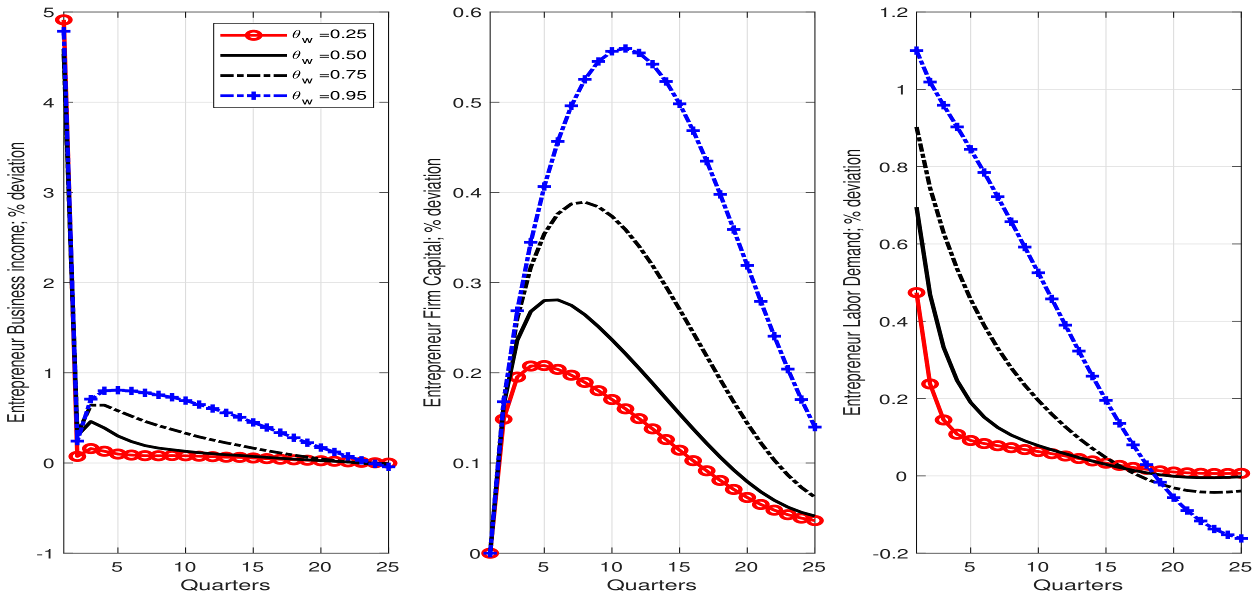


Figure 30: Entrepreneurs' responses with different Frisch Labor elasticity

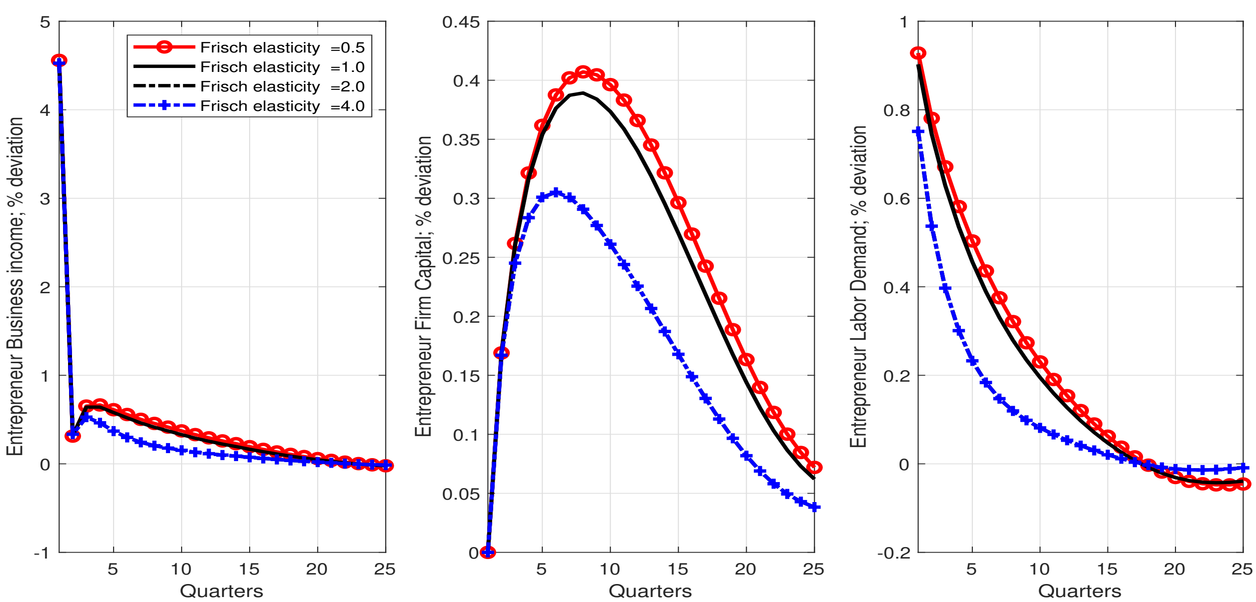


Figure 31: Entrepreneurs' responses with different shock persistence